



Intermediate PreAlgebra: Building Success

Intermediate PreAlgebra: Building Success

KIM TAMBLYN

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About the Book

This book was created to improve access to materials for Intermediate Pre-Algebra Math 0400 students at Thompson Rivers University (TRU). While I loved the previous textbook that I was using, the hardcopy was expensive, and students were finding the e-copy difficult to navigate.

Several of my colleagues in the University and Employment Preparation Department (UEPrep) had created open-source textbooks for BC Campus and encouraged the rest of us to adopt and/or create open-source materials for our classes. When I couldn't find an open access textbook that exactly fit the course outcomes and my approaches, I pursued the idea of collecting material from a variety of sources to create a suitable textbook. With the support of the Open Press Team at TRU; my chair, Christine Miller; and my dean, Yasmin Dean, I was given the time and resources to create this textbook.

I want to express my thanks to my TRU colleagues Izabela Mazur and Kim Moshenko, along with the creators of the Fundamental level 2 – 6 textbooks, Wendy Tagami, Leanne Caillier-Smith, Liz Girard, Katherine Arendt, and Mercedes de la Nuez. Drawing on their work, I was able to pull together existing open-source materials to create a new textbook. Over time, my goal is to continue to update and add materials to the textbook, including videos.

This textbook covers most of the learning outcomes listed in the BC Articulation Guide for the first part of Intermediate Level – Algebraic. My hope is that this is a useful contribution to the ABE Open-Source library and that others will be able to use it.

Acknowledgements

The Open Press



The Open Press combines TRU's open platforms and expertise in learning design and open resource development. TRU Open Press supports the creation and reuse of open educational resources, while encouraging open scholarship and research.

Land Acknowledgement

Thompson Rivers University (TRU) campuses are situated on the traditional lands of the Tk'emlúps te Secwépemc (Kamloops) and the T'exelc (Williams Lake) within Secwepemcúl'ecw, the traditional and unceded territory of the Secwépemc. The rich tapestry of this land also encompasses the territories of the St'át'imc, Nlaka'pamux, Tšilhqot'in, Nuxalk, and Dakelh. Recognizing the deep histories and ongoing presence of these Indigenous peoples, we express gratitude for the wisdom held by this land. TRU is dedicated to fostering an inclusive and respectful environment, valuing education as a shared journey. The TRU Open Press, inspired by collaborative learning on this land, upholds open access principles, and freely accessible education for all.

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Resources

Intermediate PreAlgebra: Building Success by Kim Tamblyn has been created from a combination of original content and materials compiled and adapted from a number of open text publications, including:

- Adult Literacy Fundamental Mathematics: Book

2 – 2nd Edition

- Tagami, W., & Girard, L. (2022). *Adult literacy fundamental mathematics: Book 2* (2nd ed.). BCcampus.
<https://opentextbc.ca/alfm2/>.
- Adult Literacy Fundamental Mathematics: Book 3 – 2nd Edition

- Tagami, W., & Girard, L. (2022). *Adult literacy fundamental mathematics: Book 3* (2nd ed.). BCcampus.
<https://opentextbc.ca/alfm3/>.
- Adult Literacy Fundamental Mathematics: Book 4 – 2nd Edition

- Arendt, K., de la Nuez, M., & Girard, L. (2023). *Adult literacy fundamental mathematics: Book 4* (2nd ed.). BCcampus.
<https://opentextbc.ca/alfm4/>.
- Adult Literacy Fundamental Mathematics: Book 5 – 2nd Edition

- Girard, L., Tagami, W., & Caillier-Smith, C. (2023). *Adult literacy fundamental mathematics: Book 5* (2nd ed.). BCcampus.
<https://opentextbc.ca/alfm5/>.
- Adult Literacy Fundamental Mathematics: Book 6 – 2nd Edition

- Girard, L., & Tagami, W. (2022). *Adult*

literacy fundamental mathematics: Book 6
(2nd ed.). BCcampus.
<https://opentextbc.ca/alfm6/>.

- Business/Technical Mathematics
 - Mazur, I., & Moshenko, K. (2021). *Business/technical mathematics*. BCcampus.
<https://opentextbc.ca/businessstechnicalmath/>.
- Introductory Algebra
 - Mazur, I. (2021). *Introductory algebra*. BCcampus. <https://opentextbc.ca/introalgebra/>.

Accessibility

The web version of Intermediate PreAlgebra: Building Success has been designed to meet Web Content Accessibility Guidelines 2.0, level AA. In addition, it follows all guidelines in Appendix A: Checklist for Accessibility of the Accessibility Toolkit – 2nd Edition.

Includes:

- **Easy navigation.** This resource has a linked table of contents and uses headings in each chapter to make navigation easy.
- **Accessible videos.** All videos in this resource have captions.
- **Accessible images.** All images in this resource that convey information have alternative text. Images that are decorative have empty alternative text.
- **Accessible links.** All links use descriptive link text.

Accessibility Checklist

Element	Requirements	Pass
Headings	Content is organized under headings and subheadings that are used sequentially.	Yes
Images	Images that convey information include alternative text descriptions. These descriptions are provided in the alt text field, in the surrounding text, or linked to as a long description.	Yes
Images	Images and text do not rely on colour to convey information.	Yes
Images	Images that are purely decorative or are already described in the surrounding text contain empty alternative text descriptions. (Descriptive text is unnecessary if the image doesn't convey contextual content information.)	Yes
Tables	Tables include row and/or column headers with the correct scope assigned.	Yes
Tables	Tables include a title or caption.	Yes
Tables	Tables do not have merged or split cells.	Yes
Tables	Tables have adequate cell padding.	Yes
Links	The link text describes the destination of the link.	Yes
Links	Links do not open new windows or tabs. If they do, a textual reference is included in the link text.	Yes
Links	Links to files include the file type in the link text.	Yes
Video	All videos include high-quality (i.e., not machine generated) captions of all speech content and relevant non-speech content.	Yes
Video	All videos with contextual visuals (graphs, charts, etc.) are described audibly in the video.	Yes
H5P	All H5P activities have been tested for accessibility by the H5P team and have passed their testing.	Yes
H5P	All H5P activities that include images, videos, and/or audio content meet the accessibility requirements for those media types.	Yes
Font	Font size is 12 point or higher for body text.	Yes
Font	Font size is 9 point for footnotes or endnotes.	Yes
Font	Font size can be zoomed to 200% in the webbook or eBook formats.	Yes

Known Accessibility Issues and Areas for Improvement

- None

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Other Formats Available

- In addition to the web version, this book is available in a number of file formats, including PDF, EPUB (for eReaders), and various editable files. The Digital PDF has passed the Adobe Accessibility Check.

OER Adoption Form

Please consider filling out a survey about this textbook to help us better understand how it's used and fits with the needs of our readers.

CHAPTER 1: WHOLE NUMBERS (OPERATIONS WITH RATIONAL NUMBERS)

1.1: Addition with Carrying

Chapter 1: Learning Outcomes

By the end of this chapter, you should be able to:

- Add, subtract, multiply and divide rational numbers.
- Use order of operations.

When the digits of one column add up to a two-digit number (10 or more), you must carry the digit to the next column.

Example A

$$\begin{array}{r} 27 \\ + 55 \\ \hline \end{array}$$

Step 1: Add the ones.

$$7 \text{ ones} + 5 \text{ ones} = 12 \text{ ones}$$

Rename 12 ones as 1 ten and 2 ones.

Write the 2 ones under the ones column and carry the ten to be added with the tens column.

$$\begin{array}{r} 1 \\ 27 \\ + 55 \\ \hline 2 \end{array}$$

Step 2: Add the tens.

$$1 \text{ ten} + 2 \text{ tens} + 5 \text{ tens} = 8 \text{ tens}$$

$$\begin{array}{r} 1 \\ 27 \\ + 55 \\ \hline 82 \end{array}$$

Example B

$$\begin{array}{r} 58 \\ + 76 \\ \hline \end{array}$$

Step 1: Add the one.

$$8 \text{ ones} + 6 \text{ ones} = 14 \text{ ones}$$

Rename the 14 ones as 1 ten and 4 ones.

Write the 4 ones under the ones column and carry the ten to be added with the tens column.

$$\begin{array}{r} 1 \\ 58 \\ + 76 \\ \hline 4 \end{array}$$

Step 2: Add the tens.

$$1 \text{ ten} + 5 \text{ tens} + 7 \text{ tens} = 13 \text{ tens}$$

The 1 hundred can just be written in the sum because there are no other hundreds to add it to.

$$\begin{array}{r} 1 \\ 58 \\ + 76 \\ \hline 134 \end{array}$$

Exercise 1

Find the sums. Check your work using the answer key at the end of the exercise.

$$\begin{array}{r} 62 \\ + 18 \\ \hline \end{array}$$

$$\begin{array}{r} 49 \\ + 42 \\ \hline \end{array}$$

$$\begin{array}{r} 17 \\ + 79 \\ \hline \end{array}$$

$$\begin{array}{r} 46 \\ + 37 \\ \hline \end{array}$$

$$\begin{array}{r} 44 \\ + 26 \\ \hline \end{array}$$

$$\begin{array}{r} 23 \\ + 82 \\ \hline \end{array}$$

$$\begin{array}{r} 28 \\ g. 91 \\ \hline \end{array} \quad \begin{array}{r} 89 \\ l. 74 \\ \hline \end{array} \quad \begin{array}{r} 54 \\ q. 17 \\ \hline \end{array}$$

$$\begin{array}{r} 54 \\ h. 58 \\ \hline \end{array} \quad \begin{array}{r} 37 \\ m. 15 \\ \hline \end{array} \quad \begin{array}{r} 72 \\ r. 33 \\ \hline \end{array}$$

$$\begin{array}{r} 68 \\ i. 49 \\ \hline \end{array} \quad \begin{array}{r} 55 \\ n. 27 \\ \hline \end{array} \quad \begin{array}{r} 26 \\ s. 56 \\ \hline \end{array}$$

$$\begin{array}{r} 66 \\ j. 35 \\ \hline \end{array} \quad \begin{array}{r} 29 \\ o. 76 \\ \hline \end{array} \quad \begin{array}{r} 38 \\ t. 80 \\ \hline \end{array}$$

$$\begin{array}{r} 99 \\ k. 88 \\ \hline \end{array} \quad \begin{array}{r} 35 \\ p. 69 \\ \hline \end{array}$$

Exercise 1 Answers

a. 80	h. 112	o. 105
b. 83	i. 117	p. 104
c. 91	j. 101	q. 71
d. 70	k. 187	r. 105
e. 96	l. 163	s. 82
f. 105	m. 52	t. 118
g. 119	n. 82	

Extra Practice: Who's the Pig? A Game of Chance.

How to play:

- This game is played by two people with one set of dice. Ask your instructor for one set of dice.
- Players take turns rolling the dice.
- You add the numbers on the dice to find your score.
- When it is your turn, you may roll as many times in a row as you like. Therefore, it is possible to score 100 or more points in one turn.
- However, if you roll a 1 on either die during your turn, you lose all your points for that turn, and your turn is over.
- If you roll a 1 on both dice, you lose all the points you have. You have to start all over again at 0, and your turn is over.

How to win:

- The first player to reach 100 or more points is the winner.

Example C

$$\begin{array}{r} 45 \\ 37 \\ + 69 \\ \hline \end{array}$$

Step 1: Add the ones.

$$5 \text{ ones} + 7 \text{ ones} + 9 \text{ ones} = 21 \text{ ones}$$

Rename 21 ones as 2 tens and 1 one.

Write the one in the sum under the ones column and carry the 2 tens to the tens column.

$$\begin{array}{r} 2 \\ 45 \\ 37 \\ + 69 \\ \hline 1 \\ \end{array}$$

Step 2: Add the tens.

$$2 \text{ tens} + 4 \text{ tens} + 3 \text{ tens} + 6 \text{ tens} = 15 \text{ tens}$$

15 tens is 1 hundred and 5 tens.

The one hundred can just be written in the sum because there are no other hundreds to add it to.

$$\begin{array}{r}
 2 \\
 45 \\
 37 \\
 + 69 \\
 \hline
 151
 \end{array}$$

Exercise 2

Find the sums.

a.

$$\begin{array}{r}
 67 \\
 78 \\
 + 55 \\
 \hline
 \end{array}$$

c.

$$\begin{array}{r}
 31 \\
 12 \\
 + 49 \\
 \hline
 \end{array}$$

e.

$$\begin{array}{r}
 41 \\
 52 \\
 + 65 \\
 \hline
 \end{array}$$

b.

$$\begin{array}{r}
 42 \\
 13 \\
 + 25 \\
 \hline
 \end{array}$$

d.

$$\begin{array}{r}
 23 \\
 27 \\
 + 84 \\
 \hline
 \end{array}$$

f.

$$\begin{array}{r}
 63 \\
 74 \\
 + 21 \\
 \hline
 \end{array}$$

	47		14		34
	18		24		32
g.	+ 55	l.	+ 51	q.	+ 85

	12		56		27
	24		23		51
h.	+ 89	m.	+ 67	r.	+ 96

	73		78		25
	21		45		46
i.	+ 37	n.	+ 89	s.	+ 43

	25		22		36
	60		52		47
j.	+ 47	o.	+ 64	t.	+ 52

	53		35		53
	60		11		67
k.	+ 71	p.	+ 75	u.	+ 81

	41		31		76
	59		83		62
v.	+	w.	+	x.	+
	<u>99</u>		<u>27</u>		<u>25</u>

Exercise 2 Answers

a. 200	i. 131	q. 151
b. 80	j. 132	r. 174
c. 92	k. 184	s. 114
d. 134	l. 89	t. 135
e. 158	m. 146	u. 201
f. 158	n. 212	v. 199
g. 120	o. 138	w. 141
h. 125	p. 121	x. 163

Use the same method for carrying when you add the columns of tens, hundreds, thousands, ten thousands, and so on. Look at these examples:

Example D

$$\begin{array}{r} 374 \\ + 438 \\ \hline \end{array}$$

Step 1: Add the ones.

4 ones + 8 ones = 12 ones = 1 ten and 2 ones

Write the 2 ones in the sum.

Carry the 1 ten to the tens column.

$$\begin{array}{r} 1 \\ 374 \\ + 438 \\ \hline 2 \end{array}$$

Step 2: Add the tens.

7 + 3 + 1 ten you carried = 11 tens = 1 hundred and 1 ten

Write the 1 ten.

Carry the 1 hundred to the hundreds column.

$$\begin{array}{r} 11 \\ 374 \\ + 438 \\ \hline 12 \end{array}$$

Step 3: Add the hundreds.

$3 + 4 + 1$ hundred you carried = 8 hundreds

Write 8.

$$\begin{array}{r} 11 \\ \mathbf{3}74 \\ + \mathbf{4}38 \\ \hline \mathbf{8}12 \end{array}$$

Example E

$$\begin{array}{r} 4974 \\ 2385 \\ + 6890 \\ \hline \end{array}$$

Step 1: Add the ones.

$$4 + 5 + 0 = 9 \text{ ones}$$

Write 9 ones in the sum.

$$\begin{array}{r}
 4\,974 \\
 2\,385 \\
 +\, 6\,890 \\
 \hline
 9
 \end{array}$$

Step 2: Add the tens.

$24 \text{ tens} = 2 \text{ hundreds} + 4 \text{ tens}$ (write 4 tens)

Carry the 2 hundreds to the hundreds column.

$$\begin{array}{r}
 ^2 \\
 4\,974 \\
 2\,385 \\
 +\, 6\,890 \\
 \hline
 49
 \end{array}$$

Step 3: Add the hundreds and the 2 hundreds you carried.

$22 \text{ hundreds} = 2 \text{ thousands} + 2 \text{ hundreds}$ (write 2 hundreds)

$$\begin{array}{r}
 ^2^2 \\
 4\,974 \\
 2\,385 \\
 +\, 6\,890 \\
 \hline
 249
 \end{array}$$

Step 4: Add the thousands and the 2 thousands you carried.

$14 \text{ thousands} = 1 \text{ ten thousand} + 4 \text{ thousand}$

Write 14 thousands in the sum.

$$\begin{array}{r}
 22 \\
 4974 \\
 2385 \\
 + 6890 \\
 \hline
 14249
 \end{array}$$

Example F

$$\begin{array}{r}
 246\,476 \\
 + 873\,706 \\
 \hline
 \end{array}$$

Step 1: Add the ones.

$$12 \text{ ones} = 1 \text{ ten} + 2 \text{ ones}$$

Write 2 ones in the sum and carry the 1 ten over.

$$\begin{array}{r}
 1 \\
 246\,476 \\
 + 873\,706 \\
 \hline
 2
 \end{array}$$

Step 2: Add the tens.

$$1 + 7 + 0 = 8 \text{ tens}$$

Write 8 tens in the sum, nothing to carry.

$$\begin{array}{r} ^1 \\ 246 \text{ } 476 \\ + \quad 873 \text{ } 706 \\ \hline 82 \end{array}$$

Step 3: Add the hundreds.

$$11 \text{ hundreds} = 1 \text{ thousand} + 1 \text{ hundred}$$

Write 1 hundred in the sum, carry the 1 thousand.

$$\begin{array}{r} ^1 ^1 \\ 246 \text{ } 476 \\ + \quad 873 \text{ } 706 \\ \hline 182 \end{array}$$

Step 4: Add the thousands.

$$10 \text{ thousands} = 1 \text{ ten thousand} + 0 \text{ thousands}$$

Be sure to write the 0 to hold the thousands place in the sum.

$$\begin{array}{r} ^1 ^1 ^1 \\ 246 \text{ } 476 \\ + \quad 873 \text{ } 706 \\ \hline 0182 \end{array}$$

Carry the 1 ten thousand.

Step 5: Add the ten thousands.

$$\begin{array}{r}
 \begin{array}{ccccccc}
 & 1 & 1 & 1 & & 1 & \\
 & 2 & 4 & 6 & 4 & 7 & 6 \\
 + & 8 & 7 & 3 & 7 & 0 & 6 \\
 \hline
 & 2 & 2 & 0 & 8 & 2 &
 \end{array}
 \end{array}$$

12 ten thousands = 1 hundred thousand + 2 ten thousands

Write the 2 ten thousands in the sum and carry the 1 hundred thousand.

Step 6: Add the hundred thousands.

11 hundred thousands = 1 million + 1 hundred thousand

Write 1 million and the 1 hundred thousand in the sum.

To read the answer say, “one million, one hundred twenty thousand, one hundred eight-two.”

$$\begin{array}{r}
 \begin{array}{ccccccc}
 & 1 & 1 & 1 & & 1 & \\
 & 2 & 4 & 6 & 4 & 7 & 6 \\
 + & 8 & 7 & 3 & 7 & 0 & 6 \\
 \hline
 & 1 & 1 & 2 & 0 & 1 & 8 & 2
 \end{array}
 \end{array}$$

Exercise 3

Find the sums.

$$\begin{array}{r} \text{a.} \quad \begin{array}{r} 231 \\ + 452 \\ \hline \end{array} \quad \text{g.} \quad \begin{array}{r} 5128 \\ + 4907 \\ \hline \end{array} \quad \text{m.} \quad \begin{array}{r} 5837 \\ + 2569 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \text{b.} \quad \begin{array}{r} 520 \\ + 239 \\ \hline \end{array} \quad \text{h.} \quad \begin{array}{r} 6005 \\ + 239 \\ \hline \end{array} \quad \text{n.} \quad \begin{array}{r} 2846 \\ + 1457 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \text{c.} \quad \begin{array}{r} 481 \\ + 306 \\ \hline \end{array} \quad \text{i.} \quad \begin{array}{r} 8106 \\ + 3923 \\ \hline \end{array} \quad \text{o.} \quad \begin{array}{r} 3517 \\ + 4296 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \text{d.} \quad \begin{array}{r} 306 \\ + 83 \\ \hline \end{array} \quad \text{j.} \quad \begin{array}{r} 5028 \\ + 4907 \\ \hline \end{array} \quad \text{p.} \quad \begin{array}{r} 9020 \\ + 684 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \text{e.} \quad \begin{array}{r} 5237 \\ + 2549 \\ \hline \end{array} \quad \text{k.} \quad \begin{array}{r} 6005 \\ + 273 \\ \hline \end{array} \quad \text{q.} \quad \begin{array}{r} 2648 \\ + 1238 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \text{f.} \quad \begin{array}{r} 2846 \\ + 1437 \\ \hline \end{array} \quad \text{l.} \quad \begin{array}{r} 2648 \\ + 1838 \\ \hline \end{array} \quad \text{r.} \quad \begin{array}{r} 5237 \\ + 6968 \\ \hline \end{array} \end{array}$$

Exercise 3 Answers

a. 683

b. 759

c. 787

d. 389	i. 12 029	n. 4 303
e. 7 786	j. 9 935	o. 7 813
f. 4 283	k. 6 278	p. 9 704
g. 10 035	l. 4 486	q. 3 886
h. 6 244	m. 8 406	r. 12 205

If you are having any problems with this work, ask your instructor to check your method of addition with carrying before you go any further.

If you feel that you need more practice, your instructor will give you more addition questions to do.

Adding Across

If an addition question is written with the numbers side by side, rewrite the question in columns. Put the ones under the ones, the tens under the tens, the hundreds under the hundreds, and so on.

Example G

$$\begin{array}{r}
 263 \\
 + \quad 25 \\
 \hline
 288
 \end{array}$$

Example H

$$\begin{array}{r} 316 \\ + 9\,560 \\ \hline 9\,876 \end{array}$$

Exercise 4

Rewrite each question in columns. Be careful to write ones under ones, tens under tens, hundreds under hundreds, and so on.

a. $476 + 392 + 483 =$	e. $697 + 7\,639 + 27 + 5\,396 =$
b. $986 + 483 + 524 =$	f. $1\,436 + 844 + 16\,009 =$
c. $3\,714 + 3\,189 + 4\,582 =$	g. $242\,100 + 62\,418 + 32 + 528\,002 =$
d. $466 + 5\,973 + 821 + 83 =$	h. $279\,661 + 475 + 49\,264 =$

Exercise 4 Answers

- a. 1 351
- b. 1 993
- c. 11 485
- d. 7 343

- e. 13 759
- f. 18 289
- g. 832 552
- h. 329 400

I.I: Practice Questions

1. Find the sums. Be sure to check your answers.

$$\begin{array}{r} \text{a.} \quad \begin{array}{r} 85 \\ + 57 \\ \hline \end{array} \quad \text{e.} \quad \begin{array}{r} 7\,834 \\ + 2\,169 \\ \hline \end{array} \quad \text{i.} \quad \begin{array}{r} 3\,742 \\ 4\,108 \\ + 7\,336 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \text{b.} \quad \begin{array}{r} 94 \\ + 48 \\ \hline \end{array} \quad \text{f.} \quad \begin{array}{r} 5\,976 \\ + 2\,081 \\ \hline \end{array} \quad \begin{array}{r} 3\,742 \\ 4\,108 \\ + 7\,336 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \text{c.} \quad \begin{array}{r} 982 \\ + 743 \\ \hline \end{array} \quad \text{g.} \quad \begin{array}{r} 46\,940 \\ + 86\,502 \\ \hline \end{array} \quad \text{j.} \quad \begin{array}{r} 352\,641 \\ 432\,345 \\ + 720\,250 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \text{d.} \quad \begin{array}{r} 829 \\ + 303 \\ \hline \end{array} \quad \text{h.} \quad \begin{array}{r} 41\,801 \\ + 39\,199 \\ \hline \end{array} \quad \text{k.} \quad \begin{array}{r} 432\,345 \\ + 720\,250 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r}
 18\,060 \\
 62\,549 \\
 \text{i.} \quad 1\,375 \\
 + \quad 399 \\
 \hline
 \end{array}$$

2. Add these numbers.

a. $74 + 32 + 67 + 85 =$

b. $721 + 8\,462 + 968 + 99 =$

c. $389 + 2\,517 + 2 =$

1.1: Practice Answers

1. Find the sums.

a. 142

b. 142

c. 1 725

d. 1 132

e. 10 003

f. 8 057

g. 133 442

h. 81 000

i. 15 186

j. 53 223

k. 1 505 236

l. 82 383

2. Add these numbers.

a. 258

b. 10 250

c. 2 908

Attribution

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1.2: Estimating Answers in Addition

You have learned how to round numbers. Now, you can use that skill to quickly find an approximate sum. Often, an estimate is all you need.

If you are going away for the weekend, you have to think about how much money you will need. The hotel is about \$60, meals are about \$80, gas is about \$40, and entertainment is about \$100.

You will take $\$60 + \$80 + \$40 + \$100 = \$280$

When solving word problems or working with a calculator, you should estimate your answer first to determine if your answer is sensible.

In these examples, **estimate** the answer. Round each number **BEFORE** you add.

Example A

53	rounds to	50
69	rounds to	70
22	rounds to	20
+ 88	rounds to	+ 90
		230

Example B

$$\begin{array}{rclcl} 349 & \text{rounds to} & 300 \\ + & 682 & \text{rounds to} & + & 700 \\ \hline & & & & 1\,000 \end{array}$$

Example C

$$\begin{array}{rclcl} 43\,928 & \text{rounds to} & 40\,000 \\ 29\,785 & \text{rounds to} & 30\,000 \\ 88\,319 & \text{rounds to} & 90\,000 \\ + & 243\,928 & \text{rounds to} & + & 240\,000 \\ \hline & & & & 400\,000 \end{array}$$

If you are estimating an answer, you usually estimate to the largest

Exercise 1

Estimate the sums.

	973	≈	1 000		
	496	≈	500		
Example:	+	382	≈	+	400
	1 900				

a.	519		3 519		4 113	
	439	c.	4 003	e.	1 590	
+	382		+	3 832	+	2 671

	1 234		2 727		38 985			
	4 567	d.	2 329	f.	43 691			
b.	+	7 890		+	9 818		+	8 336

Example:

	519		3 519		4 113
	439		4 003		1 590
a.	+	c.	+	e.	+
	<u>382</u>		<u>3 832</u>		<u>2 671</u>
	1 234		2 727		38 985
	4 567		2 329		43 691
b.	+	d.	+	f.	+
	<u>7 890</u>		<u>9 818</u>		<u>8 336</u>

	42 163		30 706		171 234
	30 820		29 115		102 085
g.	21 911	i.	40 082	k.	460 892
	<u>+ 60 422</u>		<u>+ 31 621</u>		<u>+ 542 329</u>
	21 472		431 391		
	46 371		554 423		
h.	98 393	j.	913 174		
	<u>+ 82 218</u>		<u>+ 282 826</u>		

Exercise 1 Answers

- $500 + 400 + 400 = 1\,300$
- $1\,000 + 5\,000 + 8\,000 = 14\,000$
- $4\,000 + 4\,000 + 4\,000 = 12\,000$
- $3\,000 + 2\,000 + 10\,000 = 15\,000$
- $4\,000 + 2\,000 + 3\,000 = 9\,000$
- $39\,000 + 44\,000 + 8\,000 = 91\,000$
- $40\,000 + 30\,000 + 20\,000 + 60\,000 = 150\,000$
- $20\,000 + 50\,000 + 100\,000 + 80\,000 = 250\,000$
- $30\,000 + 30\,000 + 40\,000 + 30\,000 = 130\,000$
- $400\,000 + 600\,000 + 900\,000 + 300\,000 = 2\,200\,000$
- $200\,000 + 100\,000 + 500\,000 = 1\,300\,000$

Estimating Answers in Addition Word Problems

When solving word problems, an estimate tells you if your answer is sensible. You can use your estimate to help you check your answers. If your answer and the estimate are not close, then you know that you should add your numbers again.

Exercise 2

Estimate the following answers. Be sure to round to the largest place value possible before adding. Remember to circle the information and underline what is being asked.

Example: During one month, Chaska spends 11 432 minutes sleeping and 5 812 minutes eating. Estimate how much time he spends sleeping and eating.

During one month, Chaska spends 11 432 minutes sleeping and 5 812 minutes eating. Estimate how much time he spends sleeping and eating.

$$11\,432 + 5\,812$$

Estimate:

$$11\,000 + 6\,000 = 17\,000$$

Chaska spent about 17 000 minutes sleeping and eating.

- a. During October, Amul drove 674 kilometres, 493 kilometres, 384 kilometres and 914 kilometres. Estimate the total kilometres Amul drove.
- b. The number of passengers using the ABE Taxi Company for the past three weeks was 3 205 passengers, 3 542 passengers and 2 821 passengers. Estimate the number of passengers that used the ABE Taxi Company.
- c. In 2008, the top three winning teams in the NHL were the Montreal Canadiens, winning 2 980 games, the Boston Bruins, winning 2 669 games, and the Toronto Maple Leafs, winning 2 535 games. Estimate the total number of games won by these three teams.
- d. The three deepest lakes in the world are Baikal Lake at 1 741 metres, Tanganyika Lake at 1 471 metres and the Caspian Sea at 1 025 metres. Estimate the total depth of the three lakes.

Exercise 2 Answers

- a. $700 + 500 + 400 + 900 = 2\,500$ kilometres
- b. $3\,000 + 4\,000 + 3\,000 = 10\,000$ passengers
- c. $3\,000 + 3\,000 + 3\,000 = 9\,000$ games
- d. $2\,000 + 1\,000 + 1\,000 = 4\,000$ metres

1.2: Practice Questions

1. Estimate the sums. Show your work.

	7 964		3 395		898 402
	971		2 709		465 766
a.	6 888	d.	18 060	g.	558 485
	+ 2 021		+ 932 335		+ 324 715

	5 365		2 364		6 182 390
	5 100		62 182		2 763 393
b.	9 982	e.	549 272 ^{h.}		1 326 879
	+ 7 752		+ 6 395		+ 2 743 912

			75 536		1 226 590
	5 211		31 807 ^{i.}		687 029
	1 982	f.	337 427		533 905
c.	3 371		+ 7 912		+ 1 359 254
	+ 2 801				

2. Estimate for each of the following word problems. Be sure to include the unit of measure in your answer.

Be sure to circle information and underline what is being asked.

- Yuan counted 854 old books and 519 new books. Estimate how many books there were altogether.
- A magazine has 34 783 subscribers. Last year, the

magazine had 26 876 subscribers. Estimate how many subscribers the magazine had in total.

- c. The area of Canada is 9 984 670 square kilometres. The area of the United States is 9 629 091 square kilometres. The area of Mexico is 1 964 375 square kilometres. Estimate the total area of the three countries.

1.2: Practice Answers

1. Estimate the sums.

a. 18 000	d. 955 000	g. 2 300 000
b. 28 000	e. 619 000	h. 13 000 000
c. 13 000	f. 553 000	i. 3 800 000

2. Estimate for each of the following word problems.

- a. 1 400 books
b. 60 000 subscribers
c. 22 000 000 square kilometres

Attribution

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1.3: Subtraction of Larger Numbers

You can find the difference between two large numbers using the subtraction facts you have been practicing. Always take away or subtract the number after the minus sign.

Use these steps to complete each subtraction question:

1. Subtract the ones from the ones.
2. Subtract the tens from the tens.
3. Subtract the hundreds from the hundreds.
4. Subtract the thousands from the thousands.
5. Subtract the ten thousands from the ten thousands, and so on.

Example A

$$\begin{array}{r} 57 \\ - 26 \\ \hline \end{array}$$

Step 1: Subtract the ones from the ones.

$$7 \text{ ones} - 6 \text{ ones} = 1 \text{ one}$$

Write the answer in line with the ones in the question.

$$\begin{array}{r} 57 \\ - 26 \\ \hline 1 \end{array}$$

Step 2: Subtract the tens from the tens.

$$5 \text{ tens} - 2 \text{ tens} = 3 \text{ tens}$$

The difference between 57 and 26 is 31.

$$\begin{array}{r} 57 \\ - 26 \\ \hline 31 \end{array}$$

Exercise 1

Find the differences. Check your work by adding and then by using the answer key at the end of the exercise.

$$\begin{array}{lll} \text{a.} & \begin{array}{r} 36 \\ - 13 \\ \hline \end{array} & \text{b.} \begin{array}{r} 72 \\ - 42 \\ \hline \end{array} & \text{c.} \begin{array}{r} 48 \\ - 22 \\ \hline \end{array} \end{array}$$

		55		98		74		
d.	—	31	k.	—	73	r.	—	53

		93		76		97		
e.	—	40	l.	—	64	s.	—	83

		76		86		89		
f.	—	71	m.	—	50	t.	—	80

		95		95		79		
g.	—	62	n.	—	35	u.	—	29

		39		28		89		
h.	—	26	o.	—	17	v.	—	80

		64		69		67		
i.	—	21	p.	—	52	w.	—	61

		85		84		48		
j.	—	64	q.	—	40	x.	—	40

Exercise 1 Answers

a. 23
b. 30
c. 26
d. 24
e. 53
f. 5
g. 33
h. 13

i. 43
j. 21
k. 25
l. 12
m. 36
n. 60
o. 11
p. 17

q. 44
r. 21
s. 14
t. 9
u. 50
v. 9
w. 6
x. 8

Checking Subtraction

You can check your subtraction. Add the answer (the difference) to the number you took away (the second number). If your subtracting was correct, the adding will result in the number you started with (the top number) in the subtraction question.

Example B

The difference is

$$\begin{array}{r} 928 \\ - 416 \\ \hline 512 \end{array}$$

Check: Add 512 to 416.

$$\begin{array}{r} 512 \\ + 416 \\ \hline 928 \end{array}$$

Exercise 2

Find the differences.

$$\begin{array}{lll} \text{a.} & \begin{array}{r} 87 \\ - 36 \\ \hline \end{array} & \text{c.} \begin{array}{r} 48 \\ - 40 \\ \hline \end{array} & \text{e.} \begin{array}{r} 75 \\ - 45 \\ \hline \end{array} \end{array}$$

$$\begin{array}{lll} \text{b.} & \begin{array}{r} 29 \\ - 21 \\ \hline \end{array} & \text{d.} \begin{array}{r} 99 \\ - 63 \\ \hline \end{array} & \text{f.} \begin{array}{r} 73 \\ - 20 \\ \hline \end{array} \end{array}$$

		92		59		70		
g.	—	21	m.	—	14	s.	—	50

		58		89		38		
h.	—	27	n.	—	63	t.	—	24

		84		88		31		
i.	—	23	o.	—	15	u.	—	10

		69		56		27		
j.	—	38	p.	—	44	v.	—	12

		45		96		74		
k.	—	23	q.	—	75	w.	—	53

		49		37		45		
l.	—	19	r.	—	17	x.	—	20

Exercise 2 Answers

a. 51

b. 8

c. 8

d. 36
e. 30
f. 53
g. 71
h. 31
i. 61
j. 31

k. 22
l. 30
m. 45
n. 24
o. 73
p. 12
q. 21

r. 20
s. 20
t. 14
u. 21
v. 15
w. 21
x. 25

Example C

$$\begin{array}{r} 696 \\ - 251 \\ \hline \end{array}$$

Step 1: Subtract the ones from the ones.

$$6 \text{ ones} - 1 \text{ one} = 5 \text{ ones}$$

$$\begin{array}{r} 696 \\ - 251 \\ \hline 5 \end{array}$$

Step 2: Subtract the tens from the tens.

$$9 \text{ tens} - 5 \text{ tens} = 4 \text{ tens}$$

$$\begin{array}{r} 696 \\ - 251 \\ \hline 45 \end{array}$$

Step 3: Subtract the hundreds from the hundreds.

6 hundreds $-$ 2 hundreds $=$ 4 hundreds

$$\begin{array}{r} 696 \\ - 251 \\ \hline 445 \end{array}$$

The difference between 696 and 251 is 445.

Exercise 3

Find the differences.

$$\begin{array}{lll} \begin{array}{r} 995 \\ - 452 \\ \hline \end{array} & \begin{array}{r} 877 \\ - 342 \\ \hline \end{array} & \begin{array}{r} 788 \\ - 615 \\ \hline \end{array} \\ \text{a.} & \text{b.} & \text{c.} \end{array}$$

	987		485		528
d.	— 243	k.	— 203	r.	— 208

	549		381		549
e.	— 131	l.	— 270	s.	— 120

	806		796		627
f.	— 204	m.	— 172	t.	— 523

	953		864		849
g.	— 603	n.	— 531	u.	— 246

	569		963		175
h.	— 403	o.	— 810	v.	— 163

	874		957		937
i.	— 650	p.	— 342	w.	— 224

	269		837		875
j.	— 159	q.	— 410	x.	— 252

Exercise 3 Answers

a. 543
b. 535
c. 173
d. 744
e. 418
f. 602
g. 350
h. 166

i. 224
j. 110
k. 282
l. 111
m. 624
n. 333
o. 153
p. 615

q. 427
r. 320
s. 429
t. 104
u. 603
v. 12
w. 713
x. 623

Exercise 4

Find the differences.

$$\begin{array}{r} 543 \\ - 132 \\ \hline \end{array}$$

$$\begin{array}{r} 328 \\ - 115 \\ \hline \end{array}$$

$$\begin{array}{r} 587 \\ - 425 \\ \hline \end{array}$$

$$\begin{array}{r} 752 \\ - 150 \\ \hline \end{array}$$

$$\begin{array}{r} 758 \\ - 341 \\ \hline \end{array}$$

$$\begin{array}{r} 857 \\ - 143 \\ \hline \end{array}$$

$\begin{array}{r} 545 \\ - 302 \\ \hline \end{array}$	$\begin{array}{r} 964 \\ - 231 \\ \hline \end{array}$	$\begin{array}{r} 757 \\ - 136 \\ \hline \end{array}$
$\begin{array}{r} 466 \\ - 115 \\ \hline \end{array}$	$\begin{array}{r} 679 \\ - 424 \\ \hline \end{array}$	$\begin{array}{r} 467 \\ - 132 \\ \hline \end{array}$

Exercise 4 Answers

a. 411	e. 162	i. 733
b. 602	f. 714	j. 255
c. 213	g. 243	k. 621
d. 417	h. 351	l. 335

Example D

$$\begin{array}{r} 4\,628 \\ - 2\,604 \\ \hline \end{array}$$

Step 1: Subtract the ones from the ones.

$$8 \text{ ones} - 4 \text{ ones} = 4 \text{ ones}$$

$$\begin{array}{r} 4628 \\ - 2604 \\ \hline 4 \end{array}$$

Step 2: Subtract the tens from the tens.

$$2 \text{ tens} - 0 \text{ tens} = 2 \text{ tens}$$

$$\begin{array}{r} 4628 \\ - 2604 \\ \hline 24 \end{array}$$

Step 3: Subtract the hundreds from the hundreds.

$$6 \text{ hundreds} - 6 \text{ hundreds} = 0 \text{ hundreds}$$

$$\begin{array}{r} 4628 \\ - 2604 \\ \hline 024 \end{array}$$

The 0 must be placed in the answer to hold the hundreds place.

Step 4: Subtract the thousands from the thousands.

$$4 \text{ thousands} - 2 \text{ thousands} = 2 \text{ thousands}$$

$$\begin{array}{r} 4628 \\ - 2604 \\ \hline 2024 \end{array}$$

The difference between 4 628 and 2 604 is 2 024.

Example E

$$\begin{array}{r} 79\,486 \\ - 42\,104 \\ \hline \end{array}$$

Step 1: Subtract the ones from the ones.

$$6 \text{ ones} - 4 \text{ ones} = 2 \text{ ones}$$

$$\begin{array}{r} 79\,486 \\ - 42\,104 \\ \hline 2 \end{array}$$

Step 2: Subtract the tens from the tens.

$$8 \text{ tens} - 0 \text{ tens} = 8 \text{ tens}$$

$$\begin{array}{r} 79\,486 \\ - 42\,104 \\ \hline 82 \end{array}$$

Step 3: Subtract the hundreds from the hundreds.

4 hundreds $-$ 1 hundreds $=$ 3 hundreds

$$\begin{array}{r} 79\,486 \\ - 42\,104 \\ \hline 37\,382 \end{array}$$

Step 4: Subtract the thousands from the thousands.

9 thousands $-$ 2 thousands $=$ 7 thousands

$$\begin{array}{r} 79\,486 \\ - 42\,104 \\ \hline 37\,382 \end{array}$$

Step 5: Subtract the ten thousands from the ten thousands.

7 ten thousands $-$ 4 ten thousands $=$ 3 ten thousands

$$\begin{array}{r} 79\,486 \\ - 42\,104 \\ \hline 37\,382 \end{array}$$

The difference between 79 486 and 42 104 is 37 382.

Exercise 5

Find the differences.

$$\begin{array}{r} 8\,646 \\ - \quad 542 \\ \hline \end{array}$$

a. $\quad$ f. $\begin{array}{r} 3\,219 \\ - \quad 2\,116 \\ \hline \end{array}$ k. $\begin{array}{r} 7\,296 \\ - \quad 5\,081 \\ \hline \end{array}$

$$\begin{array}{r} 7\,295 \\ - \quad 231 \\ \hline \end{array}$$

b. $\quad$ g. $\begin{array}{r} 6\,456 \\ - \quad 5\,234 \\ \hline \end{array}$ l. $\begin{array}{r} 9\,678 \\ - \quad 4\,316 \\ \hline \end{array}$

$$\begin{array}{r} 9\,738 \\ - \quad 215 \\ \hline \end{array}$$

c. $\quad$ h. $\begin{array}{r} 1\,758 \\ - \quad 1\,431 \\ \hline \end{array}$ m. $\begin{array}{r} 9\,489 \\ - \quad 2\,079 \\ \hline \end{array}$

$$\begin{array}{r} 6\,498 \\ - \quad 253 \\ \hline \end{array}$$

d. $\quad$ i. $\begin{array}{r} 8\,954 \\ - \quad 2\,151 \\ \hline \end{array}$ n. $\begin{array}{r} 7\,638 \\ - \quad 6\,218 \\ \hline \end{array}$

$$\begin{array}{r} 3\,674 \\ - \quad 2\,503 \\ \hline \end{array}$$

e. $\quad$ j. $\begin{array}{r} 8\,975 \\ - \quad 4\,732 \\ \hline \end{array}$ o. $\begin{array}{r} 4\,759 \\ - \quad 1\,136 \\ \hline \end{array}$

$$\begin{array}{r} 8\,275 \\ p. \quad - 4\,073 \\ \hline \end{array}$$

Exercise 5 Answers

- a. 8 104
- b. 7 064
- c. 9 523
- d. 6 245
- e. 1 171
- f. 1 103

- g. 1 222
- h. 327
- i. 6 803
- j. 4 243
- k. 2 215
- l. 5 362

- m. 7 410
- n. 1 420
- o. 3 623
- p. 4 202

If a subtraction question is written with the numbers side by side, rewrite the question in columns. Put the ones under the ones, the tens under the tens, the hundreds under the hundreds, and so on. The first number is always the top number, and the second number is always written below the first number.

Example F

$$\begin{array}{r} 687 \\ - 52 \\ \hline 635 \end{array}$$

Example G

$$\begin{array}{r} 9\,756 \\ - 420 \\ \hline 9\,336 \end{array}$$

Exercise 6

Rewrite each question in columns and find the differences.

$$\begin{array}{r} 43 \\ - 21 \\ \hline \end{array}$$

$$\begin{array}{r} 779 \\ - 54 \\ \hline \end{array}$$

$$\begin{array}{r} 965 \\ - 152 \\ \hline \end{array}$$

$$\begin{array}{r} 84 \\ - 30 \\ \hline \end{array}$$

$$\begin{array}{r} 695 \\ - 173 \\ \hline \end{array}$$

$$\begin{array}{r} 849 \\ - 212 \\ \hline \end{array}$$

$$\begin{array}{r} 975 \\ - 21 \\ \hline \end{array}$$

$$\begin{array}{r} 863 \\ - 701 \\ \hline \end{array}$$

Exercise 6 Answers

- a. 22
- b. 54
- c. 954

- d. 725
- e. 522
- f. 162

- g. 813
- h. 637

1.3: Practice Questions

1. Find the differences. Check your answers.

	39		72		64
a.	— 15	c.	— 60	e.	— 10

	58		49		85
b.	— 24	d.	— 23	f.	— 71

2. Find the differences. Check your answers.

	896		399		752
a.	— 385	c.	— 202	e.	— 231

	698		467		497
b.	— 461	d.	— 124	f.	— 341

3. Find the differences. Check your answers.

	8 627		9 875		9 751
a.	— 323	b.	— 9 251	c.	— 7 340

	34 859		37 698		96 723
d.	$\underline{\quad - \quad 1\,336 \quad}$	e.	$\underline{\quad - \quad 12\,540 \quad}$	f.	$\underline{\quad - \quad 51\,403 \quad}$

4. Subtract these numbers.

a.	$85 - 61 =$	c.	$879 - 152 =$	e.	$94\,658 - 12\,307 =$
b.	$724 - 13 =$	d.	$4\,957 - 821 =$	f.	$89\,653 - 27\,450 =$

1.3: Practice Answers

1. Find the differences.

a.	24	c.	12	e.	54
b.	34	d.	26	f.	14

2. Find the differences.

a.	511	c.	197	e.	521
b.	237	d.	343	f.	156

3. Find the differences.

a.	8304	c.	2 411	e.	25 158
b.	624	d.	33 523	f.	45 320

4. Subtract these numbers.

a.	24	c.	727	e.	82 351
b.	711	d.	4 136	f.	62 203

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1.4: Subtraction With Borrowing

When you subtract, the digit you are taking away may be larger than the top digit in that same column. You must borrow from the column on the left.

First, let's look at two examples using the place value shapes.

Example A

$$243 - 128$$

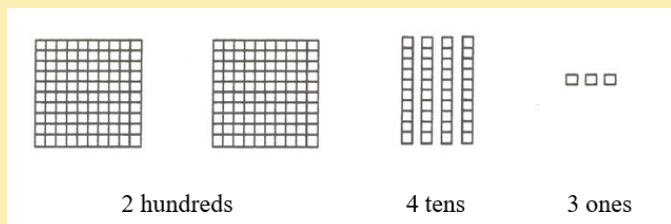


Figure 1.4.1

Step 1: 3 ones – 8 ones cannot be done

Borrow one ten and rename it as ten ones.

Add the ten ones to the three ones.

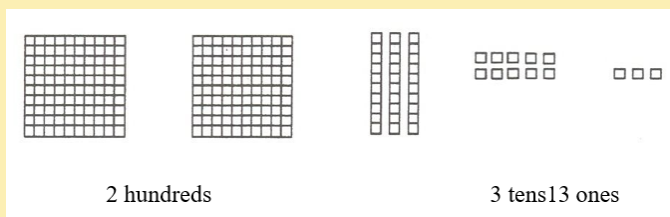


Figure 1.4.2

Now you can subtract:

$$13 \text{ ones} - 8 \text{ ones} = 5 \text{ ones}$$

Step 2: Subtract the tens.

$$3 \text{ tens} - 2 \text{ tens} = 1 \text{ ten}$$

Step 3: Subtract the hundreds.

$$2 \text{ hundreds} - 1 \text{ hundred} = 1 \text{ hundred}$$

Here is the question using numerals:

$$\begin{array}{r}
 3 13 \\
 2\cancel{4}\cancel{3} \\
 - 128 \\
 \hline
 115
 \end{array}$$

Example B

$$350 - 124$$

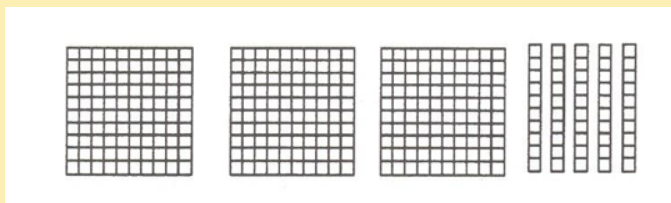


Figure 1.4.3

Step 1: 0 ones – 4 ones cannot be done

Borrow one ten and rename it as ten ones.

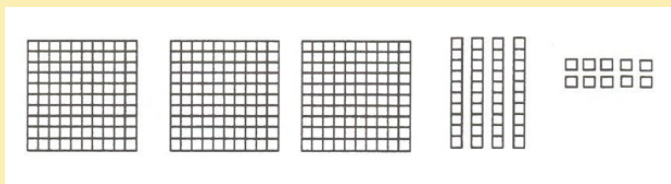


Figure 1.4.4

$$10 \text{ ones} - 4 \text{ ones} = 6 \text{ ones}$$

Step 2:

$$4 \text{ tens} - 2 \text{ tens} = 2 \text{ tens}$$

Step 3:

$$3 \text{ hundreds} - 1 \text{ hundred} = 2 \text{ hundreds}$$

This is how the question looks using numerals:

$$\begin{array}{r} ^4 ^{10} \\ 3\cancel{5}\cancel{0} \\ - 124 \\ \hline 226 \end{array}$$

Exercise 1

You may need to borrow 1 ten and rename it as 10 ones to do these subtractions.

Examples:

$$\begin{array}{r} 53 \\ - 16 \\ \hline 37 \end{array} \quad \begin{array}{r} 82 \\ - 45 \\ \hline 37 \end{array}$$

	37		40		747
a.	— 9	h.	— 38	o.	— 109

	28		45		642
b.	— 4	i.	— 20	p.	— 420

	63		70		438
c.	— 7	j.	— 21	q.	— 215

	54		645		953
d.	— 5	k.	— 26	r.	— 838

	25		258		532
e.	— 7	l.	— 14	s.	— 314

	84		786		795
f.	— 6	m.	— 47	t.	— 238

	45		895		956
g.	— 15	n.	— 29	u.	— 348

$$\begin{array}{r} 574 \\ \text{v. } - 218 \\ \hline \end{array}$$

Exercise 1 Answers

a. 28
b. 24
c. 56
d. 49
e. 18
f. 78
g. 30
h. 2

i. 25
j. 49
k. 619
l. 244
m. 739
n. 866
o. 638
p. 222

q. 223
r. 115
s. 218
t. 557
u. 608
v. 356

To check your subtraction, add the **answer** (the **difference**) to the number you took away. If your subtracting was correct, the result of the adding will equal the number you started with in the subtraction question.

Example C

Difference:

$$\begin{array}{r} 726 \\ - 317 \\ \hline 409 \end{array}$$

Check: 409 to 317

$$\begin{array}{r} 409 \\ + 317 \\ \hline 726 \end{array}$$

Exercise 2

You may need to borrow 1 ten and rename it as 10 ones to do these subtractions. Use the method for checking your answer beside each question.

$$\begin{array}{rcl} \text{Example:} & \begin{array}{r} 42 \\ - 5 \\ \hline 37 \end{array} & \text{Check: } \begin{array}{r} 37 \\ + 5 \\ \hline 42 \end{array} \end{array}$$

$$\begin{array}{r} 83 \\ \text{a. } \underline{ - 6} \end{array} \text{ Check:}$$

$$\begin{array}{r} 775 \\ \text{h. } \underline{ - 49} \end{array} \text{ check:}$$

$$\begin{array}{r} 91 \\ \text{b. } \underline{ - 7} \end{array} \text{ Check:}$$

$$\begin{array}{r} 974 \\ \text{i. } \underline{ - 26} \end{array} \text{ Check:}$$

$$\begin{array}{r} 70 \\ \text{c. } \underline{ - 4} \end{array} \text{ check:}$$

$$\begin{array}{r} 483 \\ \text{j. } \underline{ - 75} \end{array} \text{ Check:}$$

$$\begin{array}{r} 64 \\ \text{d. } \underline{ - 37} \end{array} \text{ Check:}$$

$$\begin{array}{r} 896 \\ \text{k. } \underline{ - 57} \end{array} \text{ Check:}$$

$$\begin{array}{r} 32 \\ \text{e. } \underline{ - 16} \end{array} \text{ Check:}$$

$$\begin{array}{r} 785 \\ \text{l. } \underline{ - 627} \end{array} \text{ Check:}$$

$$\begin{array}{r} 65 \\ \text{f. } \underline{ - 16} \end{array} \text{ Check:}$$

$$\begin{array}{r} 961 \\ \text{m. } \underline{ - 543} \end{array} \text{ Check:}$$

$$\begin{array}{r} 98 \\ \text{g. } \underline{ - 39} \end{array} \text{ Check:}$$

$$\begin{array}{r} 941 \\ \text{n. } \underline{ - 319} \end{array} \text{ Check:}$$

$$\begin{array}{r} 850 \\ \text{o. } \underline{} - 434 \end{array} \quad \text{Check:}$$

Exercise 2 Answers

- a. 77
- b. 84
- c. 66
- d. 27
- e. 16

- f. 49
- g. 59
- h. 726
- i. 948
- j. 408

- k. 839
- l. 158
- m. 418
- n. 622
- o. 416

Use this same method of borrowing when you subtract the hundreds, thousands, ten thousands, and so on. Look at the place value shapes as you work through these examples.

Example D

$$225 - 162$$

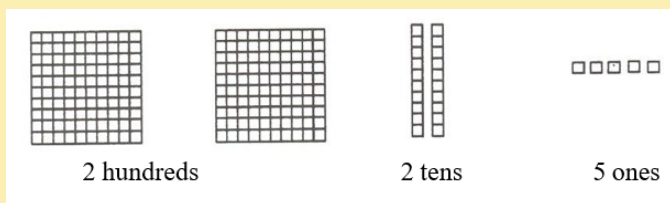


Figure 1.4.5

Step 1:

$$5 \text{ ones} - 2 \text{ ones} = 3 \text{ ones}$$

Step 2: 2 tens - 6 tens (can't be done)

Borrow one hundred and rename it as 10 tens.

Add the 10 tens to the 2 tens.

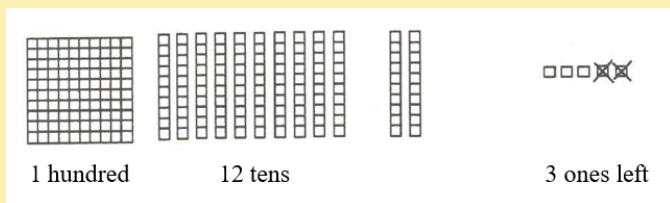


Figure 1.4.6

$$12 \text{ tens} - 6 \text{ tens} = 6 \text{ tens}$$

Step 3:

$$1 \text{ hundred} - 1 \text{ hundred} = 0 \text{ hundreds}$$

Note: The 0 in the hundreds is not needed in the answer

(063) because it is the first digit and does not have to hold the place.

$$\begin{array}{r} 112 \\ \cancel{2}\cancel{2}5 \\ - 162 \\ \hline 63 \end{array}$$

Example E

$$331 - 145$$

Step 1: 1 one – 5 ones (can't be done)

Borrow 1 ten and rename it as 10 ones, which you add onto the 1 one.

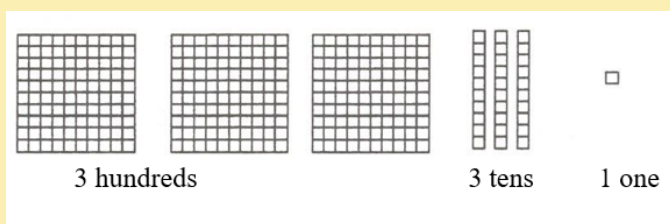


Figure 1.4.7

$$11 \text{ ones} - 5 \text{ ones} = 6 \text{ ones}$$

Step 2: 2 tens - 4 tens (can't be done)

Borrow one hundred and rename it as 10 tens, which you add onto the 2 tens.

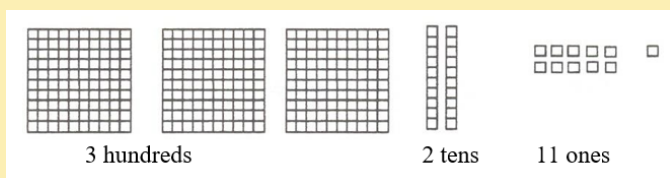


Figure 1.4.8

Step 3:

$$2 \text{ hundreds} - 1 \text{ hundred} = 1 \text{ hundred}$$

$$\begin{array}{r}
 \begin{array}{ccc}
 & 12 & \\
 2 & \cancel{2} & 11 \\
 \cancel{3} & \cancel{3} & \cancel{1}
 \end{array} \\
 - \quad 145 \\
 \hline
 186
 \end{array}$$

Check:

$$\begin{array}{r}
 11 \\
 186 \\
 + 145 \\
 \hline
 331
 \end{array}$$

Exercise 3

Subtract the following.

Examples:

$$\begin{array}{r}
 286 \\
 - 138 \\
 \hline
 148
 \end{array}
 \qquad
 \begin{array}{r}
 481 \\
 - 225 \\
 \hline
 256
 \end{array}$$

	390		532		465
a.	— 135	h.	— 290	o.	— 374

	390		82		785
b.	— 135	i.	— 79	p.	— 147

	734		262		937
c.	— 582	j.	— 39	q.	— 258

	281		427		946
d.	— 175	k.	— 183	r.	— 463

	925		452		734
e.	— 68	l.	— 173	s.	— 208

	260		692		563
f.	— 154	m.	— 473	t.	— 154

	379		634		782
g.	— 235	n.	— 273	u.	— 254

$$\begin{array}{r} 621 \\ \text{v. } - 442 \\ \hline \end{array}$$

Exercise 3 Answers

a. 255
b. 152
c. 152
d. 106
e. 857
f. 106
g. 144
h. 242

i. 3
j. 223
k. 244
l. 279
m. 219
n. 361
o. 91
p. 638

q. 679
r. 483
s. 526
t. 409
u. 528
v. 179

Exercise 4

Subtract the following.

$$\begin{array}{r} 776 \\ \text{a. } - 382 \\ \hline \end{array} \quad \begin{array}{r} 426 \\ \text{b. } - 327 \\ \hline \end{array} \quad \begin{array}{r} 957 \\ \text{c. } - 234 \\ \hline \end{array}$$

		845		863		745		
d.	—	416	k.	—	389	r.	—	649

		967		689		639		
e.	—	173	l.	—	434	s.	—	484

		406		754		811		
f.	—	257	m.	—	526	t.	—	173

		857		572		678		
g.	—	143	n.	—	493	u.	—	290

		757		714		740		
h.	—	129	o.	—	588	v.	—	272

		567		795		983		
i.	—	182	p.	—	497	w.	—	876

		952		390		839		
j.	—	278	q.	—	256	x.	—	653

Exercise 4 Answers

a. 394
b. 109
c. 723
d. 429
e. 794
f. 149
g. 714
h. 628

i. 385
j. 674
k. 474
l. 255
m. 228
n. 79
o. 126
p. 298

q. 134
r. 96
s. 155
t. 638
u. 388
v. 468
w. 107
x. 186

Now, work through this example, where you must also rename one thousand as ten hundreds to do the subtraction.

Example F

$$3\,245 - 1\,678$$

Step 1: Subtract the ones.

$$\begin{array}{r} \\ \\ \\ + \\ \hline \end{array}$$

Step 2: Subtract the tens.

$$\begin{array}{r}
 \overset{13}{\cancel{3}} \overset{15}{\cancel{4}} \cancel{5} \\
 + \\
 \hline

 \end{array}$$

Step 3: Subtract the hundreds.

$$\begin{array}{r}
 \overset{11}{\cancel{2}} \overset{13}{\cancel{3}} \overset{15}{\cancel{4}} \cancel{5} \\
 - \\
 \hline

 \end{array}$$

Step 4: Subtract the thousands.

$$\begin{array}{r}
 \overset{11}{\cancel{2}} \overset{13}{\cancel{3}} \overset{15}{\cancel{4}} \cancel{5} \\
 - \\
 \hline

 \end{array}$$

Check:

$$\begin{array}{r}
 \overset{1}{1} \overset{1}{5} \overset{1}{6} \overset{1}{7} \\
 + \\
 \hline

 \end{array}$$

Exercise 5

Find the differences.

$$\begin{array}{r} 4\,295 \\ - \quad 724 \\ \hline \end{array}$$

a. $\quad$ f. $\begin{array}{r} 3\,527 \\ - \quad 758 \\ \hline \end{array}$ k. $\begin{array}{r} 4\,289 \\ - \quad 2\,534 \\ \hline \end{array}$

$$\begin{array}{r} 8\,281 \\ - \quad 470 \\ \hline \end{array}$$

b. $\quad$ g. $\begin{array}{r} 3\,154 \\ - \quad 205 \\ \hline \end{array}$ l. $\begin{array}{r} 6\,753 \\ - \quad 1\,942 \\ \hline \end{array}$

$$\begin{array}{r} 5\,564 \\ - \quad 644 \\ \hline \end{array}$$

c. $\quad$ h. $\begin{array}{r} 2\,640 \\ - \quad 834 \\ \hline \end{array}$ m. $\begin{array}{r} 8\,684 \\ - \quad 2\,916 \\ \hline \end{array}$

$$\begin{array}{r} 6\,382 \\ - \quad 882 \\ \hline \end{array}$$

d. $\quad$ i. $\begin{array}{r} 7\,355 \\ - \quad 4\,038 \\ \hline \end{array}$ n. $\begin{array}{r} 7\,459 \\ - \quad 3\,927 \\ \hline \end{array}$

$$\begin{array}{r} 8\,513 \\ - \quad 829 \\ \hline \end{array}$$

e. $\quad$ j. $\begin{array}{r} 5\,189 \\ - \quad 2\,348 \\ \hline \end{array}$ o. $\begin{array}{r} 8\,360 \\ - \quad 6\,376 \\ \hline \end{array}$

$$\begin{array}{r}
 9\,418 \\
 p. 4\,739 16\,793 34\,092 \\
 \hline
 4\,739 7\,325 4\,538
 \end{array}$$

$$\begin{array}{r}
 75\,762 \\
 q. 9\,351 12\,533 42\,126 \\
 \hline
 75\,762 12\,533 42\,126 \\
 9\,351 9\,362 24\,762
 \end{array}$$

$$\begin{array}{r}
 72\,641 \\
 r. 8\,736 72\,209 52\,750 \\
 \hline
 72\,641 72\,209 52\,750 \\
 8\,736 9\,786 14\,789
 \end{array}$$

Exercise 5 Answers

a. 3 571	i. 3 317	q. 66 411
b. 7 811	j. 2 841	r. 63 905
c. 4 920	k. 1 755	s. 9 468
d. 5 500	l. 4 811	t. 3 171
e. 7 684	m. 5 768	u. 62 423
f. 2 769	n. 3 532	v. 29 554
g. 2 949	o. 1 984	w. 17 364
h. 1 806	p. 4 679	x. 37 961

Exercise 6

Find the differences.

Example:

$$\begin{array}{r} 4\,262 \\ - 2\,738 \\ \hline 1\,524 \end{array}$$

a. $\begin{array}{r} 3\,236 \\ - 1\,594 \\ \hline \end{array}$ e. $\begin{array}{r} 5\,623 \\ - 3\,352 \\ \hline \end{array}$ i. $\begin{array}{r} 3\,682 \\ - 963 \\ \hline \end{array}$

b. $\begin{array}{r} 4\,697 \\ - 3\,268 \\ \hline \end{array}$ f. $\begin{array}{r} 8\,428 \\ - 6\,309 \\ \hline \end{array}$ j. $\begin{array}{r} 29\,285 \\ - 18\,357 \\ \hline \end{array}$

c. $\begin{array}{r} 8\,321 \\ - 4\,543 \\ \hline \end{array}$ g. $\begin{array}{r} 9\,629 \\ - 7\,258 \\ \hline \end{array}$ k. $\begin{array}{r} 43\,325 \\ - 3\,187 \\ \hline \end{array}$

d. $\begin{array}{r} 2\,831 \\ - 289 \\ \hline \end{array}$ h. $\begin{array}{r} 5\,230 \\ - 2\,456 \\ \hline \end{array}$ l. $\begin{array}{r} 81\,328 \\ - 22\,595 \\ \hline \end{array}$

$$\begin{array}{r} 58\,234 \\ m. 23\,678 \end{array} \quad \begin{array}{r} 5\,214 \\ q. 1\,783 \end{array} \quad \begin{array}{r} 31\,582 \\ u. 14\,413 \end{array}$$

$$\begin{array}{r} 28\,243 \\ n. 9\,578 \end{array} \quad \begin{array}{r} 23\,244 \\ r. 15\,534 \end{array} \quad \begin{array}{r} 71\,629 \\ v. 12\,350 \end{array}$$

$$\begin{array}{r} 3\,245 \\ o. 1\,678 \end{array} \quad \begin{array}{r} 16\,121 \\ s. 12\,768 \end{array} \quad \begin{array}{r} 44\,610 \\ w. 13\,071 \end{array}$$

$$\begin{array}{r} 6\,254 \\ p. 1\,733 \end{array} \quad \begin{array}{r} 53\,507 \\ t. 14\,421 \end{array}$$

Exercise 6 Answers

a. 1 642
b. 1 429
c. 3 778
d. 2 542
e. 2 271
f. 2 119
g. 2 371
h. 2 774

i. 2 719
j. 10 928
k. 40 138
l. 58 733
m. 34 556
n. 18 665
o. 1 567
p. 4 521

q. 3 431
r. 7 710
s. 3 353
t. 39 086
u. 17 169
v. 59 279
w. 31 539

Zeros in Subtracting

You will have subtraction questions with a zero in the place you want to borrow from. You have to do a double borrowing.

Look carefully at the following example.

Example G

$$2405 - 368$$

Step 1: 5 ones – 8 ones (can't be done)

Borrow one ten – whoops – no tens!

Borrow one hundred and rename it as 10 tens...

$$\begin{array}{r} 3 \quad 10 \\ 2 \cancel{4} \cancel{0} 5 \\ - \quad 368 \\ \hline \end{array}$$

Step 2: Borrow a ten.

$$15 \text{ ones} - 8 \text{ ones} = 7 \text{ ones}$$

$$\begin{array}{r}
 \begin{array}{ccc} 3 & 10 & 15 \\ 2 & \cancel{4} & \cancel{0} & \cancel{5} \end{array} \\
 - \quad \quad \quad 3 & 6 & 8 \\
 \hline
 & & 7
 \end{array}$$

Step 2:

$$9 \text{ tens} - 6 \text{ tens} = 3 \text{ tens}$$

$$\begin{array}{r}
 \begin{array}{ccc} 9 & & \\ 3 & \cancel{10} & 15 \\ 2 & \cancel{4} & \cancel{0} & \cancel{5} \end{array} \\
 - \quad \quad \quad 3 & 6 & 8 \\
 \hline
 & 3 & 7
 \end{array}$$

Step 3:

$$3 \text{ hundreds} - 3 \text{ hundreds} = 0 \text{ hundreds}$$

$$\begin{array}{r}
 \begin{array}{ccc} 9 & & \\ 3 & \cancel{10} & 15 \\ 2 & \cancel{4} & \cancel{0} & \cancel{5} \end{array} \\
 - \quad \quad \quad 3 & 6 & 8 \\
 \hline
 & 0 & 3 & 7
 \end{array}$$

Step 4: 2 thousands - no thousands = 2 thousands

$$\begin{array}{r}
 \begin{array}{ccc} 9 & & \\ 3 & \cancel{10} & 15 \\ 2 & \cancel{4} & \cancel{0} & \cancel{5} \end{array} \\
 - \quad \quad \quad 3 & 6 & 8 \\
 \hline
 2 & 0 & 3 & 7
 \end{array}$$

Exercise 7

Find the differences.

$$\begin{array}{r} 102 \\ \text{a. } \underline{ - 23} \end{array} \quad \begin{array}{r} 302 \\ \text{f. } \underline{ - 218} \end{array} \quad \begin{array}{r} 4\,037 \\ \text{k. } \underline{ - 289} \end{array}$$

$$\begin{array}{r} 508 \\ \text{b. } \underline{ - 39} \end{array} \quad \begin{array}{r} 203 \\ \text{g. } \underline{ - 157} \end{array} \quad \begin{array}{r} 6\,032 \\ \text{l. } \underline{ - 764} \end{array}$$

$$\begin{array}{r} 804 \\ \text{c. } \underline{ - 37} \end{array} \quad \begin{array}{r} 601 \\ \text{h. } \underline{ - 296} \end{array} \quad \begin{array}{r} 4\,057 \\ \text{m. } \underline{ - 2\,049} \end{array}$$

$$\begin{array}{r} 607 \\ \text{d. } \underline{ - 48} \end{array} \quad \begin{array}{r} 2\,075 \\ \text{i. } \underline{ - 436} \end{array} \quad \begin{array}{r} 6\,035 \\ \text{n. } \underline{ - 2\,634} \end{array}$$

$$\begin{array}{r} 406 \\ \text{e. } \underline{ - 178} \end{array} \quad \begin{array}{r} 3\,076 \\ \text{j. } \underline{ - 594} \end{array} \quad \begin{array}{r} 9\,025 \\ \text{o. } \underline{ - 4\,603} \end{array}$$

$$\begin{array}{r}
 5\,075 \\
 p. 3\,64 5\,183 30\,429 \\
 \hline
 3\,64 5\,183 30\,429
 \end{array}$$

$$\begin{array}{r}
 50\,398 \\
 q. 4\,247 60\,831 70\,543 \\
 \hline
 50\,398 60\,831 70\,543
 \end{array}$$

$$\begin{array}{r}
 40\,683 \\
 r. 3\,162 40\,465 80\,106 \\
 \hline
 40\,683 40\,465 80\,106
 \end{array}$$

Exercise 7 Answers

a. 79	i. 1 639	q. 46 151
b. 469	j. 2 482	r. 37 521
c. 767	k. 3 748	s. 45 033
d. 559	l. 5 268	t. 53 750
e. 228	m. 2 008	u. 18 937
f. 84	n. 3 401	v. 15 476
g. 46	o. 4 422	w. 32 708
h. 305	p. 2 711	x. 32 809

Exercise 8

Find the differences.

$$\begin{array}{r} 400 \\ \text{a. } - 197 \\ \hline \end{array} \quad \begin{array}{r} 200 \\ \text{f. } - 99 \\ \hline \end{array} \quad \begin{array}{r} 2\,048 \\ \text{k. } - 281 \\ \hline \end{array}$$

$$\begin{array}{r} 307 \\ \text{b. } - 138 \\ \hline \end{array} \quad \begin{array}{r} 400 \\ \text{g. } - 43 \\ \hline \end{array} \quad \begin{array}{r} 6\,005 \\ \text{l. } - 2\,368 \\ \hline \end{array}$$

$$\begin{array}{r} 800 \\ \text{c. } - 475 \\ \hline \end{array} \quad \begin{array}{r} 208 \\ \text{h. } - 126 \\ \hline \end{array} \quad \begin{array}{r} 5\,000 \\ \text{m. } - 3\,468 \\ \hline \end{array}$$

$$\begin{array}{r} 608 \\ \text{d. } - 439 \\ \hline \end{array} \quad \begin{array}{r} 3\,000 \\ \text{i. } - 2\,678 \\ \hline \end{array} \quad \begin{array}{r} 4\,006 \\ \text{n. } - 2\,179 \\ \hline \end{array}$$

$$\begin{array}{r} 307 \\ \text{e. } - 168 \\ \hline \end{array} \quad \begin{array}{r} 7\,205 \\ \text{j. } - 2\,306 \\ \hline \end{array} \quad \begin{array}{r} 3\,007 \\ \text{o. } - 1\,930 \\ \hline \end{array}$$

	2 007		60 125		80 062
p.	— 237	s.	— 8 421	v.	— 35 087
	43 004		40 063		90 035
q.	— 2 873	t.	— 2 734	w.	— 68 746
	20 038		70 059		60 063
r.	— 9 156	u.	— 38 423	x.	— 55 895

Exercise 8 Answers

a. 203	i. 322	q. 40 131
b. 169	j. 4 899	r. 10 882
c. 325	k. 1 767	s. 51 704
d. 169	l. 3 637	t. 37 329
e. 139	m. 1 532	u. 31 636
f. 101	n. 1 827	v. 44 975
g. 357	o. 1 077	w. 21 289
h. 82	p. 1 770	x. 4 168

If a subtraction question has the numbers side by side, rewrite the question in columns. Put the ones under the ones, the tens under the tens, the hundreds under the hundreds, and so on.

Example H: $5\,625 - 2\,468$

$$5\,625 - 2\,468$$

$$\begin{array}{r}
 \overset{11}{\cancel{2}} \\
 \overset{5}{\cancel{6}} \overset{15}{\cancel{5}} \\
 - \\
 \hline

 \end{array}$$

Exercise 9

Rewrite each question in columns and find the difference.

$$\begin{array}{lll}
 \begin{array}{r} 5\,042 \\ - 3\,185 \\ \hline \end{array} & \begin{array}{r} 8\,042 \\ - 6\,368 \\ \hline \end{array} & \begin{array}{r} 2\,630 \\ - 95 \\ \hline \end{array} \\
 \text{a.} & \text{b.} & \text{c.}
 \end{array}$$

$$\begin{array}{r} \text{d.} \quad \begin{array}{r} 1\,201 \\ - \quad 159 \\ \hline \end{array} \quad \begin{array}{r} 54\,507 \\ - \quad 13\,421 \\ \hline \end{array} \quad \begin{array}{r} 64\,182 \\ - \quad 28\,934 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \text{e.} \quad \begin{array}{r} 34\,582 \\ - \quad 6\,121 \\ \hline \end{array} \quad \begin{array}{r} 7\,050 \\ - \quad 2\,144 \\ \hline \end{array} \quad \text{h.} \end{array}$$

$$\begin{array}{r} \text{f.} \quad \begin{array}{r} 44\,610 \\ - \quad 4\,527 \\ \hline \end{array} \quad \begin{array}{r} 71\,629 \\ - \quad 12\,350 \\ \hline \end{array} \quad \text{i.} \end{array}$$

Exercise 9 Answers

- a. 1 857
- b. 1 674
- c. 2 535
- d. 1 042

- e. 28 461
- f. 40 083
- g. 41 086
- h. 4 906

- i. 59 279
- j. 35 248

1.4: Practice Questions

1. Find the differences. Check your answers using addition.

$$\begin{array}{r} \text{a.} \quad \begin{array}{r} 71 \\ - \quad 32 \\ \hline \end{array} \quad \text{b.} \quad \begin{array}{r} 704 \\ - \quad 325 \\ \hline \end{array} \quad \text{c.} \quad \begin{array}{r} 400 \\ - \quad 208 \\ \hline \end{array} \end{array}$$

	8 923		9 074		86 502
d.	— 3 061	g.	— 5 482	j.	— 6 590

	5 211		8 092		47 293
e.	— 4 390	h.	— 6 578	k.	— 26 349

	8 204		49 053		73 050
f.	— 3 461	i.	— 8 954	l.	— 27 455

2. Subtract.

a.	$5\,302 - 3\,981 =$	c.	$6\,000 - 989 =$
b.	$7\,043 - 95 =$		

1.4: Practice Answers

1. Find the differences.

a.	39	e.	821	i.	40099
b.	379	f.	4743	j.	79912
c.	192	g.	3562	k.	20944
d.	5 862	h.	1514	l.	45595

2. Subtract.

a.	1321	c.	5011
b.	6948		

Attribution

All figures in this chapter are from Topic D: Subtraction with Borrowing in Adult Literacy Fundamental Mathematics: Book 2 – 2nd Edition by Tagami & Girard (2022), via BCcampus.

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1.5: Estimating Answers in Subtraction

You have learned how to round numbers. Now, you can use that skill in rounding numbers to find an approximate difference. By estimating your answer first, you can tell if your answer is sensible.

In these examples, estimate the answer. Round each number **BEFORE** you subtract.

$$\begin{array}{rclcl} & 47 & \text{rounds to} & & 50 \\ \text{Example A: } & - & 26 & \text{rounds to} & - & 30 \\ & \hline & & & & & 20 \end{array}$$

$$\begin{array}{rclcl} & 870 & \text{rounds to} & & 900 \\ \text{Example B: } & - & 342 & \text{rounds to} & - & 300 \\ & \hline & & & & & 600 \end{array}$$

$$\begin{array}{rclcl} & 24\,397 & \text{rounds to} & & 24\,000 \\ \text{Example C: } & - & 6\,148 & \text{rounds to} & - & 6\,000 \\ & \hline & & & & & 18\,000 \end{array}$$

Usually, you estimate to the largest place value that you can.

Exercise 1

Estimate the differences. Round the numbers before you

subtract. Check your work using the answer key at the end of the exercise.

Examples:

$\begin{array}{r} 9\,963 \\ - 7\,099 \\ \hline \end{array}$	$\approx$	$\begin{array}{r} 10\,000 \\ - 7\,000 \\ \hline 3\,000 \end{array}$	$\approx$	$\begin{array}{r} 70\,534 \\ - 7\,689 \\ \hline \end{array}$	$\approx$	$\begin{array}{r} 71\,000 \\ - 8\,000 \\ \hline 63\,000 \end{array}$

a. $\begin{array}{r} 687 \\ - 438 \\ \hline \end{array}$	f. $\begin{array}{r} 7\,111 \\ - 5\,982 \\ \hline \end{array}$	k. $\begin{array}{r} 54\,751 \\ - 7\,896 \\ \hline \end{array}$
b. $\begin{array}{r} 754 \\ - 236 \\ \hline \end{array}$	g. $\begin{array}{r} 6\,031 \\ - 2\,899 \\ \hline \end{array}$	l. $\begin{array}{r} 72\,450 \\ - 31\,924 \\ \hline \end{array}$
c. $\begin{array}{r} 8\,442 \\ - 1\,876 \\ \hline \end{array}$	h. $\begin{array}{r} 41\,573 \\ - 4\,846 \\ \hline \end{array}$	m. $\begin{array}{r} 81\,692 \\ - 53\,908 \\ \hline \end{array}$
d. $\begin{array}{r} 5\,630 \\ - 1\,752 \\ \hline \end{array}$	i. $\begin{array}{r} 36\,154 \\ - 9\,038 \\ \hline \end{array}$	n. $\begin{array}{r} 92\,163 \\ - 45\,517 \\ \hline \end{array}$
e. $\begin{array}{r} 5\,342 \\ - 3\,647 \\ \hline \end{array}$	j. $\begin{array}{r} 46\,124 \\ - 9\,762 \\ \hline \end{array}$	o. $\begin{array}{r} 171\,234 \\ - 82\,169 \\ \hline \end{array}$
		p. $\begin{array}{r} 102\,085 \\ - 36\,526 \\ \hline \end{array}$

Exercise 1 Answers

a. 700 − 400 = 300	k. 55\,000 − 8\,000 = 47\,000
b. 800 − 200 = 600	l. 70\,000 − 30\,000 = 40\,000
c. 8\,000 − 2\,000 = 6\,000	m. 80\,000 − 50\,000 = 30\,000
d. 6\,000 − 2\,000 = 4\,000	n. 90\,000 − 50\,000 = 40\,000
e. 5\,000 − 4\,000 = 1\,000	o. 170 000 − 80 000 = 90 000
f. 7\,000 − 6\,000 = 1\,000	p. 100 000 − 40 000 = 60 000
g. 6 000 − 3 000 = 3 000	
h. 42 000 − 5 000 = 37 000	
i. 36 000 − 9 000 = 27 000	
j. 46\,000 − 10\,000 = 36\,000	

Estimating Answers in Subtraction Word Problems

When solving word problems, an estimate tells you if your answer makes sense. You can use your estimate to help you check your answers. If your answer and the estimate are not close, then you know that you should subtract your numbers again.

Exercise 2

Estimate the following answers. Be sure to round to the largest place value possible before adding or subtracting. Remember to circle the information and underline what is being asked.

Example: On a recent petition about sales tax, Mulan had 2 865 people sign. Arnav had 1 564 people sign the petition. Estimate how many more people Mulan had sign the petition than Arnav.

On a recent petition about sales tax, Mulan had 2 865 people sign. Arnav had 1 564 people sign the petition. Estimate how many more people Mulan had sign the petition than Arnav.

2 865	Estimate:	3 000
— 1 564		— 2 000
		1 000

Mulan had 1 000 more people sign the petition.

- a. On Tuesday, a coffee shop had sales of \$8 523. On Wednesday, the same coffee shop had sales of \$6 914. Estimate the difference between Tuesday's sales and Wednesday's sales.

- b. Last week, 4 931 passengers used the ABE Taxi Company. This week, there were 3 491 passengers. Estimate how many more passengers used ABE Taxi Company last week.
- c. In Japan, people chew 52 700 tons of gum. In Russia, people chew 25 700 tons of gum. Estimate how many more tons of gum the Japanese chew.
- d. In Colombia, there are 1 897 bird species. In China, there are 1 319 bird species. Estimate how many more bird species there are in Colombia.
- e. The whale shark weighs 3 500 kilograms. The basking shark weighs 9 258 kilograms. Estimate how much more the whale shark weighs.
- f. In India, there were 155 204 post offices in 2007. In China, there were 59 886 post offices. Estimate the difference.
- g. By 2008, the Montreal Canadiens had played the most games at 5 792. The Buffalo Sabres had played 2 952. Estimate how many more games the Montreal Canadiens had played.
- h. In 2006, the population of Kelowna was 162 276. The population of Prince George was 83 225. Estimate how many more people live in Kelowna in 2006.

Exercise 2 Answers

- a. $\$9\,000 - \$7\,000 = \$2\,000$
- b. $5\,000 - 3\,000 = 2\,000$ passengers
- c. $50\,000 - 30\,000 = 20\,000$ tons

- d. $2\,000 - 1\,000 = 1\,000$ species
- e. $31\,000 - 9\,000 = 22\,000$ kilograms
- f. $160\,000 - 60\,000 = 100\,000$ post offices
- g. $6\,000 - 3\,000 = 3\,000$ games
- h. $160\,000 - 80\,000 = 80\,000$ people

1.5: Practice Questions

1. Estimate the differences. Show your work.

	73		4 071		49 053
a.	— 34	e.	— 2 986	i.	— 28 954

	67		5 946		36 174
b.	— 18	f.	— 4 281	j.	— 16 925

	896		57 201		86 502
c.	— 385	g.	— 5 892	k.	— 26 590

	467		23 006		943 982
d.	— 214	h.	— 4 999	l.	— 721 354

2. Estimate each of the following word problems. Be sure to circle information and underline what is being asked. Include the unit of measurement in your answer.
- a. A magazine has 54 823 readers. Last year, the magazine had 26 876 readers. By how much did the number of readers increase?
 - b. In 2009, the number of marriages per year in Japan was 964 702. The number of marriages per year in Egypt was 525 412. How many more marriages were there in Japan than in Egypt?
 - c. In France in 2010, there were 235 846 people with the last name Martin. There were 78 177 people with the last name Moreau. How many more Martins were there?

1.5: Practice Answers

1. Estimate the differences.

a. 40	e. 1 000	i. 20 000
b. 50	f. 2 000	j. 20 000
c. 500	g. 51 000	k. 60 000
d. 300	h. 18 000	l. 200 000

2. Estimate each of the following word problems.

- a. 20 000 readers
- b. 500 000 marriages
- c. 160 000 Martins

Attribution

This chapter has been adapted from Topic E: Estimating Answers

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– 2nd Edition (BCcampus) by Wendy Tagami and Liz Girard (2022),
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1.6: Problem Solving

Why are you studying mathematics?

Some of you are taking math because you have to, but we hope you all want to have math skills to help you in your jobs, job training, and everyday life. Numbers are an important part of our lives — we are surrounded by numbers.

Numbers are not often by themselves or set up neatly on a page for us to add or subtract. Numbers are usually in the middle of sentences and mixed in with other numbers. Sorting out the numbers you want and deciding what to do with those numbers is called problem-solving.

You are going to learn five problem-solving steps that will be useful in all your math work in courses, in jobs, and in your everyday life.

Problem Solving Steps

Step 1: Read or listen to the problem carefully. **Understand** the problem.

- Are there words that help you imagine what is happening?
- Can you draw a picture or diagram to show what is happening?
- Can you say the problem in your own words?
- What is the **question**? Underline it.

Step 2: What does the problem tell you?

- What do you know? Write down or circle the **information** you have.
- Often, you have more information than you need. Think about the question you need to answer and use only the information

that will help you answer that question. What do you want to find out?

Step 3: What must you do with the information to answer the question?

- What **arithmetic operation** should you use: addition, subtraction, multiplication or division?
- You will learn **keywords** and **patterns** to help you choose the correct operation.
- Write an equation for the problem. An equation is a number sentence, such as:

$$12 + 5 = \underline{\hspace{2cm}}$$

Step 4: Estimate the answer.

- Round the numbers so you can work with them quickly.
- Use the operation you chose in Step 3 and come to a quick answer.
- Does this estimated answer make sense? Does it answer the question in the problem? **THINK** about this before you do Step 5.

Step 5: Solve the problem using the **actual** numbers.

- Check your arithmetic calculations.
- Compare your result to your estimated answer.
- Reread the problem. Does your answer make sense?
- Write a sentence answer to the problem.

You must always say what the numbers are counting. He has 4 means nothing. We need to know 4 what... 4 children? 4 dogs? 4 dollars? These are called the units.

Table 1.6.1 shows some units with their common abbreviations.

Table 1.6.1: Common Unit Abbreviations

Unit	Abbreviation
kilometre	km
metre	m
centimetre	cm
kilogram	kg
gram	g
litre	L
hour	h
minute	min

Now, study the three example problems that show the five steps.

Example A

Jorge earned \$165 last week and \$142 this week in his job pumping gas at the service station. He spent \$15 on his girlfriend's gift. How much did he earn pumping gas?

Step 1: Read. Understand the problem. Find the question. Underline it.

How much did Jorge earn pumping gas?

Step 2: Find the needed information. Circle it.

Jorge earned \$165 and \$142.

The information about his girlfriend's gift has nothing to do with finding out how much he earned.

Step 3: What arithmetic operation to use?

We are putting together two amounts. That is addition.

The equation:

$$\$165 + \$142 = \text{what he earned.}$$

Step 4: Estimate.

$$\$165 + \$142$$

$$\$170 + \$140 = \$310$$

$$\$200 + \$100 = \$300$$

Is about \$300 a reasonable answer to the question? Is it sensible to earn \$300 for two weeks of pumping gas? Probably. \$3 000 would not be sensible, and \$30 would not be sensible.

Step 5: Solve, check, write a sentence answer.

$$\$165 + \$142 = \$307$$

Check by adding again

Is \$307 close to the estimate?

Does it make sense?

Jorge earned \$307 pumping gas.

Example B

The town of Gloryville had a population of 4 206 people before the mill had a big lay-off in May 2007. Since then, 858 people have moved away. Find the population of Gloryville now.

Step 1: Read, understand the problem, find the question underline it.

Find the population of Gloryville now.

Step 2: (Circle) needed information.

(4 206) people before

(858) people moved away

The date of the lay-off is not needed to answer the question.

Step 3: Operation.

One amount is being taken away. That is subtraction.

Equation:

$$4\,206 - 858 = \text{people in Gloryville now.}$$

Step 4: Estimate.

$$4\,206 - 858$$

$$4\,000 - 1\,000 = 3\,000$$

$$4\,200 - 900 = 3\,300$$

Step 5: Solve, check, and write a sentence answer.

$$\begin{array}{r}
 \begin{array}{ccccccc}
 & & 11 & & 9 & & \\
 & 3 & \cancel{1} & & \cancel{10} & & 16 \\
 \cancel{4} & \cancel{2} & \cancel{0} & \cancel{0} & & & \\
 - & & & & 8 & 5 & 8 \\
 \hline
 & & & & 3 & 3 & 4 & 8
 \end{array}
 \end{array}$$

Check:

$$\begin{array}{r}
 \begin{array}{ccccccc}
 & 1 & 1 & 1 & & & \\
 & 3 & 3 & 4 & 8 & & \\
 + & & & 8 & 5 & 8 & \\
 \hline
 & 4 & 2 & 0 & 6 & &
 \end{array}
 \end{array}$$

Close to estimate?

Makes sense?

Gloryville has a population now of 3 348 people.

Example C

Paul works at a lumber mill and gets paid every two weeks. He has an account at the bank. Today, he got a cheque for \$845. He and his wife decided to deposit \$600 in the account and keep the rest of the money out for a

weekend trip. How much money did Paul and his wife keep out for the weekend trip?

Step 1: Question.

How many did Paul and his wife keep for the weekend trip?

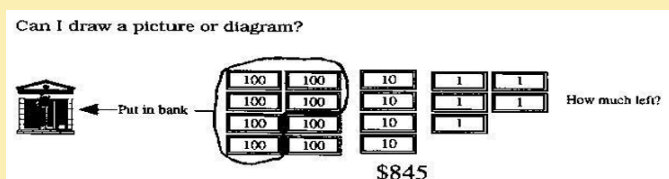


Figure 1.6.1

Step 2: Needed information.

Paul got a cheque for \$845 for two weeks of work.

He and his wife decided to put \$600 in their account.

Step 3: Operation.

One amount is being taken away. That is subtraction.

Equation:

$$\$845 - \$600 = \text{money left over for weekend trip}$$

Step 4: Estimate

$$\$845 - \$600 = \$245$$

Check:

$$\$250 + \$600 = \$845$$

Close to estimate?

Makes sense?

Paul and his wife have \$245 for the weekend trip.

Addition Problems

The problems in this section all use the addition operation to find the solution (the answer to the problem). Addition problems give two or more amounts that must be put together (added). When you read the problems, pay special attention to keywords and patterns that will help you to recognize other addition problems.

Keywords pointing to addition include:

- sum
- combine
- total
- entire
- in all
- altogether
- complete

Exercise 1

Do these problems by following the five problem-solving steps. It is good practice to write down each step while learning this method. Check your work using the answer key at the end of the exercise.

- a. It was raining so Gita decided to bake several batches of cookies and freeze them. She made 75 chocolate chip cookies, 96 of her son's favourite ginger snaps, and 42 fancy —Birds' nest cookies for when she had company. How many cookies did Gita bake altogether?
- **Step 1:** What is the question? Underline it.
 - **Step 2:** What information are you given that you need to solve the problem? Circle it.
 - **Step 3:** What arithmetic operation should you use? addition Why?
 - **Step 4:** Estimate the answer using rounded numbers.
 - **Step 5:** Solve, check, and write a sentence answer.
- b. Levi wanted to paint his apartment and needed to buy some supplies. Brushes cost \$10, sandpaper cost \$4, a paint roller and tray cost \$9, and the paint was \$55. How much did it cost for all the paint supplies?

- **Step 1:** What is the question? Underline it.
 - **Step 2:** What information are you given that you need to solve the problem? Circle it.
 - **Step 3:** What arithmetic operation should you use? addition Why?
 - **Step 4:** Estimate the answer using rounded numbers.
 - **Step 5:** Solve, check, and write a sentence answer.
- c. Altogether, the college has 475 students in the Adult Basic Education department, 320 students from university transfers, 64 students in the Early Childhood Education program, 232 students in the Forestry Department, and 125 students in trades courses. How many students are at the college?
- **Step 1:** What is the question? Underline it.
 - **Step 2:** What information are you given that you need to solve the problem? Circle it.
 - **Step 3:** What arithmetic operation should you use? addition Why?
 - **Step 4:** Estimate the answer using rounded numbers.
 - **Step 5:** Solve, check, and write a sentence answer.
- d. Zhou works part-time at the daycare centre. Last month she worked every week. She worked 24 hours in the first week, 36 hours in the second, 29 hours in the third, and only 17 hours in the fourth. Give the total number of hours that Zhou worked last month.

- **Step 1:** What is the question? Underline it.
- **Step 2:** What information are you given that you need to solve the problem? Circle it.
- **Step 3:** What arithmetic operation should you use? addition Why?
- **Step 4:** Estimate the answer using rounded numbers.
- **Step 5:** Solve, check, and write a sentence answer.

The rest of the problems in this exercise just ask you for the estimate and actual solution. You must still follow all five steps, but you do not have to write everything down. Remember that the solution to problems must include the units (what is being counted) and should be written in a sentence answer.

- e. September is hard on the family budget! Amul figured they spent \$275 for clothes and shoes for their two little daughters, \$43 for school supplies, \$24 for haircuts, and \$130 to enroll them in the Figure Skating Club. How much has Amul spent getting his children ready for school and skating?
- Estimation:
 - Actual Solution:
- f. The sign in the elevator says — 1 200 kg maximum weight. Can the elevator hold all these large football players safely? Sean weighs 91 kg, Raja is 114 kg, and Eyota is a heavyweight at 159 kg. Kiefer is even heavier at 168 kg, the two fullbacks weigh 135 kg and 148 kg, and the quarterback Juan is a muscular 87 kg.

Find their combined weight to see if they are all safe in the elevator.

- Estimation:
- Actual Solution:

- g. On their holidays, the Matthews family drove to Saskatchewan from their home in Langley. They drove 620 km the first day, 810 km the second day, and only 350 km the next day because they went to Head Smashed-in Buffalo Jump Museum. On the fourth day, they drove a long 1 208 km. How many kilometres did they drive on their trip to Saskatchewan?

- Estimation:
- Actual Solution:

Exercise 1 Answers

(The wording in the sentences will vary, but this is the idea)

- a. How many cookies altogether?
- She made 75, 96, and 42 cookies
 - All the amounts have to be put together to find a total.
 - $80 + 100 + 40 = 220$ cookies
 - $75 + 96 + 42 = 213$ cookies
 - Gita baked 213 cookies altogether

- b. How much did it cost for all the paint supplies?
- He paid \$ 10, \$4, \$9, and \$55
 - All the amounts have to be put together to find a total
 - Rounding one digit numbers isn't too helpful, but $\$10 + \$0 + \$10 + \$60 = \$80$
 - $\$10 + \$4 + \$9 + \$55 = \$78$
 - Levi paid \$78.
- c. How many students at the college?
- There are 475, 320, 232, and 125 students.
 - You must find a total.
 - $500 + 300 + 100 + 200 + 100 = 1\ 200$ students
 - $475 + 320 + 64 + 232 + 125 = 1\ 216$ students
 - The college has 1 216 students
- d. How many hours did Zhou work last month?
- She worked 24, 36, 29, and 17 hours.
 - You are looking for an amount altogether.
 - $20 + 40 + 30 + 20 = 110$ hours
 - $24 + 36 + 29 + 17 = 106$ hours
 - Zhou worked 106 hours last month
- e. \$472 altogether
- f. 902 kg altogether; safe
- g. 2 988 km

Subtraction Problems

These problems give you a chance to get the feel of subtraction problems.

Subtraction problems tell you an amount and then take something away from that amount. Money might be spent, saved, or deducted (taken off), people might move away, and items might be sold or lost. These types of subtraction problems are usually easy to recognize.

A more difficult type of subtraction problem compares two amounts. You will be asked to find the difference between the amounts. Subtract to find the difference. These problems might ask you how much more, how much less, how many fewer, how much farther, how much did it increase (go up), or what is the decrease (the amount it went down)? You might also have to find the age of something by comparing the dates.

Keywords pointing to subtraction include:

- difference
- balance
- amount left
- the saving
- how much more/greater/farther
- how much less/fewer/smaller
- how old/find the age

Exercise 2

Use the five problem steps to solve these problems. Write down each step for the first three problems.

- a. Only 368 people went to the movie theatre on Friday night, while 756 went to see the new comedy movie they were showing on Saturday. How many more people went to the theatre on Saturday than on Friday?
- **Step 1:** What is the question? Underline it.
 - **Step 2:** What information are you given that you need to solve the problem? Circle it.
 - **Step 3:** What arithmetic operation should you use? subtraction Why?
 - **Step 4:** Estimate the answer using rounded numbers.
 - **Step 5:** Solve, check, and write a sentence answer.
- b. The highway construction started in 2004 and finished in 2010. How long did the construction take?
- **Step 1:** What is the question? Underline it.
 - **Step 2:** What information are you given that you need to solve the problem? Circle it.
 - **Step 3:** What arithmetic operation should you use? subtraction Why?

- **Step 4:** Estimate the answer using rounded numbers. In a question like this, an estimation using rounded numbers is not useful because the numbers are too similar and would round to the same number. Instead, think about the question carefully and figure out an approximate answer in your head.
 - **Step 5:** Solve, check, and write a sentence answer.
- c. Aimee's gross pay was \$1 656, but she had \$331 of deductions. What is her net pay? (Gross pay is the amount we earn before anything is taken off. Net pay is the amount we take home after taxes, pension, employment insurance, etc. have been deducted.)
- **Step 1:** What is the question? Underline it.
 - **Step 2:** What information are you given that you need to solve the problem? Circle it.
 - **Step 3:** What arithmetic operation should you use? subtraction Why?
 - **Step 4:** Estimate the answer using rounded numbers.
 - **Step 5:** Solve, check, and write a sentence answer.
- d. Mike and Ann want to can 240 jars of fruit this year. They have already canned 165 jars. How many more jars do they need to do?
- Estimation:
 - Actual Solution:
- e. Jian has purchased a used car for \$3 599. He has

paid \$450 so far. How much more money does he owe?

- Estimation:
- Actual Solution:

- f. In 1956, the population of the town was 10 874. Many people left after the dam construction finished. The population in 1989 was only 7,892 people. How much less was the population in 1989 than in 1956?

- Estimation:
- Actual Solution:

Exercise 2 Answers

- a. How many more people at the theatre on Saturday than on Friday?
- 368 people on Friday; 756 on Saturday
 - You must find the difference between two amounts.
 - $800 - 400 = 400$ more people on Saturday
 - $756 - 368 = 388$ more people on Saturday
- b. How long did the construction take?
- Started in 2004; ended in 2010.
 - Find the difference between the two dates.
 - Think — from 2004 to 2010 — about 5 years
 - $2010 - 2004 = 6$ years for the road construction
- c. What is Aimee's net pay?

- Her gross pay was \$1 656 and she had \$331 taken off (deducted).
 - Subtract to find how much is left.
 - $\$1\,700 - \$300 = \$1\,400$
 - $\$1\,656 - \$331 = \$1\,325$ net pay
- net pay
- d. 75 jars
 - e. \$3 149 still owed
 - f. 2 982 people less

Mixed Addition and Subtraction Problems

Exercise 3

Use the 5 problem-solving steps. Look for keywords and patterns to help you choose the correct operation. Estimate the answer using rounded numbers if the numbers have 2 digits or more.

- a. Enrico worked 37 hours one week and 26 hours the next week. How many hours did he work?

- Estimation:
 - Actual Solution:
- b. Myung-Hee had \$85. She spent \$37 for groceries. How much did she have left?
- Estimation:
 - Actual Solution:
- c. Ann bought 25 kg of potatoes. She used 13 kg the first week. How much did she have left?
- Estimation:
 - Actual Solution:
- d. The sign in a furniture store read, \$35 off all chairs. How much will a chair cost that was \$125 before the sale?
- Estimation:
 - Actual Solution:
- e. Guillaume bought a pair of jeans for \$29 at a sale. When he got home, he found the price tag on the jeans had been \$48. How much did Guillaume save?
- Estimation:
 - Actual Solution:
- f. British Columbia has an area of 947 800 square kilometres. The area of Alberta is 666 190 square kilometres. BC is how much larger than Alberta?
- Estimation:
 - Actual Solution:
- g. Maxine paid \$26 for an electric iron and \$39 for an

ironing board. How much did she pay for both?

- Estimation:
- Actual Solution:

- h. Ang bought a used TV set for \$125. She made a down payment of \$40. How much does she still owe on the set?

- Estimation:
- Actual Solution:

- i. Paulo had \$325 in the bank. He wrote a cheque for \$76. How much money did he have left in the bank?

- Estimation:
- Actual Solution:

- j. Mizu weighs 99 kg. Akula weighs 81 kg. How much heavier is Mizu than Akula?

- Estimation:
- Actual Solution:

- k. Kenji has three children. One weighs 25 kg, another weighs 20 kg, and the last weighs 17 kg. How much do they weigh together?

- Estimation:
- Actual Solution:

- l. Rafael bought a boat priced at \$8 400. He was given \$1 250 as a trade-in on his old boat. How much does he owe on the new boat?

- Estimation:

- Actual Solution:
- m. Last week, Luis earned \$212. The week before, he earned \$198. This week, he earned \$133. How much did he earn in all?
- Estimation:
 - Actual Solution:
- n. Jakob went on a trip of 739 km. The first day, he drove 561 km. How many kilometres did he have left to drive?
- Estimation:
 - Actual Solution:
- o. In 2005, Jacques' net income was \$29 675. In 2006, his net income was \$30 207. How much more did he earn in 2006?
- Estimation:
 - Actual Solution:

Exercise 3 Answers

a. 63 hours	i. \$249 left in the bank
b. \$48 left	j. 18 kg heavier
c. 12 kg of potatoes left	k. 62 kg altogether
d. \$90 for the chair	l. \$7 150 still owed
e. \$19 saved	m. \$543 in all
f. 281 610	n. 178 km left to drive
g. \$65 in all	o. \$532 more
h. \$85 still owed	

Two-Operation Questions

Sometimes, you may need to use two operations to solve a question. We work from left to right when solving questions that involve two operations. If addition is first, you must do the addition first, then the subtraction. If subtraction is first, you must do the subtraction first, then do the addition.

Example D

$$342 + 325 - 146$$

Step 1:

$$342 + 325 = 667$$

Step 2: Use your answer and subtract 146.

$$\begin{array}{r} 667 \\ - 146 \\ \hline 521 \end{array}$$

$$342 + 325 - 146 = 521$$

Example E

$$475 - 284 + 362$$

Step 1:

$$\begin{array}{r} 475 \\ - 284 \\ \hline 191 \end{array}$$

Step 2: Use your answer and add 362.

$$\begin{array}{r} 191 \\ + 362 \\ \hline 553 \end{array}$$

$$475 - 284 + 362 = 553$$

Exercise 4

Find the sum or difference for each question.

a. $312 + 541 - 135 =$	i. $5\,247 + 2\,216 - 4\,673 =$
b. $427 + 231 - 384 =$	j. $1\,285 + 4\,672 - 1\,401 =$
c. $687 - 434 + 256 =$	k. $7\,354 - 4\,038 + 2\,348 =$
d. $754 - 576 + 393 =$	l. $4\,187 - 2\,574 + 1\,846 =$
e. $1\,456 + 218 - 295 =$	m. $5\,314 + 7\,053 - 597 =$
f. $2\,461 + 723 - 349 =$	n. $4\,315 + 3\,197 - 2\,106 =$
g. $3\,857 - 665 + 1\,234 =$	o. $46\,124 - 9\,762 + 2\,534 =$
h. $4\,367 - 843 + 5\,679 =$	p. $70\,534 - 7\,689 + 1\,824 =$

Exercise 4 Answers

a. 718	i. 2 790
b. 274	j. 4 556
c. 509	k. 5 664
d. 571	l. 3 459
e. 1 379	m. 11 770
f. 2 835	n. 5 406
g. 4 426	o. 38 896
h. 9 203	p. 64 669

Two-Operation Problems

Sometimes, you may need to use more than one operation to solve a word problem or a real-life problem.

Example F

Janet bought a submarine sandwich for \$5, a soft drink for \$1, and some carrot cake for \$3. She gave the cashier a twenty-dollar bill. How much money did she get back as change?

Step 1: Question: How much change from \$20?

Step 2: Information: Spent \$5, \$1, and \$3. Gave the cashier \$20.

Step 3: Operations.

Add the amounts she spent to find the total:

$$\$5 + \$1 + \$3 =$$

Subtract the amount she spent from \$20:

$$\$20 - \text{total of what she spent} = \text{change}$$

Step 4: Estimate.

Numbers are only one digit, so do not round them. But a quick addition tells you that her change will be about \$10.

Step 5: Solve.

$$\begin{aligned} \$5 + \$1 + \$3 &= \$9 \text{ total spent} \\ \$20 - \$9 &= \$11 \end{aligned}$$

Janet will get \$11 in change.

Exercise 5

Use the 5 problem-solving steps. Look for keywords and patterns to help you choose the correct operation. Estimate the answer using rounded numbers if the numbers have 2 digits or more. Show all your work.

- a. Maureen weighed 72 kg and decided to go on a diet for her New Year's Resolution. She lost 3 kg in January, 2 kg in February, and 4 kg in March. How much did she weigh after her three-month diet?
 - Estimation:
 - Actual Solution:
- b. The local Girl Guides and Brownies had a goal to sell 2 850 boxes of Girl Guide cookies. In the first week, the Brownies sold 975 boxes, while the Guides sold 1 138 boxes. How many more boxes do they need to sell to reach their goal?
 - Estimation:
 - Actual Solution:
- c. Pat is ready to start first-year college; she received a Passport to Education award from the provincial government, which was \$625. She got a Rotary Club Scholarship of \$250 and a science scholarship of \$400. Her first year's tuition and books are going to

cost \$2 000. Pat will use all her awards and scholarships. How much more money will she need to pay?

- Estimation:
- Actual Solution:

- d. The elementary school had 83 girls and 95 boys enrolled in September. Five of the girls and three of the boys moved away in September. How many children were still enrolled in the school at the end of September?

- Estimation:
- Actual Solution:

- e. Franco is on a 1 200-calorie-a-day diet. He had 320 calories at breakfast and 468 calories at lunch. How many calories does he have left for dinner?
- f. Lilo had a total of 150 hats in four boxes. In box one, there were 72 hats. In box two, there were 28 hats. In box three, there were 47 hats. How many hats were in box four?
- g. Miguel wanted to buy a Blu-ray player for \$225. He got \$65 for his birthday. He won \$75. How much more money does Miguel need?
- h. Kehara and Omar decided to visit their grandmother, who lives 160 kilometres away. They travelled 50 kilometres and stopped for gas. They travelled another 30 kilometres and stopped for lunch. How much farther is it to their grandmother's house?
- i. Kuen had \$7 342 in his bank account. He decided to

buy a new television for \$1 139. Kuen was able to save another \$697. How much does Kuen have in his bank account?

- j. Giles wishes to buy three gifts that cost \$15, \$9 and \$12. He has \$11 of the money he needs. How much more money does he need to earn to buy the gifts?
- k. Colette bought items costing \$34, \$19, \$65 and \$129. She used a coupon worth \$75. How much money does she still owe?
- l. Sahale had 25 metres of fencing. He wanted to fence his garden, which was 53 metres long and 38 metres wide. How much more fencing does Sahale need to buy? (Hint: To put a fence around means the perimeter. Draw a picture before you begin.)

Exercise 5 Answers

a. 63 kg	g. \$85 more
b. 737 boxes of cookies more	h. 80 kilometres
c. \$725 more	i. \$6 900
d. 170 children still enrolled	j. \$25 more
e. 412 calories	k. \$172
f. 3 hats	l. 157 metres

1.6: Practice Questions

1. Solve the following word problems. Show all your work.
 - a. Alice weighed 86 kg. She went on a diet. Now, she weighs 69 kg. How much did she lose?

- Estimation:
- Actual Solution:

- b. Jacques spent \$49 on a pair of jeans, \$18 for a shirt, \$12 for a belt, and \$3 for socks. How much did he spend altogether?

- Estimation:
- Actual Solution:

- c. A bookshelf has 94 books on the top, 86 on the middle, and 79 on the bottom. How many books are there on the three shelves?

- Estimation:
- Actual Solution:

- d. Mahad bought a new car for \$9 989. He traded in his old car for \$1 785. How much more was the new one than the value of his trade-in?

- Estimation:
- Actual Solution:

- e. Kian and Toran picked apples for their uncle. Kian picked 509 kg, and Toran picked 436 kg.

- i. How many more kilograms of apples did Kian pick than Toran?

- Actual Solution:
- Estimation:

- ii. How many kilograms of apples did they pick together?

- Estimation:
- Actual Solution:

- f. During an election, Dominique counted 4 721 votes and 8 956 votes. The number of spoiled ballots was 1 639. How many were good votes?

1.6: Practice Answers

1. Solve the following word problems.
 - a. $86 \text{ kg} - 69 \text{ kg} = 17 \text{ kg}$
 - b. $\$49 + \$18 + \$12 + \$3 = \$82$
 - c. $94 + 86 + 79 = 259 \text{ books}$
 - d. $\$9\,989 - \$1\,785 = \$8\,204$
 - i. $509 \text{ kg} - 436 \text{ kg} = 73 \text{ kg more}$
 - ii. $509 \text{ kg} + 436 \text{ kg} = 945 \text{ kg altogether}$
 - e. 12 038 votes

Attribution

All figures in this chapter are from Topic F: Problem Solving in Adult Literacy Fundamental Mathematics: Book 2 – 2nd Edition by Tagami & Girard (2022), via BCcampus.

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1.7: Two- & Three-Digit Multipliers

<https://opentextbc.ca/alm3/> – From Book 3, Unit 1: Multiplication, include Topics B, C, D. Do not include the Unit 1 Review. From Unit 2: Division, include Topics B, C, D, E, F. Do not include Unit 2 Review.

When the multiplier is more than one digit, you use the same process and get partial products. You repeat the steps until you have multiplied by every digit, then add the partial products together.

Multiplying by Two-Digit Multipliers

Example A

$$24 \times 23 =$$

Step 1: Multiply by the ones digit in the multiplier.

Multiply 3 ones by 24 using the method you already know.
The first partial product is 72.

$$\begin{array}{r} 1 \\ 24 \\ \times 23 \\ \hline 72 \end{array}$$

Step 2: Multiply by the tens digit in the multiplier.

First, put a 0 to hold the ones place in your partial product. We are multiplying by a ten, so we hold the ones place.

$$2 \text{ tens} \times 4 \text{ ones} = 8 \text{ tens}$$

Write the 8 tens under the tens in your first partial product. It is very important to keep the columns straight – ones under one, tens under tens.

$$2 \text{ tens} \times 2 \text{ tens} = 4 \text{ hundreds}$$

Write the 4 hundreds in your partial product. The second partial product is 480.

$$\begin{array}{r} 1 \\ 24 \\ \times 23 \\ \hline 72 \\ 480 \end{array}$$

Step 3: Add the partial products together, being sure to add ones to ones, tens to tens, hundreds to hundreds. The sum is the final product.

Draw a line under the partial products. Add. Check your addition.

$$\begin{array}{r}
 1 \\
 24 \\
 \times 23 \\
 \hline
 72 \\
 + 480 \\
 \hline
 552
 \end{array}$$

Example B

$$36 \times 425 =$$

Step 1: Multiply by the ones digit in the multiplier.

$$6 \times 425 = 2\,550$$

$$\begin{array}{r}
 1\,3 \\
 425 \\
 \times 36 \\
 \hline
 2550
 \end{array}$$

Step 2: Multiply by the tens digit in the multiplier. First put a 0 to hold the ones place in the second partial product.

$$3 \text{ tens} \times 5 \text{ tens} = 15 \text{ tens} = 1 \text{ hundred and } 5 \text{ tens}$$

Write the 5 tens in the second partial product and carry the 1 hundred.

Now, you can see why it is best to cross out the numbers you carry as soon as you have added them to the product.

$$3 \text{ tens} \times 2 \text{ tens} = 6 \text{ hundreds}$$

$$6 \text{ hundreds} + 1 \text{ hundred (carried)} = 7 \text{ hundreds}$$

There is nothing to carry.

$$3 \text{ tens} \times 4 \text{ hundreds} = 12 \text{ thousands}$$

$$\begin{array}{r} 1 \\ 13 \\ 425 \\ \times \quad 36 \\ \hline 2550 \\ 12750 \end{array}$$

Step 3: Add the partial products together.

$$\begin{array}{r} 1 \\ 13 \\ 425 \\ \times \quad 36 \\ \hline 2550 \\ + \quad 12750 \\ \hline 15300 \end{array}$$

Multiplication Tips

Here are some tips to keep in mind while multiplying:

- Keeping the columns straight with ones under ones, tens under tens, and hundreds under hundreds is very important. Working on large-squared graphing paper using one digit per square is often helpful.
- $\text{tens} \times \text{tens} = \text{hundreds}$
 $\text{tens} \times \text{hundreds} = \text{thousands}$

Exercise 1

Multiply, being very careful to keep the columns straight when you write your partial products. Check your work using the answer key at the end of the exercise.

$$\begin{array}{r}
 84 \\
 \times 12 \\
 \hline
 168 \\
 + 840 \\
 \hline
 1008
 \end{array}$$

Example:

$$\begin{array}{r}
 73 \\
 \times 12 \\
 \hline
 \end{array}$$

a.

$$\begin{array}{r}
 92 \\
 \times 31 \\
 \hline
 \end{array}$$

f.

$$\begin{array}{r}
 42 \\
 \times 94 \\
 \hline
 \end{array}$$

k.

$$\begin{array}{r}
 50 \\
 \times 42 \\
 \hline
 \end{array}$$

b.

$$\begin{array}{r}
 91 \\
 \times 49 \\
 \hline
 \end{array}$$

g.

$$\begin{array}{r}
 80 \\
 \times 86 \\
 \hline
 \end{array}$$

l.

$$\begin{array}{r}
 62 \\
 \times 31 \\
 \hline
 \end{array}$$

c.

$$\begin{array}{r}
 72 \\
 \times 48 \\
 \hline
 \end{array}$$

h.

$$\begin{array}{r}
 31 \\
 \times 79 \\
 \hline
 \end{array}$$

m.

$$\begin{array}{r}
 61 \\
 \times 42 \\
 \hline
 \end{array}$$

d.

$$\begin{array}{r}
 53 \\
 \times 30 \\
 \hline
 \end{array}$$

i.

$$\begin{array}{r}
 54 \\
 \times 40 \\
 \hline
 \end{array}$$

n.

$$\begin{array}{r}
 91 \\
 \times 53 \\
 \hline
 \end{array}$$

e.

$$\begin{array}{r}
 41 \\
 \times 53 \\
 \hline
 \end{array}$$

j.

$$\begin{array}{r}
 61 \\
 \times 48 \\
 \hline
 \end{array}$$

o.

		60			84
p.	×	31	r.	×	56
		55			53
q.	×	73	s.	×	38

Exercise 1 Answers

a. 876	h. 3 456	o. 2 928
b. 2 100	i. 1 590	p. 1 860
c. 1 922	j. 2 173	q. 4 015
d. 2 562	k. 3 948	r. 4 704
e. 4 823	l. 6 880	s. 2 014
f. 2 852	m. 2 449	
g. 4 459	n. 2 160	

When the multiplier has a zero in the ones place, use the following shortcut.

Example C

$$\begin{array}{r} 48 \\ \times 80 \\ \hline \end{array}$$

Step 1: Place one zero in the product, which will hold the ones place.

$$0 \text{ ones} \times 48 = 0$$

Step 2: Multiply by the tens digit and write the product beside the zero.

$$\begin{array}{r} 48 \\ \times 80 \\ \hline 3840 \end{array}$$

Example D

$$\begin{array}{r} 97 \\ \times 20 \\ \hline 1940 \end{array}$$

Exercise 2

Find the products. Use the shortcut for multipliers with a zero in them.

Example:

$$\begin{array}{r} 76 \\ \times 70 \\ \hline 5320 \end{array}$$

$$\begin{array}{r} \text{a.} \quad \begin{array}{r} 52 \\ \times 70 \\ \hline \end{array} \quad \text{h.} \quad \begin{array}{r} 398 \\ \times 70 \\ \hline \end{array} \quad \text{o.} \quad \begin{array}{r} 248 \\ \times 70 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \text{b.} \quad \begin{array}{r} 91 \\ \times 70 \\ \hline \end{array} \quad \text{i.} \quad \begin{array}{r} 432 \\ \times 70 \\ \hline \end{array} \quad \text{p.} \quad \begin{array}{r} 6287 \\ \times 70 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \text{c.} \quad \begin{array}{r} 83 \\ \times 70 \\ \hline \end{array} \quad \text{j.} \quad \begin{array}{r} 863 \\ \times 70 \\ \hline \end{array} \quad \text{q.} \quad \begin{array}{r} 9025 \\ \times 70 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \text{d.} \quad \begin{array}{r} 49 \\ \times 70 \\ \hline \end{array} \quad \text{k.} \quad \begin{array}{r} 907 \\ \times 70 \\ \hline \end{array} \quad \text{r.} \quad \begin{array}{r} 8907 \\ \times 70 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \text{e.} \quad \begin{array}{r} 61 \\ \times 70 \\ \hline \end{array} \quad \text{l.} \quad \begin{array}{r} 503 \\ \times 70 \\ \hline \end{array} \quad \text{s.} \quad \begin{array}{r} 300 \\ \times 70 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \text{f.} \quad \begin{array}{r} 16 \\ \times 70 \\ \hline \end{array} \quad \text{m.} \quad \begin{array}{r} 452 \\ \times 70 \\ \hline \end{array} \quad \text{t.} \quad \begin{array}{r} 9075 \\ \times 70 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \text{g.} \quad \begin{array}{r} 36 \\ \times 70 \\ \hline \end{array} \quad \text{n.} \quad \begin{array}{r} 943 \\ \times 70 \\ \hline \end{array} \quad \text{u.} \quad \begin{array}{r} 3952 \\ \times 70 \\ \hline \end{array} \end{array}$$

$$\begin{array}{r} \text{v.} \quad \times \quad 1528 \\ \hline \quad \quad \quad 70 \end{array} \quad \begin{array}{r} \text{w.} \quad \times \quad 7106 \\ \hline \quad \quad \quad 70 \end{array}$$

Exercise 2 Answers

a. 520	i. 8 640	q. 541 500
b. 3 640	j. 43 150	r. 712 560
c. 4 980	k. 27 210	s. 27 000
d. 2 450	l. 20 120	t. 181 500
e. 1 830	m. 36 160	u. 118 560
f. 1 440	n. 66 010	v. 106 960
g. 2 880	o. 22 320	w. 71 060
h. 3 980	p. 251 480	

Multiplying by Three-Digit Multipliers

To multiply by three-digit multipliers, use the same method with one more part.

Example E

$$417 \times 368 =$$

$$\begin{array}{r} 417 \\ \times 368 \\ \hline 3336 \\ 25020 \\ + 125100 \\ \hline 153456 \end{array}$$

Step 1: Multiply by the 8 ones.

Step 2: Multiply the 6 tens; hold the ones place with 0.

Step 3: Multiply by the 3 hundreds. Put 00 to hold the ones and tens places in the third partial product.

$3 \text{ hundreds} \times 7 \text{ ones} = 21 \text{ hundreds} = 2 \text{ thousands and } 1 \text{ hundred}$

Write the 1 hundred and carry the 2 thousands.

$3 \text{ hundreds} \times 1 \text{ ten} = 3 \text{ thousands}$

$3 \text{ thousands} \times 2 \text{ thousands (carried)} = 5 \text{ thousands}$

$3 \text{ hundreds} \times 4 \text{ hundreds} = 12\textit{ten} \text{ thousands}$

Step 4: Add the partial products.

Exercise 3

Find the products.

	416		489		269			
a.	$\times$	213	f.	$\times$	578	k.	$\times$	912

	375		498		752			
b.	$\times$	291	g.	$\times$	123	l.	$\times$	697

	361		267		983			
c.	$\times$	475	h.	$\times$	854	m.	$\times$	357

	275		613		835			
d.	$\times$	863	i.	$\times$	368	n.	$\times$	148

	984		725		386			
e.	$\times$	469	j.	$\times$	547	o.	$\times$	296

Exercise 3 Answers

a. 88 608	f. 282 642	k. 245 328
b. 109 125	g. 61 254	l. 524 144
c. 171 475	h. 228 018	m. 350 931
d. 237 325	i. 225 584	n. 123 580
e. 461 496	j. 396 575	o. 114 256

You know to hold the ones place with a zero if the multiplier has a zero in the ones place. Use the same skill if the multiplier has a zero in the tens place.

Example F

$$927 \times 405 =$$

$$\begin{array}{r} 927 \\ \times 405 \\ \hline 4635 \\ + 370800 \\ \hline 375435 \end{array}$$

Step 1: Multiply by the 5 ones.

Step 2: Multiply by the 0 tens.

Hold the ones place with a 0.

$$0 \times 927 = 0$$

Place one zero in the tens place in the second partial product.

Step 3: Multiply by the 4 hundreds. The ones and tens places are already held by zeros. Start this partial product in the hundreds place on the same line.

Step 4: Add the partial products.

Exercise 4

Find the products.

Example:

$$\begin{array}{r} 22 \\ 698 \\ \times 301 \\ \hline 698 \\ + 209400 \\ \hline 210098 \end{array}$$

Multiplying by 10, 100, & 1 000

Exercise 5

Do the following questions and see if you can find the pattern.

$$\begin{array}{r} \text{a.} \quad \times \quad 83 \\ \times \quad 10 \\ \hline \end{array}$$

830

$$\begin{array}{r} \text{e.} \quad \times \quad 70 \\ \times \quad 10 \\ \hline \end{array}$$

$$\begin{array}{r} \text{i.} \quad \times \quad 45 \\ \times \quad 100 \\ \hline \end{array}$$

$$\begin{array}{r} \text{b.} \quad \times \quad 46 \\ \times \quad 10 \\ \hline \end{array}$$

$$\begin{array}{r} \text{f.} \quad \times \quad 129 \\ \times \quad 10 \\ \hline \end{array}$$

$$\begin{array}{r} \text{j.} \quad \times \quad 26 \\ \times \quad 100 \\ \hline \end{array}$$

$$\begin{array}{r} \text{c.} \quad \times \quad 97 \\ \times \quad 10 \\ \hline \end{array}$$

$$\begin{array}{r} \text{g.} \quad \times \quad 1852 \\ \times \quad 10 \\ \hline \end{array}$$

$$\begin{array}{r} \text{k.} \quad \times \quad 432 \\ \times \quad 100 \\ \hline \end{array}$$

$$\begin{array}{r} \text{d.} \quad \times \quad 123 \\ \times \quad 10 \\ \hline \end{array}$$

$$\begin{array}{r} \text{h.} \quad \times \quad 29871 \\ \times \quad 10 \\ \hline \end{array}$$

$$\begin{array}{r} \text{l.} \quad \times \quad 679 \\ \times \quad 100 \\ \hline \end{array}$$

	2482		23		2118
m.	$\times$	100	q.	$\times$	1000
	9037		452		2431
n.	$\times$	100	r.	$\times$	1000
	46207		207		23681
o.	$\times$	100	s.	$\times$	1000
	97512		348		48203
p.	$\times$	100	t.	$\times$	1000

Exercise 5 Answers

a. 830	i. 4 500	q. 23 000
b. 460	j. 2 600	r. 452 000
c. 970	k. 43 200	s. 207 000
d. 1 230	l. 67 900	t. 348 000
e. 700	m. 248 200	u. 2 118 000
f. 1 290	n. 903 700	v. 2 431 000
g. 18 520	o. 4 620 700	w. 23 681 000
h. 298 710	p. 9 751 200	x. 48 203 000

The pattern is:

When multiplying by 10, 100, 1 000, 10 000, and so on, place as many zeros to the right of the number as there are zeros in the 10, 100, 1 000, etc.

- To multiply by 10, put one zero after the number.
- To multiply by 100, put two zeros after the number.
- To multiply by 1 000, put three zeros after the number.

Exercise 6

Find the products using the short method. Do not rewrite the questions.

a. $12 \times 10 =$	i. 120	q. $1000 \times 87 =$
b. $10 \times 3175 =$	j. $100 \times 278 =$	r. $521 \times 1000 =$
c. $162 \times 10 =$	k. $9134 \times 100 =$	s. $1000 \times 68935 =$
d. $10 \times 53821 =$	l. $651 \times 100 =$	t. $1000 \times 8902 =$
e. $10 \times 123 =$	m. $100 \times 5169 =$	u. $1576 \times 1000 =$
f. $27342 \times 10 =$	n. $100 \times 24815 =$	v. $31584 \times 1000 =$
g. $10 \times 98 =$	o. $10 \times 905 =$	w. $1000 \times 426 =$
h. $1134 \times 10 =$	p. $45683 \times 10 =$	x. $72 \times 1000 =$

Exercise 6 Answers

a. 120	i. 1500	q. 87 000
b. 31 750	j. 27 800	r. 521 000
c. 1 620	k. 913 400	s. 68 935 000
d. 538 210	l. 65 100	t. 8 902 000
e. 1 230	m. 516 900	u. 1 576 000
f. 273 420	n. 2 481 500	v. 31 584 000
g. 980	o. 9 050	w. 426 000
h. 11 340	p. 456 830	x. 72 000

1.7: Practice Questions

1. Multiply these numbers.

	$\begin{array}{r} 47 \\ \times 39 \\ \hline \end{array}$		$\begin{array}{r} 678 \\ \times 39 \\ \hline \end{array}$		$\begin{array}{r} 8047 \\ \times 236 \\ \hline \end{array}$
a.		e.		i.	

	$\begin{array}{r} 58 \\ \times 93 \\ \hline \end{array}$		$\begin{array}{r} 4579 \\ \times 86 \\ \hline \end{array}$		$\begin{array}{r} 4238 \\ \times 197 \\ \hline \end{array}$
b.		f.		j.	

	$\begin{array}{r} 48 \\ \times 100 \\ \hline \end{array}$		$\begin{array}{r} 8703 \\ \times 93 \\ \hline \end{array}$		$\begin{array}{r} 8200 \\ \times 444 \\ \hline \end{array}$
c.		g.		k.	

	$\begin{array}{r} 982 \\ \times 1000 \\ \hline \end{array}$		$\begin{array}{r} 7390 \\ \times 85 \\ \hline \end{array}$		$\begin{array}{r} 7265 \\ \times 409 \\ \hline \end{array}$
d.		h.		l.	

1.7: Practice Answers

1. Multiply these numbers.

a. 1 833	e. 26 442	i. 1 899 092
b. 5 394	f. 393 794	j. 834 886
c. 4 800	g. 809 379	k. 3 640 800
d. 982 000	h. 628 150	l. 3 012 285

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1.8: Estimating Products

Before you practice estimating products, look at the easy multiplication that can be done when the factors end in zeros.

You already know how to quickly multiply by 10, 100, 1 000, and so on.

You can find the products in questions like these using regular multiplication, but here is a shortcut:

1. Count all the zeros at the end of the numbers in both factors.
2. Write down that many zeros at the end of the product.
3. Multiply the other digits in the usual way and put them before the zeros.

Example A

$$30 \times 500 =$$

1. How many zeros at the end of the factors? **3**
2. Write them down. **000**
3. Multiply the other digits and put them before the zeros. **$3 \times 5 = 15$**

$$30 \times 500 = 15\,000$$

Example B

$$400 \times 3\,000 =$$

1. How many zeros at the end of the factors? **5**
2. Write them down. **00 000**
3. Multiply the other digits and put them before the zeros. **$4 \times 3 = 12$**

$$400 \times 3\,000 = 1\,200\,000$$

Exercise 1

Find the products using the shortcut. Check your work using the answer key at the end of the exercise.

Example:

$$\begin{array}{r} 300 \\ \times 20 \\ \hline 6000 \end{array}$$

$$\begin{array}{r} 6000 \\ \times 200 \\ \hline \end{array}$$

a. $\times$

$$\begin{array}{r} 5000 \\ \times 50 \\ \hline \end{array}$$

h.

$$\begin{array}{r} 600 \\ \times 600 \\ \hline \end{array}$$

o.

$$\begin{array}{r} 210 \\ \times 20 \\ \hline \end{array}$$

b.

$$\begin{array}{r} 70 \\ \times 80 \\ \hline \end{array}$$

i.

$$\begin{array}{r} 40000 \\ \times 800 \\ \hline \end{array}$$

p.

$$\begin{array}{r} 800 \\ \times 600 \\ \hline \end{array}$$

c.

$$\begin{array}{r} 3000 \\ \times 700 \\ \hline \end{array}$$

j.

$$\begin{array}{r} 2400 \\ \times 70 \\ \hline \end{array}$$

q.

$$\begin{array}{r} 400 \\ \times 500 \\ \hline \end{array}$$

d.

$$\begin{array}{r} 50000 \\ \times 900 \\ \hline \end{array}$$

k.

$$\begin{array}{r} 390 \\ \times 40 \\ \hline \end{array}$$

r.

$$\begin{array}{r} 6000 \\ \times 90 \\ \hline \end{array}$$

e.

$$\begin{array}{r} 9000 \\ \times 8000 \\ \hline \end{array}$$

l.

$$\begin{array}{r} 7200 \\ \times 5000 \\ \hline \end{array}$$

s.

$$\begin{array}{r} 50000 \\ \times 6000 \\ \hline \end{array}$$

f.

$$\begin{array}{r} 60000 \\ \times 90 \\ \hline \end{array}$$

m.

$$\begin{array}{r} 7000 \\ \times 7000 \\ \hline \end{array}$$

t.

$$\begin{array}{r} 80000 \\ \times 30 \\ \hline \end{array}$$

g.

$$\begin{array}{r} 90000 \\ \times 2000 \\ \hline \end{array}$$

n.

$$\begin{array}{r} 61000 \\ \times 400 \\ \hline \end{array}$$

u.

$$\begin{array}{r} 5200 \\ \times 300 \\ \hline \end{array} \quad \begin{array}{r} 40 \\ \times 60 \\ \hline \end{array}$$

Exercise 1 Answers

a. 1 200 000	i. 5 600	q. 168 000
b. 4 200	j. 2 100 000	r. 15 600
c. 480 000	k. 45 000 000	s. 36 000 000
d. 200 000	l. 72 000 000	t. 49 000 000
e. 540 000	m. 5 400 000	u. 24 400 000
f. 300 000 000	n. 180 000 000	v. 1 560 000
g. 2 400 000	o. 360 000	w. 2 400
h. 250 000	p. 32 000 000	

To find an estimated product, round the factors before you multiply.
If a factor has only one digit, do not round it.

Example C

$$\begin{array}{r} 6258 \\ \times 3 \\ \hline \end{array}$$

6 258 rounds to 6 000. Leave 3 as is.

$$\begin{array}{r} 6000 \\ \times \quad 3 \\ \hline 18000 \end{array}$$

18 000 is the estimated product.

Example D

$$\begin{array}{r} 491 \\ \times \quad 24 \\ \hline \end{array}$$

491 rounds to 500. 24 rounds to 20.

$$\begin{array}{r} 500 \\ \times \quad 20 \\ \hline 10000 \end{array}$$

10 000 is the estimated product.

I.8: Practice Questions

1. Multiply these numbers.

a.	$\begin{array}{r} 600 \\ \times 70 \\ \hline \end{array}$	c.	$\begin{array}{r} 5000 \\ \times 600 \\ \hline \end{array}$	e.	$\begin{array}{r} 400 \\ \times 50 \\ \hline \end{array}$
b.	$\begin{array}{r} 9000 \\ \times 30 \\ \hline \end{array}$	d.	$\begin{array}{r} 3000 \\ \times 500 \\ \hline \end{array}$	f.	$\begin{array}{r} 8000 \\ \times 1000 \\ \hline \end{array}$

2. Find an estimated product.

a.	$\begin{array}{r} 87 \\ \times 23 \\ \hline \end{array}$	c.	$\begin{array}{r} 9421 \\ \times 75 \\ \hline \end{array}$	e.	$\begin{array}{r} 365 \\ \times 455 \\ \hline \end{array}$
b.	$\begin{array}{r} 268 \\ \times 25 \\ \hline \end{array}$	d.	$\begin{array}{r} 2632 \\ \times 49 \\ \hline \end{array}$	f.	$\begin{array}{r} 7264 \\ \times 590 \\ \hline \end{array}$

I.8: Practice Answers

1. Multiply these numbers.

a. 42 000	c. 3 000 000	e. 20 000
b. 270 000	d. 1500 000	f. 8 000 000

2. Find an estimated product.

- a. $90 \times 20 = 1\,800$ $9\,000 \times 80 = 720\,000$ $400 \times 500 = 200\,000$
b. $300 \times 30 = 9\,000$ $3\,000 \times 50 = 150\,000$ $7\,000 \times 600 = 4\,200\,000$

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1.9: Multiplication Problems

Multiplication problems usually give information for one unit and ask for information for many units. For example, you may be given an average distance for one hour and asked for a distance for six hours. You may be given the price for one item and asked for the price of 96 items.

Remember that multiplication is the fast addition of equal amounts. Keywords for addition may also point to multiplication.

Keywords pointing to multiplication include:

- altogether
- combined
- in all
- total
- the average is
- product
- how many
- how much
- each

Exercise 1

Do these problems by following the five problem-solving steps. Remember to circle the information and underline what is being asked. Be sure to write down your estimation

before you find the actual solution. Check your work using the answer key at the end of the exercise.

- a. Maria spent \$151 a month to ride to work. How much did it cost her to ride to work for the whole year? (1 year = 12 months)
- b. Fred travels 121 km a day on his delivery route. How far does he travel in 5 working days?
- c. Manuel buys 340 L of gas a month. In 6 months, how many litres of gas does Manuel buy?
- d. An apartment building has 16 apartments, each rented for \$870 a month. What is the total monthly rental income from this building?
- e. A farmer sells potatoes at \$53 per 50 kg sack. How much will he get for 75 sacks?
- f. Bob gets paid \$12 per hour. If he worked 39 hours last week and 24 hours this week, how much did he earn for the two weeks? (two operations)
- g. It costs \$35 260 per working day to run the factory. How much does it cost to run this factory for a month of 23 working days?
- h. The train has an average speed of 75 km an hour. How far does this train travel in 14 hours?
- i. Lee's sports car averages 18 km per litre. How far can she drive on 12 L of gasoline?
- j. Frank bought a used car and paid for it over 15 months. He made 15 monthly payments of \$325 each. How much did he pay?

Exercise 1 Answers

a. \$1 812	e. \$3 975	i. 216 km
b. 605 km	f. 63 hours, \$756	j. \$4 875
c. 2 040 L	g. \$810, 980	
d. \$13 920	h. 1 050 km	

1.9: Practice Answers

1. Solve these problems. Show all your work.
 - a. A freight train has 70 cars. Each car can hold 22 680 kilograms of cargo. How much cargo can the train hold in all?
 - b. The highway distance between Fernie and Edmonton is 621 kilometres. How many kilometres will a bus travel in 68 trips from Fernie to Edmonton?
 - c. A tanker truck made 275 trips in one year. The truck hauled 23 800 litres each time. How many litres did the truck haul during the year?
 - d. The college cafeteria hopes to serve 425 people each day. Estimate how many meals will be served if the cafeteria is open for 175 days.

1.9: Practice Answers

1. Solve these problems. Show all your work.

- a. 1 587 600 kilograms
- b. 42 228 kilometres
- c. 6 545 500 litres
- d. $400 \times 200 = 80\,000$ meals

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1.10: Divisibility

For example, 18 is divisible by 3 because $18 \div 3 = 6$. 18 is not divisible by 5 because $18 \div 5 = 3$ with a remainder of 3.

Divisibility by 2

A number is divisible by 2 if it ends in 0, 2, 4, 6 or 8.

Example A

438, 540, 256, 12, and 154 are all divisible by 2 because each number ends in a 0, 2, 4, 6, or 8.

Example B

351, 945, 849, 347, and 193 are not divisible by 2 because each number does not end in a 0, 2, 4, 6 or 8.

Exercise 1

Circle the numbers that are divisible by 2. Check your work using the answer key at the end of the exercise.

a. 22	e. 274	i. 2 437
b. 35	f. 345	j. 7 548
c. 17	g. 639	k. 6 754
d. 10	h. 456	l. 5 543

Exercise 1 Answers

The following numbers are divisible by 2: a, d, e, h, j, k.

Divisibility by 3

A number is divisible by 3 if the sum (total) of the digits is divisible by 3.

Example C

63

$$6 + 3 = 9$$

9 is divisible by three, so 63 is divisible by three.

Example D

148

$$1 + 4 + 8 = 13$$

13 is not divisible by three, so 148 is not divisible by 3.

Example E

5 892

$$5 + 8 + 9 + 2 = 24$$

Add again:

$$2 + 4 = 6$$

6 is divisible by 3, so 5 892 is divisible by 3.

Exercise 2

Circle the numbers that are divisible by 3.

- a. 27
- b. 35
- c. 81
- d. 94

- e. 274
- f. 581
- g. 564
- h. 316

- i. 3 175
- j. 1 458
- k. 1 890
- l. 3 934

Exercise 2 Answers

The following numbers are divisible by 3: a, c, g, j, k.

Divisibility by 5

A number is divisible by 5 if the number ends in 0 or 5.

Example F

290 is divisible by 5 because it ends in 0.

Example G

132 is not divisible by 5 because it does not end in 0 or 5.

Exercise 3

Circle the numbers that are divisible by 5.

a. 45	e. 800	i. 6 009
b. 84	f. 753	j. 6 375
c. 72	g. 672	k. 7 020
d. 90	h. 355	l. 1 704

Exercise 3 Answers

The following numbers are divisible by 5: a, d, e, h, j, k.

Exercise 4

For each number, identify if it is divisible by 2, 3, and 5.

a. 474	f. 831	k. 906
b. 615	g. 525	l. 634
c. 412	h. 350	m. 430
d. 865	i. 710	n. 275
e. 300	j. 429	

Exercise 4 Answers

- Divisible by 2: a, c, e, h, i, k, l, m.
- Divisible by 3: a, b, e, f, g, j, k.
- Divisible by 5: b, d, e, g, h, i, m, n.

Exercise 5

For each number, identify if it is divisible by 2, 3, and 5.

a. 3 585	b. 7 548	c. 5 890

d. 6 318
e. 3 905
f. 5 280
g. 1 760

h. 8 007
i. 6 752
j. 7 375
k. 5 523

l. 2 625
m. 8 956
n. 9 150

Exercise 5 Answers

- Divisible by 2: b, c, d, f, g, i, m, n.
- Divisible by 3: a, b, d, f, h, k, l, n.
- Divisible by 5: a, c, e, f, g, j, l, n.

Divisibility by 9

A number is divisible by 9 if the sum (total) of the digits is divisible by 9.

Example H

135

$$1 + 3 + 5 = 9$$

9 is divisible by 9, so 135 is divisible by 9.

Example I

7 578

$$7 + 5 + 7 + 8 = 27$$

27 is divisible by 9, so 7 578 is divisible by 9.

Example J

57 896

$$5 + 7 + 8 + 9 + 6 = 35$$

35 is not divisible by 9, so 57 896 is not divisible by 9.

Exercise 6

Circle the numbers that are divisible by 9.

a. 538	e. 6 213	i. 34 937
b. 783	f. 5 742	j. 39 402
c. 954	g. 7 083	k. 74 124
d. 762	h. 5 738	l. 45 683

Exercise 6 Answers

The following numbers are divisible by 9: b, c, f, g, j, k

Exercise 7

For each number, identify if it is divisible by 2, 3, 5, and 9.

a. 453	e. 712	i. 990
b. 320	f. 425	j. 210
c. 216	g. 630	
d. 726	h. 375	

Exercise 7 Answers

- Divisible by 2: b, c, d, e, g, i, j.
- Divisible by 3: a, c, d, g, h, i, j.
- Divisible by 5: b, f, g, h, i, j.
- Divisible by 9: c, g, i.

Exercise 8

For each number, identify if it is divisible by 2, 3, 5, and 9.

a. 837	e. 465	i. 8 022
b. 360	f. 1 002	j. 7 425
c. 648	g. 3 520	
d. 981	h. 6 435	

Exercise 8 Answers

- Divisible by 2: b, c, f, g, i.
- Divisible by 3: a, b, c, d, f, h, i, j.
- Divisible by 5: b, e, g, h, j
- Divisible by 9: a, b, c, d, h, j.

Exercise 9

For each number, identify if it is divisible by 2, 3, 5, and 9.

a. 1 200	e. 2 745	i. 1 368
b. 7 164	f. 4 320	j. 6 720
c. 3 681	g. 7 350	
d. 8 205	h. 4 000	

Exercise 9 Answers

- Divisible by 2: a, b, f, g, h, i, j.
- Divisible by 3: a, b, c, d, e, f, g, i, j.
- Divisible by 5: a, d, e, f, g, h, j.
- Divisible by 9: b, c, e, f, i.

1.10: Practice Questions

1. From the list of numbers, write the numbers that are divisible by the specified number.
48, 925, 1 467, 2 645, 5 534, 7 512, 31 183, 52 361
 - a. Which numbers are divisible by 2?
 - b. Which numbers are divisible by 3?
 - c. Which numbers are divisible by 5?
 - d. Which numbers are divisible by 9?
2. For each number, identify if it is divisible by 2, 3, 5, and 9.
 - a. 1 200
 - b. 7 164
 - c. 3 681
 - d. 8 205
 - e. 2 745

1.10: Practice Answers

1. From the list of numbers, write the numbers that are divisible by the specified number.
 - a. 48, 7 512, 5 534
 - b. 48, 1 467, 7 512
 - c. 925, 2 645
 - d. 1 467
2. For each number, identify if it is divisible by 2, 3, 5, and 9.
 - Divisible by 2: a, b.
 - Divisible by 3: a, b, c, d, e.

- Divisible by 5: a, d, e.
- Divisible by 9: b, c, e.

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I.II: Dividing Larger Numbers by One-Digit Divisors

Several methods are used to divide larger numbers. This book will only teach one method. If you have learned a different method for dividing, ask your instructor to review it with you. You can use the practice exercises in this workbook using whichever method you prefer.

One-Digit Divisors Without Remainders

Division has four steps, which are repeated until the dividend is completely divided. Work through the three examples which show these steps.

Here are the division steps:

1. Divide.
2. Multiply.
3. Subtract and compare the remainder to the divisor.
4. Bring down the next digit in the dividend and repeat.

Example A

$$294 \div 7 =$$

Rewrite as $7 \overline{)294}$

Step 1: Divide.

- a. Find a trial quotient using multiplication tables or division facts.
- b. Look at the dividend one digit at a time.
 - i. The first digit is a 2, which is really 2 hundreds.
 - ii. Will 7 go into 2? Can you divide 2 by 7? No.
 - iii. Look at the first 2 digits, 29, which is really 29 tens.
 - iv. Will 7 go into 29? Yes. $4 \times 7 = 28$.
- c. The first number in the quotient is 4. Place the 4 in the quotient directly above the 9 tens. The 4 is 4 tens in the quotient.

$$\begin{array}{r} 4 \\ 7 \overline{)294} \end{array}$$

Step 2: Multiply.

$$4 \times 7 = 28$$

Write the 28 under the 29. Draw a line.

$$\begin{array}{r} 4 \\ 7 \overline{)294} \\ \underline{28} \end{array}$$

Step 3: Subtract.

$$29 - 28 = 1 \text{ (ten)}$$

Check $1 < 7$.

$$\begin{array}{r} 4 \\ 7 \overline{)294} \\ \underline{-28} \\ 1 \end{array}$$

Step 4: Bring down the next number in the dividend.

- a. Bring down the next number in the dividend (4) and put it beside the result of the subtraction you just completed (1).
- b. Together, they make 14. This 14 is the number that you must now divide.

$$\begin{array}{r} 4 \\ 7 \overline{)294} \\ \underline{-28} \\ 14 \end{array}$$

Repeat Step 1: Divide.

$$14 \div 7 = 2$$

Put the 2 in the quotient right after the 4 in the ones place.

$$\begin{array}{r} 42 \\ 7 \overline{)294} \\ -28 \\ \hline 14 \end{array}$$

Repeat Step 2: Multiply.

$$2 \times 7 = 14$$

Write the 14 under the 14. Draw a line.

Repeat Step 3: Subtract.

$$14 - 14 = 0.$$

There is 0 remainder.

Check: $0 < 7$.

$$\begin{array}{r} 42 \\ 7 \overline{)294} \\ -28 \\ \hline 14 \\ 14 \\ \hline \end{array}$$

There are no more numbers in the dividend to bring down, so you are done.

$$294 \div 7 = 42$$

Example B

$$128 \div 2 =$$

$$\begin{array}{r} 64 \\ 2 \overline{)128} \\ \underline{-12} \\ 08 \\ \underline{-8} \\ 0 \end{array}$$

Step 1: Divide.

- a. Can 2 go into 1? No.
- b. Can 2 go into 12? Yes.
- c. How many times?
 - $2 \times 6 = 12$
 - $12 \div 2 = 6$
- d. The first number in the trial quotient is 6. Put the 6 in the quotient directly above the 2 tens dividend.

Step 2: Multiply.

$$6 \times 2 = 12$$

Write the 12 under the 12. Draw a line.

Step 3: Subtract.

$$12 - 12 = 0$$

Check: $0 < 12$

Step 4: Bring down the next number in the dividend.

Bring down the next number in the dividend (8) and put it beside the result of the subtraction you just completed (0).

8 is now the number to be divided.

Repeat Steps 1 to 4.

Repeat Step 1: Divide.

$$8 \div 2 = 4$$

Repeat Step 2: Multiply.

$$4 \times 2 = 8$$

Repeat Step 3: Subtract.

$$8 - 8 = 0$$

Check: $0 < 2$

Repeat Step 4: Bring down the next digit.

There are no more digits in the dividend.

$$128 \div 2 = 64$$

Exercise 1

Find the quotients (divide, multiply, subtract, compare).
Check your work using the answer key at the end of the exercise.

a. $4 \overline{)364}$

b. $2 \overline{)144}$

c. $5 \overline{)455}$

d. $7 \overline{)651}$

e. $8 \overline{)144}$

f. $2 \overline{)166}$

g. $7 \overline{)588}$

h. $2 \overline{)196}$

i. $5 \overline{)230}$

j. $8 \overline{)584}$

k. $6 \overline{)366}$

l. $4 \overline{)244}$

m. $5 \overline{)375}$

n. $8 \overline{)200}$

o. $2 \overline{)628}$

p. $7 \overline{)357}$

q. $9 \overline{)837}$

r. $8 \overline{)248}$

s. $8 \overline{)312}$

t. $7 \overline{)462}$

u. $5 \overline{)295}$

v. $6 \overline{)384}$

w. $2 \overline{)276}$

x. $4 \overline{)372}$

Exercise 1 Answers

a. 91

b. 72

c. 91

d. 93

e. 18

f. 83

g. 84

h. 98

i. 46

j. 73

k. 61

l. 61

m. 75

n. 25

o. 314

p. 51

q. 93

r. 31

s. 39

t. 66

u. 59

v. 64

w. 138

x. 93

Example C

$$856 \div 8 =$$

$$8 \overline{)856}$$

Step 1: Divide.

Does 8 go into 8? Yes.

$$8 \div 8 = 1$$

$$\begin{array}{r} 1 \\ 8 \overline{)856} \end{array}$$

Step 2: Multiply.

$$1 \times 8 = 8$$

$$\begin{array}{r} 1 \\ 8 \overline{)856} \\ 8 \end{array}$$

Step 3: Subtract.

$$8 - 8 = 0$$

Check $0 < 8$.

$$\begin{array}{r}
 1 \\
 8 \overline{)856} \\
 \underline{-8} \\
 0
 \end{array}$$

Step 4: Bring down the next digit.

5 is now the number to be divided.

$$\begin{array}{r}
 1 \\
 8 \overline{)856} \\
 \underline{-8} \\
 05
 \end{array}$$

Repeat Steps 1 to 4.

Repeat Step 1: Divide.

Does 8 go into 5? No.

You must put a zero to hold the place in the quotient. If a digit is brought down, a digit must be placed in the quotient.

Repeat Step 2: Multiply.

$$0 \times 8 = 0$$

$$\begin{array}{r}
 10 \\
 8 \overline{)856} \\
 \underline{-8} \\
 05
 \end{array}$$

Repeat Step 3: Subtract.

$$5 - 0 = 5$$

Check $5 < 8$.

$$\begin{array}{r} 10 \\ 8 \overline{)856} \\ -8 \\ \hline 05 \\ -0 \\ \hline 5 \end{array}$$

Repeat Step 4: Bring down the next digit.

56 is now the number to be divided.

$$\begin{array}{r} 10 \\ 8 \overline{)856} \\ -8 \\ \hline 05 \\ -0 \\ \hline 56 \end{array}$$

Repeat Steps 1 to 4 again.

Repeat Step 1: Divide.

Does 8 go into 56? Yes.

$$56 \div 8 = 7$$

Write 7 in the quotient in the ones place above the 6 in the dividend.

$$\begin{array}{r}
 107 \\
 8 \overline{)856} \\
 \underline{-8} \\
 05 \\
 \underline{-0} \\
 56
 \end{array}$$

Repeat Step 2: Multiply.

$$7 \times 8 = 56$$

$$\begin{array}{r}
 107 \\
 8 \overline{)856} \\
 \underline{-8} \\
 05 \\
 \underline{-0} \\
 56 \\
 56
 \end{array}$$

Repeat Step 3: Subtract.

$$56 - 56 = 0$$

Check $0 < 8$.

$$\begin{array}{r}
 107 \\
 8 \overline{)856} \\
 \underline{-8} \\
 05 \\
 \underline{-0} \\
 56 \\
 \underline{-56} \\
 0
 \end{array}$$

Repeat Step 4: Bring down the next digit.

No more digits.

$$856 \div 8 = 107$$

Exercise 2

Find the quotients (divide, multiply, subtract, compare).

a. $7 \overline{)721}$

d. $3 \overline{)621}$

g. $7 \overline{)714}$

b. $9 \overline{)954}$

e. $4 \overline{)824}$

h. $2 \overline{)416}$

c. $3 \overline{)927}$

f. $9 \overline{)972}$

i. $5 \overline{)540}$

j. $6 \overline{)654}$

k. $8 \overline{)832}$

l. $4 \overline{)436}$

m. $5 \overline{)515}$

n. $2 \overline{)814}$

o. $6 \overline{)648}$

p. $8 \overline{)856}$

q. $5 \overline{)525}$

r. $7 \overline{)763}$

s. $9 \overline{)945}$

t. $3 \overline{)315}$

u. $8 \overline{)872}$

v. $4 \overline{)416}$

w. $6 \overline{)618}$

x. $2 \overline{)612}$

Exercise 2 Answers

a. 103

b. 106

c. 309

d. 207

e. 206

f. 108

g. 102

h. 208

i. 108

j. 109

k. 104

l. 109

m. 103

n. 407

o. 108

p. 107

q. 105

r. 109

s. 105

t. 105

u. 109

v. 104

w. 103

x. 306

One-Digit Divisors With Remainders

Divide the same way that you have been learning. Often, there is a remainder after the last subtraction. Write it with the quotient as you already know how to do.

Example D

$$259 \div 8 =$$

$$\begin{array}{r} 32 \text{ R}3 \\ 8 \overline{)259} \\ \underline{-24} \\ 19 \\ \underline{-16} \\ 3 \end{array}$$

$$259 \div 8 = 32 \text{ R}3$$

Exercise 3

Divide and show any remainders.

a. $20 \overline{)93}$

b. $3 \overline{)52}$

c. $5 \overline{)94}$

d. $7 \overline{)74}$

e. $4 \overline{)95}$

f. $9 \overline{)96}$

g. $6 \overline{)97}$

h. $8 \overline{)99}$

i. $9 \overline{)98}$

j. $4 \overline{)59}$

k. $6 \overline{)76}$

l. $3 \overline{)79}$

m. $7 \overline{)96}$

n. $5 \overline{)57}$

o. $2 \overline{)47}$

p. $8 \overline{)91}$

q. $7 \overline{)89}$

r. $6 \overline{)82}$

s. $5 \overline{)67}$

t. $2 \overline{)85}$

u. $4 \overline{)71}$

v. $3 \overline{)65}$

w. $9 \overline{)92}$

x. $8 \overline{)94}$

Exercise 3 Answers

a. 46 R1

b. 17 R1

c. 18 R4

d. 10 R4

e. 23 R3

f. 10 R6

g. 16 R1

h. 12 R3

i. 10 R8

j. 14 R3

k. 12 R4

l. 26 R1

m. 13 R5

n. 11 R2

o. 23 R1

p. 11 R3

q. 12 R5

r. 13 R4

s. 13 R2

t. 42 R1

u. 17 R3

v. 21 R2

w. 10 R2

x. 11 R6

Check Division by Multiplying

To check your division, do the following:

1. Multiply the quotient by the divisor.
2. Add on any remainder.

The product will equal the dividend if your arithmetic is correct.

Example E

$$\begin{array}{r} 52 \text{ R}1 \\ 9 \overline{)469} \\ \underline{-45} \\ 19 \\ \underline{-18} \\ 1 \end{array} \qquad \begin{array}{r} 52 \\ \times 9 \\ \hline 468 \end{array}$$
$$468 + 1 = 469$$

Exercise 4

Divide and check your answer by multiplying.

a. $7 \overline{)709}$

c. $5 \overline{)538}$

e. $9 \overline{)406}$

b. $2 \overline{)423}$

d. $4 \overline{)609}$

f. $6 \overline{)125}$

g. $3 \overline{)605}$

h. $9 \overline{)928}$

i. $3 \overline{)962}$

j. $4 \overline{)805}$

k. $8 \overline{)301}$

l. $2 \overline{)807}$

m. $6 \overline{)725}$

n. $7 \overline{)320}$

o. $9 \overline{)140}$

p. $8 \overline{)483}$

q. $2 \overline{)197}$

r. $6 \overline{)307}$

s. $5 \overline{)504}$

t. $8 \overline{)709}$

u. $7 \overline{)876}$

v. $4 \overline{)101}$

w. $3 \overline{)269}$

x. $5 \overline{)473}$

Exercise 4 Answers

a. 101 R2

b. 211 R1

c. 107 R3

d. 152 R1

e. 45 R1

f. 20 R5

g. 201 R2

h. 103 R1

i. 320 R2

j. 201 R1

k. 37 R5

l. 403 R1

m. 120 R5

n. 45 R5

o. 15 R5

p. 60 R3

q. 98 R1

r. 51 R1

s. 100 R4

t. 88 R5

u. 125 R1

v. 25 R1

w. 89 R2

x. 94 R3

I.II: Practice Questions

1. Find the quotient.

a. $6 \overline{)96}$

c. $7 \overline{)91}$

e. $5 \overline{)94}$

b. $4 \overline{)92}$

d. $2 \overline{)93}$

f. $3 \overline{)52}$

2. Divide.

a. $7 \overline{)182}$

c. $6 \overline{)162}$

e. $4 \overline{)184}$

b. $8 \overline{)736}$

d. $5 \overline{)295}$

f. $9 \overline{)576}$

3. Divide and show your check for each answer.

a. $9 \overline{)705}$

c. $7 \overline{)899}$

e. $8 \overline{)876}$

b. $4 \overline{)257}$

d. $5 \overline{)538}$

f. $6 \overline{)628}$

I.II: Practice Answers

1. Find the quotient.

a. 16

c. 13

e. 18 R4

b. 23

d. 46 R1

f. 17 R1

2. Divide.

a. 26

c. 27

e. 46

b. 92

d. 59

f. 64

3. Divide and show your check for each answer.

a. 78 R3

b. 64 R1

c. 128 R3

d. 107 R3

e. 109 R4

f. 104 R4

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1.12: Dividing by Two & Three Digit Divisors

Finding Trial Quotients

When dividing by 2-digit numbers, you will need to estimate the quotient. This guess is called a trial quotient.

Example A

$$624 \div 24$$

Rewrite as $24 \overline{)624}$

Step 1: Divide.

Think: $2 \overline{)6}$ is 3.

So, $24 \overline{)62}$ is about 3.

Step 2: Multiply and subtract.

$$\begin{array}{r} 3 \\ 24 \overline{)624} \\ \underline{72} \end{array}$$

Since $72 > 62$, 3 is too large.

Step 3: Try a smaller number, multiply and subtract.

$$\begin{array}{r} 2 \\ 24 \overline{)624} \\ \underline{48} \\ 14 \end{array}$$

Since $4 < 24$, 2 is correct.

Step 4: Finish the problem.

Example B

$$630 \div 15$$

Rewrite as $15 \overline{)630}$

Step 1: Divide.

15 rounds to 20.

Think: $2 \overline{)6}$ is 3.

So, $15 \overline{)63}$ is about 3.

Step 2: Multiply and subtract.

$$\begin{array}{r} 3 \\ 15 \overline{)630} \\ \underline{45} \\ 18 \end{array}$$

Since $18 > 15$, 3 is too small.

Step 3: Try a larger number, multiply and subtract.

$$\begin{array}{r} 4 \\ 24 \overline{)630} \\ \underline{60} \\ 3 \end{array}$$

Since $3 < 15$, 4 is correct.

Step 4: Finish the problem.

Exercise 1

Each question has a trial quotient that is either too large or too small. Multiply. Write too large or too small on the line. Then, write the correct trial quotient beside. Check your work using the answer key at the end of the exercise.

$$\begin{array}{r} 2 \\ 25 \overline{)475} \end{array}$$

$$\begin{array}{r} 3 \\ 15 \overline{)682} \end{array}$$

$$\begin{array}{r} 5 \\ 18 \overline{)813} \end{array}$$

$$\begin{array}{r} 2 \\ 25 \overline{)810} \end{array}$$

$$\begin{array}{r} 3 \\ 33 \overline{)891} \end{array}$$

$$\begin{array}{r} 3 \\ 18 \overline{)819} \end{array}$$

$$\begin{array}{r} 3 \\ 27 \overline{)727} \end{array}$$

$$\begin{array}{r} 2 \\ 35 \overline{)652} \end{array}$$

$$\begin{array}{r} 3 \\ 25 \overline{)650} \end{array}$$

$$\begin{array}{r} 4 \\ 34 \overline{)176} \end{array}$$

$$\begin{array}{r} 4 \\ 12 \overline{)420} \end{array}$$

$$\begin{array}{r} 2 \\ 43 \overline{)801} \end{array}$$

$$\begin{array}{r} 3 \\ 31 \overline{)899} \end{array}$$

$$\begin{array}{r} 4 \\ 18 \overline{)648} \end{array}$$

$$\begin{array}{r} 4 \\ 27 \overline{)946} \end{array}$$

$$\begin{array}{r} 3 \\ 23 \overline{)943} \end{array}$$

$$\begin{array}{r} 3 \\ 24 \overline{)578} \end{array}$$

$$\begin{array}{r} 2 \\ 29 \overline{)406} \end{array}$$

$$\begin{array}{r} 2 \\ 48 \overline{)892} \end{array}$$

$$\begin{array}{r} 2 \\ 28 \overline{)534} \end{array}$$

$$\begin{array}{r} 3 \\ 37 \overline{)939} \end{array}$$

$$\begin{array}{r} 2 \\ 28 \overline{)854} \end{array}$$

Exercise 1 Answers

- a. too large, 1
- b. too small, 4
- c. too large, 4
- d. too small, 3
- e. too large, 2
- f. too small, 4
- g. too large, 2
- h. too large 1

- i. too large, 2
- j. too small, 5
- k. too large, 3
- l. too large, 1
- m. too. large 2
- n. too large, 3
- o. too large, 3
- p. too small, 4

- q. too large, 2
- r. too large, 1
- s. too large, 1
- t. too large 1
- u. too large, 2
- v. too small, 3

Example C

$$78 \overline{)2706}$$

Since 78 rounds to 80, think $8 \overline{)27}$. 8 goes into 27 $\approx$ 3. 3 would be a good trial quotient.

$$\begin{array}{r} 3 \\ 78 \overline{)2706} \\ \underline{234} \\ 36 \end{array}$$

Since $36 < 78$, 3 is a good trial quotient.

Example D

$$27 \overline{)2205}$$

Since 27 rounds to 30, think $3\overline{)22}$. 3 goes into $22 \approx 7$. 7 would be a good trial quotient.

$$\begin{array}{r} 7 \\ 27 \overline{)2205} \\ \underline{189} \\ 31 \end{array}$$

Since $31 > 27$, so 7 is too small. A better trial quotient would be 8.

Exercise 2

Find the first digit in the trial quotient.

Example: $43 \overline{)1772}$ $4 \overline{)17}$ $1 < 4$

$$\begin{array}{r} 4 \\ 4 \overline{)17} \\ \underline{16} \\ 1 \end{array}$$

a. $64 \overline{)3276}$ b. $28 \overline{)6008}$ c. $33 \overline{)2731}$

d. $59 \overline{)4164}$

e. $75 \overline{)2420}$

f. $54 \overline{)3316}$

g. $38 \overline{)2759}$

h. $46 \overline{)387}$

i. $35 \overline{)3316}$

j. $83 \overline{)7237}$

k. $77 \overline{)6763}$

l. $93 \overline{)3724}$

m. $52 \overline{)4690}$

n. $86 \overline{)2089}$

o. $26 \overline{)1417}$

p. $72 \overline{)1462}$

q. $27 \overline{)6939}$

r. $32 \overline{)7840}$

s. $24 \overline{)7605}$

t. $16 \overline{)8640}$

u. $45 \overline{)3060}$

v. $38 \overline{)2158}$

w. $42 \overline{)1491}$

Exercise 2 Answers

a. 5

b. 2

c. 8

d. 7

e. 3

f. 6

g. 7

h. 8

i. 4

j. 8

k. 8

l. 4

m. 9

n. 2

o. 5

p. 2

q. 2

r. 2

s. 3

t. 5

u. 6

v. 5

w. 3

Two Digit Divisors

Dividing by large divisors is a challenge!

You must **estimate** how many times one number will divide into another. Do these questions using a pencil and have an eraser close by. You will use the same steps that you already know.

Example E

$$964 \div 75 =$$

Step 1: Divide.

1. Does 75 go into 9? **NO**
2. Does 75 go into 96? **YES**
3. Estimate:
 - Round 75 to 80 – think “8”
 - Round 96 to 100 – think “10”
 - How many 8’s in 10? ($8 \times 1 = 8$, $10 - 8 = 2$)
 - The estimate for the first digit in the trial quotient is 1.
4. Write **1** in the quotient above the 6 tens.

$$\begin{array}{r} 1 \\ 75 \overline{)964} \end{array}$$

Step 2: Multiply.

$$1 \times 75 = 75$$

Write 75 under 96.

$$\begin{array}{r} 1 \\ 75 \overline{)964} \\ \underline{75} \end{array}$$

Step 3: Subtract.

$$96 - 75 = 21$$

Check $21 < 75$? **YES!**

$$\begin{array}{r} 1 \\ 75 \overline{)964} \\ \underline{75} \\ 21 \end{array}$$

Step 4: Bring down the next digit in the dividend. 214 is now the number to be divided.

$$\begin{array}{r} 1 \\ 75 \overline{)964} \\ \underline{75 \downarrow} \\ 214 \end{array}$$

Repeat Steps 1 to 4.

Repeat Step 1: Divide.

1. Does 75 go into 214? **YES**
2. Estimate 75 as 80 – think “8”:

- Estimate 214 as 200 – think “20”
 - 8 goes into 20 2 times ($8 \times 2 = 16$, so 20 $\div$ 8 $\approx$ 2)
3. The estimate for the second digit in the trial quotient is 2. Write 2 in the quotient above the 4 in the dividend.

$$\begin{array}{r}
 12 \\
 75 \overline{)964} \\
 \underline{75 \downarrow} \\
 214
 \end{array}$$

Repeat Step 2: Multiply.

$$2 \times 75 = 150$$

Write 150 under the 214.

$$\begin{array}{r}
 12 \\
 75 \overline{)964} \\
 \underline{75 \downarrow} \\
 214 \\
 \underline{150}
 \end{array}$$

Repeat Step 3: Subtract and check that the remainder is less than the divisor.

$$\begin{array}{r}
 12 \\
 75 \overline{)964} \\
 \underline{75 \downarrow} \\
 214 \\
 \underline{150} \\
 \hline
 \end{array}$$

Repeat Step 4: Bring down the next digit.

No more digits in dividend.

Check:

$$\begin{array}{r}
 75 \\
 \times 12 \\
 \hline
 150 \\
 750 \\
 \hline
 900 \\
 + 64 \\
 \hline
 964
 \end{array}$$

Exercise 3

Carefully divide these questions. Be careful to keep the hundreds in line with the hundreds, the tens with the tens, and so on. You might want to use squared paper for long division.

a. $10 \overline{)720}$

b. $12 \overline{)564}$

c. $21 \overline{)882}$

d. $22 \overline{)946}$

e. $32 \overline{)1632}$

f. $23 \overline{)943}$

g. $62 \overline{)2528}$

h. $71 \overline{)2414}$

i. $24 \overline{)7578}$

j. $82 \overline{)2958}$

k. $18 \overline{)6250}$

l. $25 \overline{)1550}$

m. $19 \overline{)9595}$

n. $47 \overline{)3854}$

o. $58 \overline{)6500}$

p. $24 \overline{)9648}$

q. $49 \overline{)1312}$

r. $67 \overline{)7683}$

Exercise 3 Answers

a. 72

b. 47

c. 42

d. 43

e. 51

f. 41

g. 40 R48

h. 34

i. 24 R2

j. 36 R6

k. 347 R4

l. 62

m. 505

n. 82

o. 112 R4

p. 402

q. 26 R38

r. 114 R45

If the estimate for your trial quotient is too large, the result of the multiplication will be larger than the numbers in the dividend.

1. **Divide:** Trial estimate is 4.

2. **Multiply:**

- $4 \times 23 = 92$

- 92 is larger than 78, so 4 is too large an estimate. Erase it.

Try 3. $3 \times 23 = 69$

- 3 is the correct estimate and you can complete the division.

$$\begin{array}{r} 23 \overline{)784} \\ \underline{69} \\ 94 \\ \underline{92} \\ 2 \end{array}$$

34 R2

If the estimate is too small, the result of the subtraction will be larger than the divisor.

1. **Divide:** Trial estimate is 5.

2. **Multiply:** $5 \times 72 = 360$

3. **Subtract:**

- $448 - 72 = 360$

- Check $88 < 72$? **NO**, 88 is greater than 72.

- So 5 is too small. Erase it and use a larger number. 6 will be a better estimate.

4. **Divide:** $448 \div 72 \approx 6$

5. **Multiply:** $6 \times 72 = 432$

6. **Subtract:**

- $448 - 432 = 16$

- Check $16 < 72$? **YES!**

7. **Bring down the next digit** and complete the division.

$$\begin{array}{r}
 72 \overline{)4487} \\
 \underline{360} \\
 88
 \end{array}
 \qquad
 \begin{array}{r}
 5 \\
 72 \overline{)4487} \\
 \underline{360} \\
 88
 \end{array}
 \qquad
 \begin{array}{r}
 62 \text{ R}23 \\
 72 \overline{)4487} \\
 \underline{432} \downarrow \\
 167 \\
 \underline{144} \\
 23
 \end{array}$$

Exercise 4

Divide and check your work by multiplying.

a. $18 \overline{)648}$	f. $94 \overline{)8126}$	k. $79 \overline{)2765}$
b. $26 \overline{)6766}$	g. $20 \overline{)4060}$	l. $42 \overline{)8442}$
c. $52 \overline{)1968}$	h. $47 \overline{)1728}$	m. $57 \overline{)9144}$
d. $84 \overline{)8640}$	i. $33 \overline{)1886}$	n. $96 \overline{)20160}$
e. $72 \overline{)2883}$	j. $25 \overline{)5750}$	o. $75 \overline{)23550}$

Exercise 4 Answers

a. 36	f. 86 R42	k. 35
b. 260 R6	g. 203	l. 201
c. 37 R44	h. 36 R36	m. 160 R24
d. 102 R72	i. 57 R5	n. 210
e. 40 R3	j. 230	o. 314

Dividing by 10, 100, 1 000...What Is The Pattern?

When You Divide by 10

The ones digit in the dividend becomes the remainder.

$$10 \overline{)324} = 32R4$$

The other numbers in the dividend stay the same, but each digit is one place value less.

- The hundreds become tens.
- The tens become ones.
- The ones become the remainder.

Exercise 5

Find the quotients. Look for the pattern.

a. $10 \overline{)46}$

d. $10 \overline{)832}$

g. $10 \overline{)2457}$

b. $10 \overline{)75}$

e. $10 \overline{)674}$

h. $10 \overline{)3685}$

c. $10 \overline{)136}$

f. $10 \overline{)952}$

Exercise 5 Answers

- a. 4 R6
- b. 7 R5
- c. 13 R6

- d. 83 R2
- e. 67 R4
- f. 95 R2

- g. 245 R7
- h. 368 R5

When You Divide by 100

The ones and tens digits in the dividend become the remainder.

The other digits in the dividend stay the same but each digit is **two** places less:

- The thousands become tens.
- The hundreds become ones.
- The tens and ones become the remainder.

Exercise 6

Find the quotients. Look for the pattern when you divide.

a. $100 \overline{)386}$

b. $100 \overline{)995}$

c. $100 \overline{)269}$

$$\begin{array}{lll} \text{d. } 100 \overline{)175} & \text{f. } 100 \overline{)4671} & \text{h. } 100 \overline{)43821} \\ \text{e. } 100 \overline{)2948} & \text{g. } 100 \overline{)92045} & \end{array}$$

Exercise 6 Answers

- a. 3 R86
- b. 9 R95
- c. 2 R69

- d. 1 R75
- e. 29 R48
- f. 46 R71

- g. 920 R45
- h. 438 R21

When You Divide by 1000

The ones, tens, and hundreds digits become the remainder.

The other digits stay the same but are **three** place values less:

- The thousands become ones.
- The ten thousands become tens.
- The hundred thousands become hundreds.

Exercise 7

Find the quotients.

$$\begin{array}{lll} \text{a. } 1000 \overline{)2398} & \text{c. } 1000 \overline{)4835} & \text{e. } 1000 \overline{)82405} \\ \text{b. } 1000 \overline{)6475} & \text{d. } 1000 \overline{)63291} & \text{f. } 1000 \overline{)293591} \end{array}$$

Exercise 7 Answers

$$\begin{array}{lll} \text{a. } 2 \text{ R}398 & \text{c. } 4 \text{ R}835 & \text{e. } 82 \text{ R}405 \\ \text{b. } 6 \text{ R}475 & \text{d. } 63 \text{ R}291 & \text{f. } 293 \text{ R}591 \end{array}$$

Three Digit Divisors

If the divisor has three digits, use the method you know for two-digit divisors, but estimate the divisor to the nearest hundred to find the trial quotient. Be very **careful** to put the first digit in the quotient in the correct place.

Example F

$$17902 \div 381 =$$

Step 1: Divide.

1. Does 381 go into 1? **NO**
2. Does 381 go into 17? **NO**
3. Does 381 go into 179? **NO**
4. Does 381 go into 1 790? **YES**
5. Estimate 381 as 400. Think 4.
6. Estimate 1 790 as 1 800. Think 18.
7. 4 goes into 18 $\approx$ 4 times ($4 \times 4 = 16$)
8. Your estimate is 4. Write 4 in the quotient above the 0 in the dividend.

$$\begin{array}{r} 4 \\ 381 \overline{)17902} \end{array}$$

Step 2: Multiply.

$$4 \times 381 = 1524$$

$$\begin{array}{r} 4 \\ 381 \overline{)17902} \\ \underline{1524} \end{array}$$

Step 3: Subtract.

$$1\ 790 - 1\ 524 = 266$$

Check: $266 < 381$? **YES!**

$$\begin{array}{r} 4 \\ 381 \overline{)17902} \\ \underline{1524} \\ 266 \end{array}$$

Step 4: Bring down the 2.

2 662 is now the number to be divided

$$\begin{array}{r} 4 \\ 381 \overline{)17902} \\ \underline{1524} \downarrow \\ 2662 \end{array}$$

Repeat Steps 1 to 4.

Repeat Step 1: Divide.

$$2\ 662 \div 381 =$$

1. Estimate 381 as 400. Think of 4.
2. Estimate 2 662 as 2 700. Think 27.
3. 4 goes into 27 $\approx$ 6 times ($4 \times 6 = 24$).
4. Place this estimate in the quotient above the 2

$$\begin{array}{r}
 4 \\
 381 \overline{)17902} \\
 \underline{1524 \downarrow} \\
 2662
 \end{array}$$

Repeat Step 2: Multiply.

$$6 \times 381 = 2\,286$$

$$\begin{array}{r}
 46 \\
 381 \overline{)17902} \\
 \underline{1524 \downarrow} \\
 2662 \\
 \underline{2286}
 \end{array}$$

Repeat Step 3: Subtract.

$$2\,662 - 2\,286 = 376$$

Check: $376 < 381$? **YES!**

$$\begin{array}{r}
 46 \\
 381 \overline{)17902} \\
 \underline{1524 \downarrow} \\
 2662 \\
 \underline{2286} \\
 376
 \end{array}$$

Repeat Step 4: No more digits to bring down.

$$17\,902 \div 381 = 46 \text{ R}376$$

Exercise 8

Divide and check your answers. These questions are hard work!

a. $115 \overline{)8682}$ c. $325 \overline{)66321}$ e. $860 \overline{)262412}$
b. $205 \overline{)2384}$ d. $241 \overline{)13284}$ f. $659 \overline{)270190}$

Exercise 8 Answers

a. 75 R57	c. 204 R21	e. 305 R112
b. 11 R129	d. 55 R29	f. 410

I.12: Practice Questions

1. Divide and check your work for questions b) and f) using multiplication.

a. $185 \div 10 =$	e. $67 \overline{)5963}$	h. $265 \overline{)133624}$
b. $185 \div 10 =$	f. $53 \overline{)4856}$	i. $606 \overline{)26094}$
c. $408 \div 50 =$	g. $91 \overline{)8736}$	j. $1000 \overline{)83652}$
d. $72 \overline{)6768}$		

I.12: Practice Answers

1. Divide and check your work for questions b) and f) using multiplication.

a. 18 R5	e. 89	i. 43 R36
b. 8 R8	f. 91 R33	j. 83 R652
c. 38	g. 96	
d. 94	h. 504 R64	

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1.13: Estimating Quotients

In this chapter, you will learn a shorter way to divide numbers that **both** end with zeros. First, do the following exercise and notice the pattern in the quotients.

Exercise 1

Divide. Check your work using the answer key at the end of the exercise.

a. $2\overline{)6}$

e. $5\overline{)25}$

i. $14\overline{)28}$

b. $20\overline{)60}$

f. $50\overline{)250}$

j. $140\overline{)280}$

c. $200\overline{)600}$

g. $500\overline{)2500}$

k. $1400\overline{)2800}$

d. $2000\overline{)6000}$

h. $5000\overline{)25000}$

l. $14000\overline{)28000}$

Exercise 1 Answers

a. 3

e. 5

i. 2

b. 3

f. 5

j. 2

c. 3

g. 5

k. 2

d. 3

h. 5

l. 2

Here is the shortcut:

When dividing numbers that both end with zeros, cross off the

same number of zeros from the end of the divisor and the dividend.
This is sometimes called cancelling zeros.

Example A

$$4800 \div 60 = 480\cancel{0} \div 6\cancel{0}$$
$$\begin{array}{r} 80 \\ 6 \overline{)480} \end{array}$$

Example B

$$23000 \div 500 = 230\cancel{00} \div 5\cancel{00}$$

$$\begin{array}{r}
 46 \\
 5 \overline{)230} \\
 \underline{20 \downarrow} \\
 30 \\
 \underline{30} \\
 0
 \end{array}$$

Example C

$$\begin{array}{r}
 \cancel{2,000} \overline{) \cancel{680,000}}
 \end{array}$$

$$\begin{array}{r}
 340 \\
 2 \overline{)680} \\
 \underline{6 \downarrow} \\
 08 \\
 \underline{8 \downarrow} \\
 00 \\
 \underline{0} \\
 0
 \end{array}$$

If you are interested in the facts of arithmetic that make this shortcut work, ask your instructor for an explanation.

Exercise 2

Quickly find the quotients. Remember to cancel the same number of zeros in the divisor and dividend.

a. $30 \overline{)90}$	d. $400 \overline{)20000}$	g. $60000 \overline{)2400000}$
b. $40 \overline{)1600}$	e. $500 \overline{)35000}$	h. $800000 \overline{)400000000}$
c. $300 \overline{)1200}$	f. $700 \overline{)42000}$	

Exercise 2 Answers

a. 3
b. 40
c. 4

d. 50
e. 70
f. 60

g. 40
h. 500

Rounding Division Questions to Estimate

We round numbers and estimate to get a quick answer.

In division, round the divisor and dividend **BEFORE** you divide:

- If the divisor only has one digit, do not round it,
- Round the dividend to make the arithmetic easier for yourself.

Example D

Look at the two ways of rounding this question.

$$1796 \div 32 =$$

The divisor (32) will round to 30. This dividend (1796) can be rounded to 1800 or to 2000. Let's try each:

Round to 1800.

$$\begin{array}{r} 60 \\ 3\cancel{0}\overline{)180\cancel{0}} \end{array}$$

Round to 2000.

$$\begin{array}{r} 66 \text{ R}2 \\ 3\cancel{0}\overline{)200\cancel{0}} \\ \underline{18 \downarrow} \\ 20 \\ \underline{18} \\ 2 \end{array}$$

Rounding 1796 to 1800 is easier arithmetic because $18 \div 3$ works out evenly, so $180 \div 3$ works out evenly. Both estimates are correct.

Example E

$$2\,688 \div 28 =$$

- Round the divisor (28) to 30.

- Round the dividend (2688) to 2 700

Round to 2 700:

$$\begin{array}{r} 90 \\ 3\cancel{} \overline{)270\cancel{}} \end{array}$$

Round to 3 000:

$$\begin{array}{r} 100 \\ 3\cancel{} \overline{)300\cancel{}} \end{array}$$

Both estimates are correct and both are easy to do.

Example F

$$2\,893 \div 47 =$$

- Round the divisor (47) to 50.
- Round the dividend (2 893) to 2 900 or 3 000.

Which rounded dividend will be easier to divide by 50?

Answer: The 3 000 because 5 goes evenly into 30.

$$\begin{array}{r} 60 \\ 5\cancel{0}\overline{)300\cancel{0}} \end{array}$$

Exercise 3

Give an estimated quotient. Show your rounding.

a. $365\overline{)27692}$	e. $56\overline{)4792}$	i. $19\overline{)1672}$
b. $23\overline{)34459}$	f. $75\overline{)7648}$	j. $200\overline{)20000}$
c. $45\overline{)4590}$	g. $81\overline{)4049}$	k. $557\overline{)41680}$
d. $16\overline{)6729}$	h. $68\overline{)5636}$	

Exercise 3 Answers

a. $28\,000 \div 400 = 70$	e. $4\,800 \div 60 = 80$	i. $2\,000 \div 20 = 100$
b. $34\,000 \div 20 = 1\,700$	f. $8\,000 \div 80 = 100$	j. $20\,000 \div 200 = 100$
c. $5\,000 \div 50 = 100$	g. $4\,000 \div 80 = 50$	k. $42\,000 \div 600 = 70$
d. $7\,000 \div 20 = 350$	h. $5\,600 \div 70 = 80$	

1.13: Practice Questions

1. Give an estimated quotient. Show your work.

a. $98 \overline{)8541}$

c. $241 \overline{)26348}$

e. $24 \overline{)1776}$

b. $27 \overline{)2963}$

d. $55 \overline{)3276}$

f. $59 \overline{)11830}$

1.13: Practice Answers

1. Give an estimated quotient. Show your work.

a. $8\,500 \div 100 = 85$ $26\,000 \div 200 = 130$ $2\,000 \div 20 = 100$

b. $3\,000 \div 30 = 100$ $3\,000 \div 60 = 50$ $12\,000 \div 60 = 200$

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1.14: Division Problems

Review 1.6: Problem Solving.

One common type of division problem gives a total amount for **several** things and asks you to **find** what the amount would be for **one**. See Table 1.14.1 for common division problems.

Table 1.14.1: Common Division Problem Types

What the Problem Tells You	What the Problem Asks You To Find
kilometres driven in 8 hours (h)	km driven in 1 h
cost for 15 kg (or litres, etc.)	cost for one kg
pay for 40 hours	pay for one hour
rent for one year (12 months)	rent for one month
work done in eight hours	work done in one hour
kilometres driven on 55 L of gas	km driven on 1 L of gas

The word *per* is a Latin word meaning “for each.” For example, “Find the kilometres *per* hour” may be used to mean, “Find the kilometres driven in one hour.” A slash (/) also means per e.g. km/h.

“Find the average” is another way of asking you to find the amount for one.

It may be difficult to decide which number is the dividend and which is the divisor. These suggestions should help:

1. Look at the question in the problem. What do you have to find out? Look for the words “per” and “for one.”
2. How will the answer be written? That is your clue. If the

answer is km/h, the division equation will be $\text{total km} \div \text{h}$.

Study these examples:

- $\text{total of kilometres} \div \text{number of hours} = \text{km/h}$
- $\text{total of kilometres} \div \text{number of litres} = \text{km/L}$
- $\text{total cost} \div \text{unit} = \text{cost per unit}$
- $\text{total pay} \div \text{hours (or days, etc.)} = \text{pay per hour}$
- $\text{total rent} \div \text{number of months} = \text{rent/month}$
- $\text{total things done} \div \text{total time} = \text{number done/unit of time}$

3. Do a quick estimate.
4. Look at your estimate and re-read the problem. Does your answer make sense?

To find the average, divide the total amount by the number of items that make up the total. You may first have to add the different items together to find the total.

Average = $\frac{\text{Total amount}}{\text{number of items that make the total}}$

Example A

You bowled 5 games with scores of 124, 187, 164, 205, 130.
What was your average score?

Step 1: Find the total by adding.

$$124 + 187 + 164 + 205 + 130 = 810$$

Step 2: Divide the total by number of items.

$$810 \div 5 \text{ games} = 162 \text{ per game}$$

Example B

Joan and Rick have been keeping track of their household costs. They want to plan a monthly budget. Their grocery bills over six months were \$428, \$605, \$397, \$530, \$590, and \$474. What is their average monthly grocery cost?

Step 1: Find the total amount.

$$\$428 + \$605 + \$397 + \$530 + \$590 + \$474 = \$3\,024$$

Step 2: Divide total amount by number of items.

$$\$3\,024 \div 6 = \$504 \text{ average cost per month}$$

Some details to remember:

- 1 minute = 60 seconds
- 1 hour = 60 minutes
- 1 year = 365 days
- 1 year = 12 months
- 1 year = 52 weeks

Keywords that point to division:

- find the average

- separated
- find the _____ per _____
- split
- find the unit price
- shared

Exercise 1

Solve the following word problems. Use the five problem solving steps. Be sure to write down an estimate and check that the estimate makes sense **BEFORE** you find the actual solution. Check your work using the answer key at the end of the exercise.

- A machine shop can stamp out 360 car parts during an 8-hour working day. How many parts is that per hour?
- Izyan paid \$560 for 4 tires. How much did each tire cost?
- Bjork earned \$8 840 in 4
 - How much did he earn each month?
 - How much did he earn per week? (4 months is 17 weeks)
- Theron used 9 L of gasoline to drive 207 km. How many kilometres did he drive per litre?
- The total cost of the car Elena bought is \$14 880,

- including taxes and interest. She will pay for it in 24 equal payments. How much will each payment be?
- f. Diego worked 8 hours a day for five days and earned \$360. How much was he paid per hour? (This is a 2-step problem – you must first find the total number of)
- g. a) Dae-Hyun and Mi-Ok can afford no more than \$14 940 per year for rent, electricity, and phone. How much can they pay per month?

Exercise 1 Answers

a. 45 parts per hour	c. 23 km/L
b. \$140 per tire	d. \$620 per payment
i. \$2210 per month	e. \$9 per hour
ii. \$520 per week	f. \$1245 per month

A second type of division problem gives the total amount and the size of each group.

You will find the number of groups. You will notice that **both numbers have the same units**. The **answer** to the problem **will give another unit**. This other unit will be asked for in the problem.

Example C

One necklace uses **125 beads**. How many necklaces can Susan make for the craft fair if she has **6 250 beads**?

Step 1: Find how many groups of 125 there are in 6 250.

$$\begin{array}{r} 6\,250 \div 125 = \\ 50 \\ 125 \overline{)6250} \\ \underline{625} \\ 0 \end{array}$$

She can make **50 necklaces**.

Example D

If you drive an average speed of 80 km an hour, how many hours will it take you to drive 560 km?

Step 1: Find how many groups of 80 km there are in 560 km.

$$560 \div 80 = 7$$

The 560 km trip will take 7 hours.

Exercise 2

Pay attention to wording and situations as you solve the following word problems. Use the 5 problem-solving steps. Be sure to write down an estimate and check that the estimate makes sense **BEFORE** you find the actual solution.

- a. A train travels 90 km per hour. How many hours will it take the train to go 540 km?
- b. A car gets 16 km per litre of gasoline. How many litres will the car need to go 128 km?
- c. About 8 m is needed for one parking space. How many parking spaces can be made along a street that is 232 m long?
- d. If you spend an average of 8 minutes on one math problem, how many problems can you finish in one hour? Will you have any time left? How much?
- e. The Skating Club members decided to sell homemade fudge to raise money. The boxes they

bought will hold 45 pieces of fudge. If everyone makes a double batch of fudge they will have 2 590 pieces. How many boxes can they fill? How many pieces of fudge are left over for them to eat?

- f. A class of 334 students is going to Victoria by bus. Each bus holds 43 passengers. How many buses do they need? Will there be any empty seats? (Be careful with this one!)

Exercise 2 Answers

a. 6 hours	d. 7 problems, yes, 4 min
b. 8 L	e. 57 boxes, 25 pieces left over
c. 29 parking spaces	f. 8 buses, 10 empty seats

Unit Pricing

A unit price is the price for one of something. To find the unit price, divide the total cost by the number of things bought.

Example E

5 shirts cost \$60

To find the cost per shirt:

$$\begin{array}{r} \$60 \div 5 = \\ 12 \\ 5 \overline{)60} \end{array}$$

The unit price is \$12.

Example F

6 L of oil for \$18

To find the cost per L:

$$\begin{array}{r} \$18 \div 6 = \\ 3 \\ 6 \overline{)18} \end{array}$$

The unit price is \$3.

Exercise 3

Solve the cost per unit price.

a. 2 CDs for \$26	f. 2 cans of peanuts for \$8	4 tie downs for \$20
b. 3 cans of dog food for \$6	g. 2 ice cream for \$12	l. 3 boxes of diapers for \$51
c. 4 air fresheners for \$8	h. 4 boxes of chocolate bars for \$48	m. 3 work shirts for \$45
d. 2 cat treats for \$4	i. 2 WD-40 for \$6	n. 8 pairs of socks for \$64
e. 2 pizzas for \$22	j. 3 paint rollers for \$9	

Exercise 3 Answers

a. \$13	f. \$4	k. \$5
b. \$2	g. \$6	l. \$17
c. \$2	h. \$12	m. \$15
d. \$2	i. \$3	n. \$8
e. \$11	j. \$3	

Best Buy

The **best buy** is the **lowest unit price**.

Example G

You can buy 4 L of canola oil for \$8 or 10 L for \$30. Which is the best buy?

Step 1: $\$8 \div 4 =$ can be rewritten and solved:

$$\begin{array}{r} 2 \\ 4 \overline{)8} \end{array}$$

Step 2: $\$30 \div 10 =$ can be rewritten and solved:

$$\begin{array}{r} 3 \\ 10 \overline{)30} \end{array}$$

4 L of canola oil for \$8 is a better buy since the unit price is \$2 per L, while 10 L for \$30 has a unit price of \$3 per L.

Exercise 4

Solve the unit price and then underline the best buy.

- a. 2 L of engine oil for \$8 or 5 L of engine oil for \$15.
- b. 4 tires for \$240 or 2 tires for \$110.
- c. 6 jars of salad dressing for \$24 or 3 jars of salad dressing for \$15.
- d. 7 kg of dog food for \$21 or 16 kg for \$32.
- e. 3 DVDs for \$54 or 7 DVDs for \$119.
- f. 3 L of laundry soap for \$6 or 17 L of laundry soap for \$68.

Exercise 4 Answers

- a. \$4, \$3, 5 L for \$15 is the best buy.
- b. \$60, \$55, 2 tires for \$110 is the best buy.
- c. \$4, \$5, 6 salad dressing for \$24 is the best buy.
- d. \$3, \$2, 16 kg for \$32 is the best buy.
- e. \$18, \$17, 7 DVDs for \$119 is the best buy.
- f. \$2, \$4, 3 L for \$6 is the best buy.

1.14: Practice Questions

1. Solve the following word problems.
 - a. Enrique drove the 1 920 km from Dease Lake to Creston in 24 hours. What was his average speed in kilometres per hour?
 - b. The Evergreen Company employs 26 people. Its total payroll for last month was \$84 162. What was the average monthly pay cheque per person?
 - c. The proud gardener grew a total crop of 135 cucumbers on 15 plants. What was the average crop per plant?

- d. In a recent truckload sale, electric stoves were sold for \$432. The gross income from the stove sale was \$42 336. How many stoves were sold?
- e. The 39 farmers in Jones Valley had a total income last year of \$2 905 500. What was their average income?
- f. A store has an inventory (stock on hand) of chairs with a total value of \$1 738. Each chair is to be sold at \$79. How many of these chairs are there?

1.14: Practice Answers

- 1. Solve these problems.
 - a. 80 km/h
 - b. \$3 237 per month
 - c. 9 cucumbers per plant
 - d. 98 stoves
 - e. \$74 500
 - f. 22 chairs

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CHAPTER 2: INTEGERS

2.1: Integers

Chapter 2: Learning Outcomes

By the end of this chapter, you should be able to:

- Add, subtract, multiply and divide rational numbers.
- Use order of operations.
- Graph rational numbers on the number line.
- Define absolute value.

Use Negatives & Opposites

If you have ever experienced a temperature below zero or accidentally overdrawn your checking account, you are already familiar with negative numbers.

Negative numbers are numbers less than 0. The negative numbers are to the left of zero on the number line (see Figure 2.1.1).

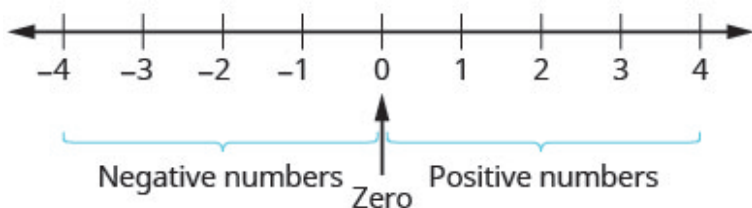


Figure 2.1.1

The arrows on the ends of the number line indicate that the numbers keep going forever. There is no biggest positive number and no smallest negative number.

Is zero a positive or a negative number? Numbers larger than zero are positive, and numbers smaller than zero are negative. Zero is neither positive nor negative.

Consider how numbers are ordered on the number line:

- Going from left to right – the numbers increase in value.
- Going from right to left – the numbers decrease in value.
- See Figure 2.1.2

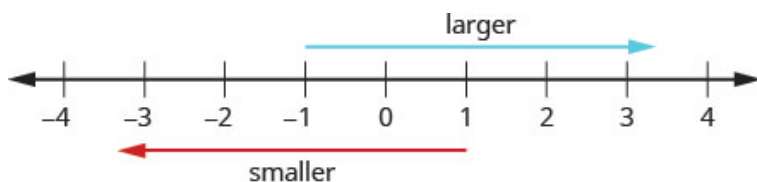


Figure 2.1.2

Number Line Notation

Remember that we use the notation:

- $a < b$ (read “ a is less than b ”) when a is to the left of b on the number line.
- $a > b$ (read “ a is greater than b ”) when a is to the right of b on the number line.

Now, we need to extend the number line (which only showed the whole numbers) to include negative numbers. The numbers marked by points in Figure 2.1.3 are called the integers. The integers are the numbers ...-3, -2, -1, 0, 1, 2, 3....

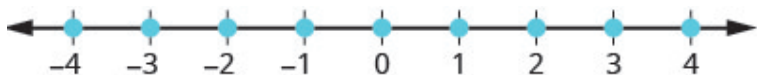


Figure 2.1.3

Example A

Order each of the following pairs of numbers, using $<$ or $>$:

- a. $14 \underline{\hspace{1cm}} 6$
- b. $-1 \underline{\hspace{1cm}} 9$
- c. $-1 \underline{\hspace{1cm}} -4$
- d. $2 \underline{\hspace{1cm}} -20$

It may be helpful to refer to the number line shown in Figure 2.14.

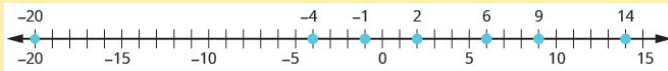


Figure 2.14

a. $14 \underline{\hspace{1cm}} 6$

14 is to the right of 6 on the number line.

$$\begin{array}{r} 14 \underline{\hspace{1cm}} 6 \\ 14 > 6 \end{array}$$

b. $-1 \underline{\hspace{1cm}} 9$

-1 is to the left of 9 on the number line.

$$\begin{array}{r} -1 \underline{\hspace{1cm}} 9 \\ -1 < 9 \end{array}$$

c. **-1** **-4**

-1 is to the right of -4 on the number line.

$$\begin{array}{r} -1 \underline{\hspace{1cm}} -4 \\ -1 > -4 \end{array}$$

d. **2** **-20**

2 is to the right of -20 on the number line.

$$\begin{array}{r} 2 \underline{\hspace{1cm}} -20 \\ 2 > -20 \end{array}$$

Exercise 1

Order each of the following pairs of numbers, using < or >:

a. $15 \underline{\hspace{1cm}} 7$
b. $-2 \underline{\hspace{1cm}} 5$

c. $-3 \underline{\hspace{1cm}} -7$

d. $5\_\_\_ - 17$

Exercise 1 Answers

- a. $>$
b. $<$

- c. $>$
d. $>$

You may have noticed that, on the number line, the negative numbers are a mirror image of the positive numbers, with zero in the middle. Because the numbers 2 and -2 are the same distance from zero, they are called **opposites**. The opposite of 2 is -2, and the opposite of -2 is 2.

Opposites:

The opposite of a number is the number that is the same distance from zero on the number line but on the opposite side of zero.

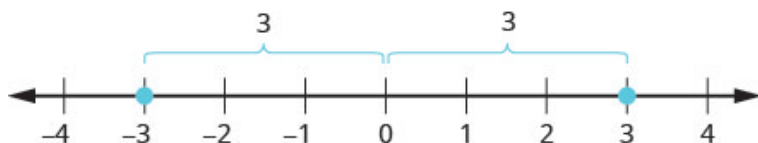


Figure 2.1.5

Opposite notation:

$-a$ means the opposite of the number a .

The notation $-a$ is read as “the opposite of a .”

The whole numbers and their opposites are called the integers. The integers are the numbers ...-3, -2, -1, 0, 1, 2, 3....

Integers:

The whole numbers and their opposites are called the **integers**.

The integers are the numbers -3, -2, -1, 0, 1, 2, 3.

Simplify Expressions With Absolute Value

We saw that numbers, such as 3 and -3, are opposites because they are the same distance from 0 on the number line. They are both two units from 0. The distance between 0 and any number on the number line is called the **absolute value** of that number.

Absolute value:

The absolute value of a number is its distance from 0 on the number line.

The absolute value of a number n is written as $|n|$.

For example,

- -5 is 5 units away from 0, so $|-5| = 5$
- 5 is 5 units away from 0, so $|5| = 5$

Figure 2.1.6 illustrates this idea. The integers 5 and are 5 units away from 0.

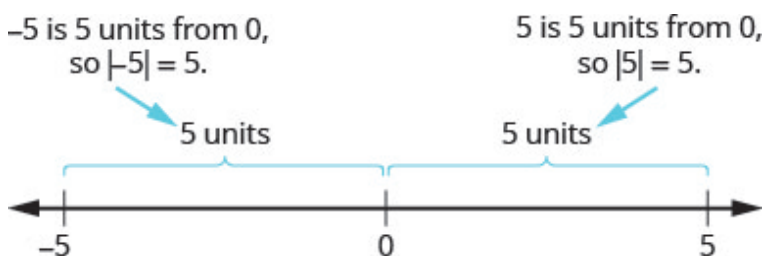


Figure 2.1.6

The absolute value of a number is never negative (because distance cannot be negative). The only number with absolute value equal to zero is the number zero itself, because the distance from 0 to 0 on the number line is zero units.

Property of absolute value:

$|n|$ for all numbers.

Absolute values are always greater than or equal to zero!

Mathematicians say it more precisely, “absolute values are always non-negative.” Non-negative means greater than or equal to zero.

Example B

Simplify:

a. $|3|$

b. $|-44|$

c. $|0|$

The absolute value of a number is the distance between the number and zero. Distance is never negative, so the absolute value is never negative.

a. $|3| = 3$

b. $|-44| = 44$

c. $|0| = 0$

Exercise 2

Simplify:

a. $|4|$
b. $|28|$

c. $|0|$

Exercise 2 Answers

a. 4
b. 28

c. 0

In the next example, we will order expressions with absolute values. Remember, positive numbers are always greater than negative numbers!

Example C

Fill in $<$, $>$, or $=$ for each of the following pairs of numbers:

a. $|-5| \_ |-5|$

b. $8 \_ |-8|$

c. $-9 \_ |-9|$

d. $-16 \_ |-16|$

a. $|-5| \_ |-5|$

Simplify order: $|-5| \_ \_\_\_\_ - |-5|$

$$5 \_\_\_\_ - 5$$

$$5 > -5$$

$$|-5| > - |-5|$$

b. $8 \_ |-8|$

Simplify order: $8 \_\_\_\_ - |-8|$

$$8 \_\_\_\_ - 8$$

$$8 > -8$$

$$8 > - |-8|$$

c. $-9 \_ | -9|$

Simplify order: $-9 \_ | -9|$

$$-9 \_ - | -9|$$

$$-9 = -9$$

$$-9 = - | -9|$$

d. $- - 16 \_ | -16|$

Simplify order: $- - 16 \_ | -16|$

$$16 \_ - 16$$

$$16 > -16$$

$$-(-16) > - | -16|$$

Exercise 3

Fill in $<$, $>$, or $=$ for each of the following pairs of numbers:

a. $ -9 _ - -9 $	c. $-8 _ - -8 $
b. $2 _ - -2 $	d. $-9 _ - -9 $

Exercise 3 Answers

- a. $>$
- b. $>$

- c. $<$
- d. $>$

We now add absolute value bars to our list of grouping symbols. When we use the order of operations, we first simplify inside the absolute value bars as much as possible, and then we take the absolute value of the resulting number.

Grouping symbols include:

- Parentheses — $()$
- Brackets — $[\]$
- Braces — $\{ \}$
- Absolute value — $| |$

In the next example, we simplify the expressions inside absolute value bars first, just like we do with parentheses.

Example D

Simplify: $24 - |19 - 3(6 - 2)|$.

Step 1: Work inside parentheses first: subtract 2 from 6

$$24 - |19 - 3(4)|$$

Step 2: Multiply $3(4)$.

$$24 - |19 - 12|$$

Step 3: Subtract inside the absolute value bars.

$$24 - |7|$$

Step 4: Take the absolute value.

$$24 - 7$$

Step 5: Subtract.

$$17$$

Exercise 4

Simplify: $19 - |11 - 4(3 - 1)|$.

Exercise 4 Answer

16

Add Integers

Most students are comfortable with the addition and subtraction facts for positive numbers. However, doing addition or subtraction with both positive and negative numbers may be more challenging.

We will use two colour counters to model the addition and subtraction of negatives so that you can visualize the procedures instead of memorizing the rules.

We let one colour (blue) represent positive. The other colour (red) will represent the negatives. If we have one positive counter and one negative counter, the value of the pair is zero. They form a neutral pair. The value of this neutral pair is zero (see Figure 2.1.7).

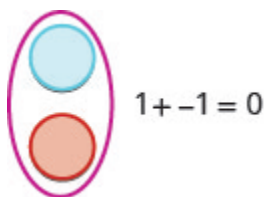


Figure 2.1.7

We will use the counters to show how to add the four addition facts using the numbers (5, -5) and (3, -3).

$$5 + 3 \quad -5 + (-3) \quad -5 + 3 \quad 5 + (-3)$$

The first example adds 5 positives and 3 positives – both positives.

The second example adds 5 negatives and 3 negatives – both negatives.

In each case, we got 8 – either 8 positives or 8 negatives.

When the signs were the same, the counters were all the same colour, so we added them (see Figure 2.1.8).



Figure 2.1.8

So, what happens when the signs are different? Let's add $(-5 + 3)$ and $(5 + (-3))$.

When we use counters to model the addition of positive and negative integers, it is easy to see whether there are more positive or negative counters. So, we know whether the sum will be positive or negative (see Figure 2.1.9).

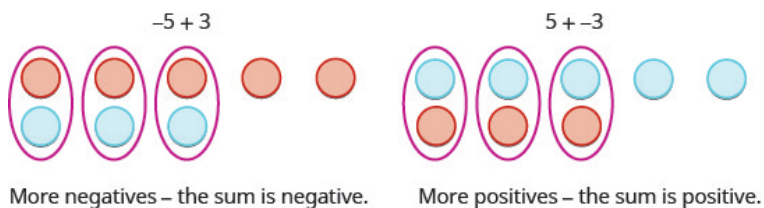


Figure 2.1.9

Example E

Add:

- a. $-1 + 5$
- b. $1 + (-5)$

a. $-1 + 5$

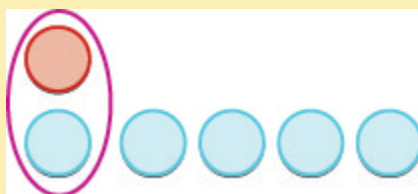


Figure 2.1.10

There are more positives, so the sum is positive.

$$-1 + 5 = 4$$

b. $1 + (-5)$

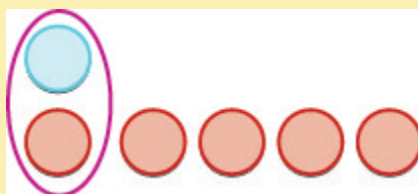


Figure 2.1.11

There are more negatives, so the sum is negative.

$$1 + (-5) = -4$$

Exercise 5

Add.

a. $-2 + 4$

b. $2 + (-4)$

Exercise 5 Answers

a. 2

b. -2

Now that we have added small positive and negative integers with a model, we can visualize the model in our minds to simplify problems with any numbers.

When you need to add numbers such as $(37 + (-53))$, you really don't want to have to count out 37 blue counters and 53 red counters. With the model in your mind, can you visualize what you would do to solve the problem?

Picture 37 blue counters with 53 red counters lined up underneath. Since there would be more red (negative) counters than blue (positive) counters, the sum would be *negative*. How many more red counters would there be? Because $(53 - 37 = 16)$, there are 16 more red counters.

Therefore, the sum of $37 + (-53)$ is -16.

$$37 + (-53) = -16$$

Let's try another one. We'll add $-74 + (-27)$. Again, imagine 74 red

counters and 27 more red counters, so we'd have 101 red counters. This means the sum is -101.

$$(-74) + (-27) = -101$$

Let's look again at the results of adding the different combinations of (5, -5) and (3, -3).

Addition of positive & negative integers:

$$5 + 3$$

$$8$$

both positive, sum positive

$$-5 + (-3)$$

$$-8$$

both negative, sum negative

When the signs are the same, the counters would be all the same colour, so add them.

$$-5 + 3$$

$$-2$$

different signs, more negatives, sum negative

$$5 + (-3)$$

$$2$$

different signs, more positives, sum positive

When the signs are different, some of the counters will make neutral pairs, so subtract to see how many are left.

Visualize the model as you simplify the expressions in the following examples.

Example F

Simplify:

a. $19 + (-47)$

b. $-14 + (-36)$

a. $19 + (-47)$

Since the signs are different, we subtract 19 from 47. The answer will be negative because there are more negatives than positives.

$$19 + (-47)$$

$$\text{Add.} \qquad -28$$

b. $-14 + (-36)$

Since the signs are the same, we add. The answer will be negative because there are only negatives.

$$-14 + (-36)$$

$$\text{Add.} \qquad -50$$

Exercise 6

Simplify:

a. $-31 + (-19)$ b. $15 + (-32)$

Exercise 6 Answers

a. -50 b. -17

The techniques used up to now extend to more complicated problems, like the ones we have seen before. Remember to follow the order of operations!

Example G

Simplify: $-5 + 3(-2 + 7)$

Step 1: Simplify inside the parentheses.

$$-5 + 3(5)$$

Step 2: Multiply.

$$-5 + 15$$

Step 3: Add left to right.

$$10$$

Exercise 7

Simplify: $-2 + 5(-4 + 7)$

Exercise 7 Answer

$$13$$

Subtract Integers

We will continue to use counters to model the subtraction. Remember, the blue counters represent positive numbers, and the red counters represent negative numbers.

Perhaps when you were younger, you read “ $5 - 3$ ” as “5 take away 3.” When you use counters, you can think of subtraction the same way!

We will model the four subtraction facts using the numbers 5 and 3.

$$5 - 3 \qquad -5 - (-3) \qquad -5 - 3 \qquad 5 - (-3)$$

In the first example, we subtract 3 positives from 5 positives and end up with 2 positives.

In the second example, we subtract 3 negatives from 5 negatives and end up with 2 negatives.

Each example used counters of only one colour, and the “take away” model of subtraction was easy to apply (see Figure 2.1.12).



Figure 2.1.12

What happens when we have to subtract one positive and one negative number? We will need to use both white and red counters as well as some neutral pairs. Adding a neutral pair does not change the value. It is like changing quarters to nickels — the value is the same, but it looks different.

To subtract $-5 - 3$, we restate it as -5 take away 3.

We start with 5 negatives. We need to take away 3 positives, but we do not have any positives to take away.

Remember, a neutral pair has a value of zero. If we add 0 to 5 its value is still 5. We add neutral pairs to the 5 negatives until we get 3 positives to take away.

$-5 - 3$ means -5 take away 3 .

Step 1: We start with 5 negatives (see Figure 2.1.13).



Figure 2.1.13

Step 2: We now add the neutrals needed to get 3 positives (see Figure 2.1.14).

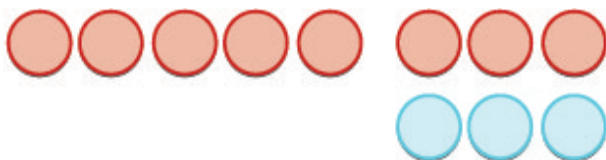


Figure 2.1.14

Step 3: We remove the 3 positives (see Figure 2.1.5).

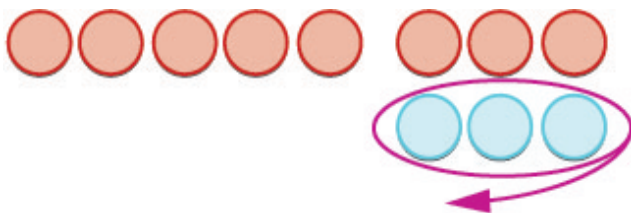


Figure 2.1.15

Step 4: We are left with 8 negatives (see Figure 2.1.16).



Figure 2.1.16

The different of -5 and 3 is -8 .

$$-5 - 3 = -8$$

Now, the fourth case, $5 - (-3)$. We start with 5 positives. We need to take away 3 negatives, but there are no negatives to take away. So, we add neutral pairs until we have 3 negatives to take away.

$5 - (-3)$ means 5 take away -3 .

Step 1: We start with 5 positives (see Figure 2.1.17).



Figure 2.1.17

Step 2: We now add the needed neutral pairs (see Figure 2.1.18).



Figure 2.1.18

Step 3: We remove the 3 negatives (see Figure 2.1.19).

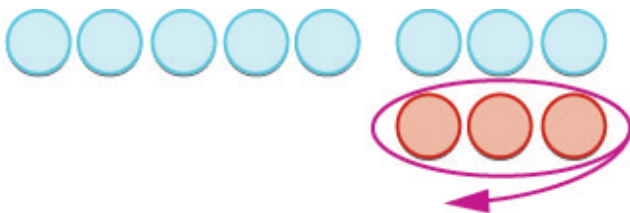


Figure 2.1.19

Step 4: We are left with 8 positives (see Figure 2.1.20).



Figure 2.1.20

The difference of 5 and -3 is 8.

$$5 - (-3) = 8$$

Example H

Subtract:

a. $-3 - 1$

b. $3 - (-1)$

a. $-3 - 1$

Take 1 positive from the one added neutral pair (see Figure 2.1.21).



Figure 2.1.21

$$-3 - 1 = -4$$

b. $3 - (-1)$

Take 1 negative from the one added neutral pair (see Figure 2.1.22).



Figure 2.1.22

$$3 - (-1) = 4$$

Exercise 8

Subtract:

a. $-6 - 4$

b. $6 - (-4)$

Exercise 8 Answers

a. -10

b. 10

Have you noticed that you can subtract signed numbers by adding the opposite? In Example H, $-3 - 1$ is the same as $-3 + (-1)$ and $3 - (-1)$ is the same as $3 + 1$. You will often see this idea, the subtraction property, written as follows:

Subtraction property:

$$a - b = a + (-b)$$

Subtracting a number is the same as adding its opposite.

Look at these two examples in Figure 2.1.23.

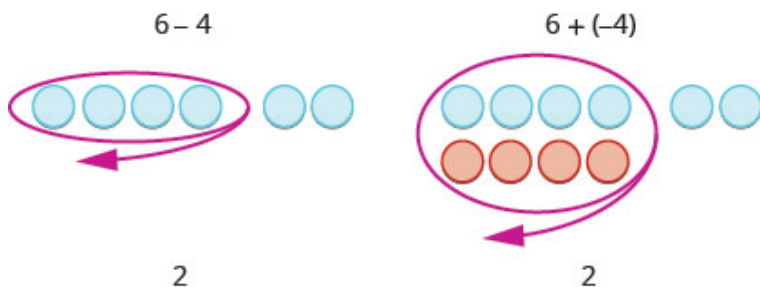


Figure 2.1.23

$6 - 4$ gives the same answer as $6 + (-4)$.

Of course, when you have a subtraction problem that has only positive numbers, like $6 - 4$, you just do the subtraction. You already knew how to subtract $6 - 4$ long ago. But *knowing* that $6 - 4$ gives the same answer as $6 + (-4)$ helps when you are subtracting negative numbers. Make sure that you understand how $6 - 4$ and $6 + (-4)$ give the same results!

Look at what happens when we subtract a negative in Figure 2.1.24.

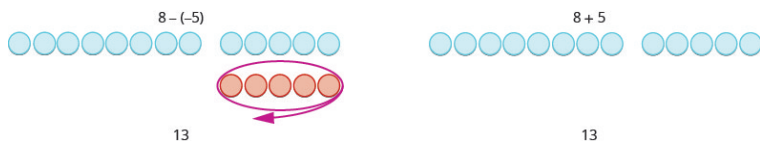


Figure 2.1.24

$8 - (-5)$ gives the same answer as $8 + 5$

Subtracting a negative number is like adding a positive!

You will often see this written as $a - (-b) = a + b$.

What happens when there are more than three integers? We just use the order of operations as usual.

Example 1

Simplify: $7 - (-4 - 3) - 9$.

Step 1: Simplify inside the parentheses first.

$$7 - (-4 - 3) - 9$$

Step 2: Subtract left to right.

$$7 - (-7) - 9$$

Step 3: Subtract.

$$\begin{array}{r} 14 - 9 \\ 7 - (-4 - 3) - 9 = 5 \end{array}$$

Exercise 9

Simplify: $8 - (-3 - 1) - 9$.

Exercise 9 Answer

3

Multiply Integers

Since multiplication is mathematical shorthand for repeated addition, our model can easily be applied to show the multiplication of integers. Let's look at this concrete model to see what patterns we notice.

We will use the same examples that we used for addition and subtraction. In Figure 2.1.25, we will use the model to help us discover the pattern.

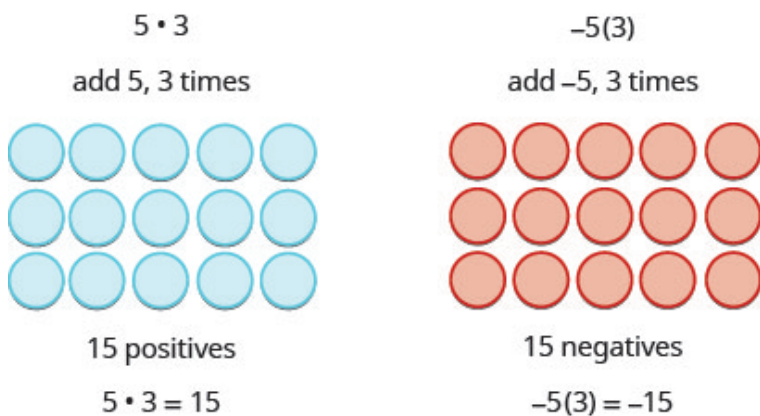


Figure 2.1.25

We remember that $a \cdot b$ means to add a , b times. Here, we are using the model just to help us discover the pattern.

The next two examples in Figure 2.1.26 are more interesting.

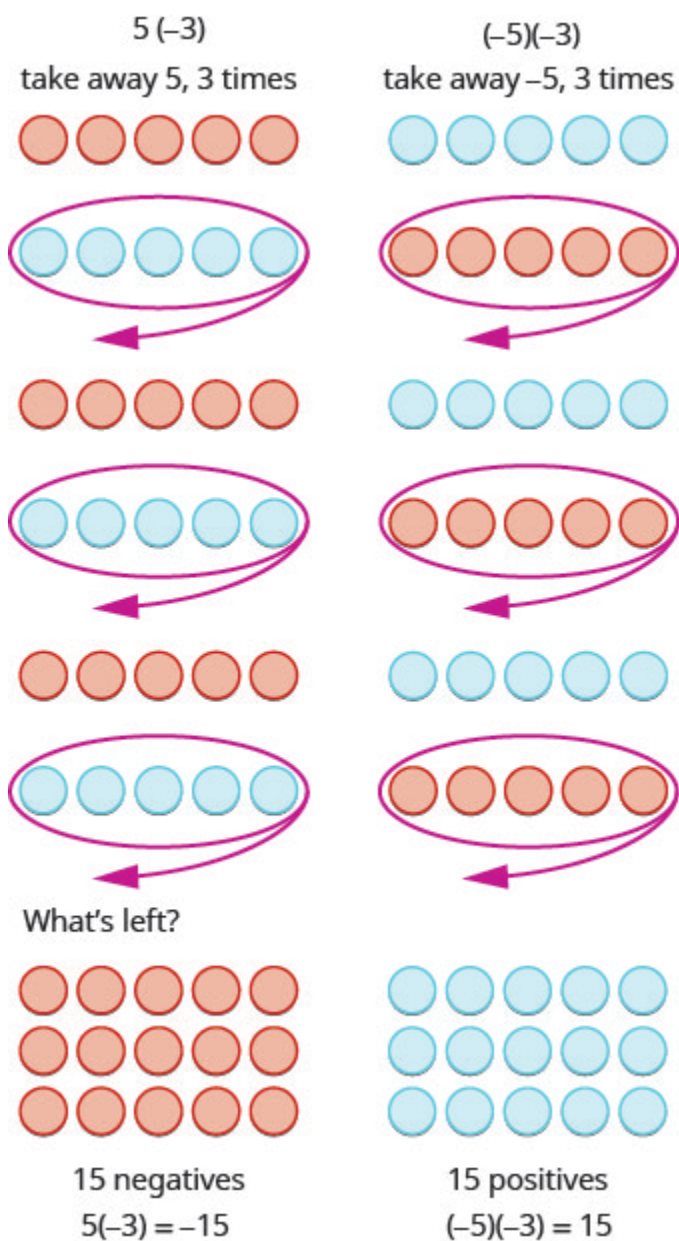


Figure 2.1.26

What does it mean to multiply 5 by -3 ? It means subtract 5, 3 times. Looking at subtraction as “taking away,” it means to take away 5, 3 times. However, there is nothing to take away, so we start by adding neutral pairs to the workspace. Then we take away 5 three times.

In summary:

$$\begin{array}{ll} 5 \cdot 3 = 15 & -5(3) = -15 \\ 5(-3) = -15 & (-5)(-3) = 15 \end{array}$$

Notice that for the multiplication of two signed numbers, when the:

- Signs are the **same**, the product is **positive**.
- Signs are **different**, the product is **negative**.

We will put this all together below.

Multiplication of signed numbers:

For the multiplication of two signed numbers:

- Two positives = Positive product
 - E.g. $7 \cdot 4 = 28$
- Two negatives = Positive product
 - E.g. $-8(-6) = 48$
- Positive multiplied by a negative = Negative product
 - E.g. $7(-9) = 63$
- Negative multiplied by a positive = Negative product

$$\circ \quad \text{E.g. } -5 \cdot 10 = -50$$

Example J

Multiply:

a. $-9 \cdot 3$

b. $-2(-5)$

c. $4(-8)$

d. $7 \cdot 6$

a. $-9 \cdot 3$

Multiply, noting that the signs are different so the product is negative.

$$-9 \cdot 3 = -27$$

b. $-2(-5)$

Multiply, noting that the signs are the same so the product is positive.

$$-2(-5) = 10$$

c. $4(-8)$

Multiply, with different signs.

$$4(-8) = -32$$

d. **$7 \cdot 6$**

Multiply, with same signs.

$$7 \cdot 6 = 42$$

Exercise 10

Multiply:

a. $a - 6 \cdot 8$

c. $9(-7)$

b. $-4(-7)$

d. $5 \cdot 12$

Exercise 10 Answers

a. -48

c. -63

b. 28

d. 60

When we multiply a number by 1, the result is the same number.
What happens when we multiply a number by -1?

Let's multiply a positive number and then a negative number by -1
to see what we get.

	$-1 \cdot 4$	$-1(-3)$
Multiply.	-4	3
	-4 is the opposite of 4.	3 is the opposite of -3 .

Each time we multiply a number by -1 , we get its opposite!

Multiplication by -1 :

$$-1a = -a$$

Multiplying a number by -1 gives its opposite.

Example K

Multiply:

a. $-1 \cdot 7$

b. $-1(-11)$

a. $-1 \cdot 7$

Multiply, noting that the signs are different so the product is negative.

$$-1 \cdot 7$$

$$-7$$

-7 is the opposite of 7.

b. $-1(-11)$

Multiply, noting that the signs are the same so the product is positive.

$$-1(-11)$$

$$11$$

11 is the opposite of -11 .

Exercise 11

Multiply:

a. $-1 \cdot 9$

b. $-1 \cdot (-17)$

Exercise 11 Answers

a. -9

b. 17

Divide Integers

What about division? Division is the inverse operation of multiplication. So, $15 \div 3 = 5$ because $5 \cdot 3 = 15$. In words, this expression says that 15 can be divided into three groups of five each because adding five three times gives 15.

Look at some examples of multiplying integers to figure out the rules for dividing integers.

$$\begin{array}{llll} 5 \cdot 3 = 15 \text{ so } 15 \div 3 = 5 & -5(3) = -15 \text{ so } -15 \div 3 = -5 \\ (-5)(-3) = 15 \text{ so } 15 \div (-3) = -5 & 5(-3) = -15 \text{ so } -15 \div (-3) = 5 \end{array}$$

Division follows the same rules as multiplication!

For division of two signed numbers, when the:

- Signs are the **same**, the quotient is **positive**.
- Signs are **different**, the quotient is **negative**.

Remember that we can always check the answer of a division problem by multiplying.

Multiplication and division of signed numbers:

For multiplication and division of two signed numbers:

- If the signs are the same, the result is positive.
 - Two positives = Positive
 - Two negatives = Positive
- If the signs are different, the result is negative.
 - Positive and negative = Negative
 - Negative and positive = Negative

Example L

Divide:

a. $-27 \div 3$

b. $-100 \div (-4)$

a. **$-27 \div 3$**

Divide. With different signs, the quotient is negative.

$$\text{null} - 27 \div 3 = -9$$

b. **$-100 \div (-4)$**

Divide. With signs that are the same, the quotient is positive.

$$-100 \div (-4) = 25$$

Exercise 12

Divide:

a. $-42 \div 6$

b. $-117 \div (-3)$

Exercise 12 Answers

a. -7

b. 39

Simplify Expressions With Integers

What happens when there are more than two numbers in an expression? The order of operations still applies when negatives are included. Remember My Dear Aunt Sally?

Let's try some examples. We'll simplify expressions that use all four operations with integers — addition, subtraction, multiplication, and division. Remember to follow the order of operations.

Example M

Simplify: $7(-2) + 4(-7) - 6$

Step 1: Multiply first.

$$7(-2) + 4(-7) - 6$$

Step 2: Add.

$$-14 + (-28) - 6$$

Step 3: Subtract.

$$-42 - 6$$

$$7(-2) + 4(-7) - 6 = -48$$

Exercise 13

Simplify: $8(-3) + 5(-7) - 4$

Exercise 13 Answer

-63

Example N

Simplify:

a. $(-2)^4$

b. -2^4

a. $(-2)^4$

Step 1: Write in expanded form.

$$(-2)(-2)(-2)(-2)$$

Step 2: Multiply.

$$4(-2)(-2)$$

Step 3: Multiply.

$$-8(-2)$$

Step 4: Multiply.

$$16$$

b. -2^4

Step 1: Write in expanded form.

$$-(2 \cdot 2 \cdot 2 \cdot 2)$$

Step 2: Multiply.

$$-(4 \cdot 2 \cdot 2)$$

Step 3: Multiply.

$$-(8 \cdot 2)$$

Step 4: Multiply.

$$16$$

Notice the difference in parts a) and b):

- In part a), the exponent means to raise what is in the parentheses, the (-2) to the 4^{th} power.
- In part b), the exponent means to raise just the 2 to the 4^{th} power and then take the opposite.

Exercise 14

Simplify:

a. $(-3)^4$

b. -3^4

Exercise 14 Answers

a. 81

b. -81

The next example reminds us to simplify inside parentheses first.

Example O

Simplify: $12 - 3(9 - 12)$

Step 1: Subtract in parentheses first.

$$12 - 3(-3)$$

Step 2: Multiply.

$$12 - (-9)$$

Step 3: Subtract.

$$21$$

Exercise 15

Simplify: $17 - 4(8 - 11)$

Exercise 15 Answer

29

Example P

Simplify: $8(-9) \div (-2)^3$

Step 1: Exponents first.

$$8(-9) \div (-8)$$

Step 2: Multiply.

$$-72 \div (-8)$$

Step 3: Divide.

9

Exercise 16

Simplify: $12(-9) \div (-3)^3$

Exercise 16 Answer

4

Example Q

Simplify: $-30 \div 2 + (-3)(-7)$

Step 1: Multiply and divide left to right, so divide first.

$$-30 \div 2 + (-3)(-7)$$

Step 2: Multiply.

$$-15 + 21$$

Step 3: Add.

$$6$$

Exercise 17

Simplify: $-27 \div 3 + (-5)(-6)$

Exercise 17 Answer

21

Additional online resources:

For additional instruction and practice with adding and subtracting integers, explore the links below. You will need to enable Java in your web browser to use the applications.

- Color Chips – Addition (Utah State University, n.d.a).
- Color Chips – Subtraction (Utah State University, n.d.b).

Key Concepts

Addition of Positive & Negative Integers

$$5 + 3$$

$$8$$

both positive,
sum positive

$$-5 + (-3)$$

$$-8$$

both negative,
sum negative

$$-5 + 3$$

$$-2$$

different signs,
more negatives
sum negative

$$5 + (-3)$$

$$2$$

different signs,
more positives
sum positive

Property of Absolute Value

$|n| \geq 0$ for all numbers. Absolute values are always greater than or equal to zero!

Subtraction of Integers

$$5 - 3$$

$$2$$

5 positives

take away 3 positives

2 positives

$$-5 - (-3)$$

$$-2$$

5 negatives

take away 3 negatives

2 negatives

$$-5 - 3$$

$$-8$$

5 negatives, want to

subtract 3 positives

need neutral pairs

$$5 - (-3)$$

$$8$$

5 positives, want to

subtract 3 negatives

need neutral pairs

Subtraction Property

Subtracting a number is the same as adding its opposite.

Multiplication & Division of Two Signed Numbers

- Same signs — Product is positive
- Different signs — Product is negative

Glossary

- **Absolute value** – The absolute value of a number is its distance from 0 on the number line. The absolute value of a number n is written as $|n|$.
- **Integers** – The whole numbers and their opposites are called the integers: ...-3, -2, -1, 0, 1, 2, 3....
- **Opposite** – The opposite of a number is the number that is the same distance from zero on the number line but on the opposite side of zero: -a means the opposite of the number. The notation -a is read “the opposite of a.”

2.1: Practice Questions

1. Order each of the following pairs of numbers, using $<$ or $>$.

a. 9__4

c. $-8\underline{\hspace{1cm}} - 2$

b. -3_6

d. 1 — 10

2. Simplify.

a. $|-32|$

c. **16**

b. $|0\rangle$

3. Fill in $<$, $>$, or $=$ for each of the following pairs of numbers.

a. $-6 \underline{\hspace{1cm}} \mid -6 \mid$

b. $- \mid -3 \mid \underline{\hspace{1cm}} - 3$

4. Simplify.

- a. $-(-5)$ and $-|-5|$ c. $|15 - 7| - |14 - 6|$
 b. $8 - |-7|$ d. $18 - |2(8 - 3)|$

5. Simplify each expression.

- a. $-21 + (-59)$ b. $-14 + (-12) + 4$ c. $19 + 2(-3 + 8)$
 d. $48 + (-16)$ e. $135 + (-110) + 83$

6. Simplify.

- a. $8 - 2$ d. $44 - 28$ g. $27 + 18$
 b. $-5 - 4$ e. $44 + (-28)$
 c. $8 - (-4)$ f. $27 - (-18)$

7. Simplify.

- a. $15 - (-12)$ e. $-45 - (54)$ j. $-(6 - 8) - (2 - 4)$
 b. $48 - 87$ f. $8 - 3 - 7$ k. $25 - [10 - (3 - 12)]$
 c. $-17 - 42$ g. $-5 - 4 + 7$ l. $6.3 - 4.3 - 7.2$
 d. $-103 - (-52)$ h. $-14 - (-27) + 9$ m. $5^2 - 6^2$
 i. $(2 - 7) - (3 - 8)(2)$

8. Multiply.

- a. $-4 \cdot 8$ c. $9(-7)$
 b. $-1 \cdot 6$ d. $-1(-14)$

9. Divide.

- a. $-24 \div 6$ c. $-52 \div (-4)$
 b. $-180 \div 15$

10. Simplify each expression.

a. $5(-6) + 7(-2) - 3$	d. $-3(-5)(6)$	g. $65 \div (-5) + (-28) \div (-7)$
b. $(-2)^6$	e. $(8 - 11)(9 - 12)$	h. $9 - 2[3 - 8(-2)]$
c. -4^2	f. $26 - 3(2 - 7)$	k. $(-3)^2 - 24 \div (8 - 2)$

11. Solve the following exercises.

- Temperature:** On January 15, the high temperature in Lytton, British Columbia, was 84° . That same day, the high temperature in Fort Nelson, British Columbia was -12° . What was the difference between the temperature in Lytton and the temperature in Embarrass?
- Checking Account:** Ester has \$124 in her checking account. She writes a check for \$152. What is the new balance in her checking account?
- Checking Account:** Kevin has a balance of $-\$38$ in his checking account. He deposits \$225 to the account. What is the new balance?
- Provincial Budgets:** For 2019 the province of Quebec estimated it would have a budget surplus of \$5.6 million. That same year, Alberta estimated it would have a budget deficit of \$7.5 million. Use integers to write the budget of:
 - Quebec
 - Alberta

2.1 Practice Answers

1. Order each of the following pairs of numbers, using $<$ or $>$.

a. $>$	c. $<$
b. $<$	d. $>$

2. Simplify.

- a. 32
- b. 0

c. 16

3. Fill in $<$, $>$, or $=$ for each of the following pairs of numbers.

a. $<$

b. $=$

4. Simplify.

- a. 5, -5
- b. 56

- c. 0
- d. 8

5. Simplify each expression.

- a. -80
- b. 32
- c. -22

- d. 108
- e. 29

6. Simplify.

- a. 6
- b. -9
- c. 12

- d. 16
- e. 16
- f. 45

g. 45

7. Simplify.

- a. 27
- b. -39
- c. -59
- d. -51
- e. -99

- f. -2
- g. -2
- h. 22
- i. -15
- j. 4

- k. 6
- l. -5.2
- m. -11

8. Multiply.

- a. -32
- b. -6

- c. -63
- d. 14

9. Divide.

a. -4	c. 13
b. -12	

10. Simplify each expression.

a. -47	d. 90	g. -9
b. 64	e. 9	h. -29
c. -16	f. 41	i. 5

11. Solve the following exercises.

- a. 96°
- b. $-\$28$
- c. $\$187$
 - i. $\$5.6$ million
 - ii. $-\$7.5$ million

References

Utah State University. (n.d.a). *Color chips - addition* [Interactive activity]. National Library of Virtual Manipulatives. http://nlvm.usu.edu/en/nav/frames_asid_161_g_1_t_1.html?from=topic_t_1.html.

Utah State University. (n.d.b). *Color chips - subtraction* [Interactive activity]. National Library of Virtual Manipulatives. http://nlvm.usu.edu/en/nav/frames_asid_162_g_2_t_1.html?from=topic_t_1.html.

Attributions

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CHAPTER 3: FRACTIONS

3.1: Introducing Common Fractions

Chapter 3: Learning Outcome

By the end of this chapter, you should be able to:

- Write fractions as decimals and decimals as fractions.
- Add, subtract, multiply and divide rational numbers.
- Use order of operations.
- Graph rational numbers on the number line.

This section gives you the background details needed for working with **common fractions**.

Parts of Common Fractions

Common fractions are written with two numbers, one above the other, with either a straight or slanted line in between. The straight-line style is the one used most (see Figure 3.1.1).

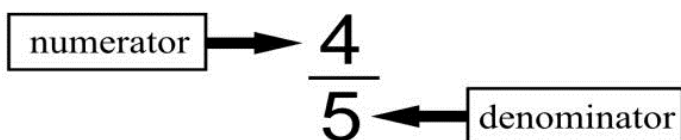


Figure 3.1.1

The **denominator** is the bottom number. It tells how many equal parts are in the whole thing. The **numerator** is the top number. It tells how many of the equal parts we are dealing with.

Example A



Figure 3.1.2

The whole thing is the bunch of bananas (see Figure 3.1.2). The whole thing has 5 equal parts (the bananas). The denominator is **5**.

How many bananas have been eaten? **1**

What fraction of the bananas have been eaten?

$\frac{1}{5}$ of the bananas

Example B



Figure 3.1.3

The whole thing is a carton of one dozen eggs (see Figure 3.1.3). One dozen has 12 equal parts, so if we are talking about the carton, the denominator is **12**.

How many eggs are still in the carton? **7**

What fraction of the eggs are left? $\frac{7}{12}$ of the eggs

A fraction is always looking at things as **parts of a whole**. In the

previous example of the eggs, the **whole** is 12 eggs. The **part** is the 7 eggs that are left. 7 is part of the whole of 12.

Example C



Figure 3.1.4

This pizza is **one whole** pizza (see Figure 3.1.4). The pizza is cut into 8 pieces. This means the whole is 8. How many

parts are left? (The pieces that are shaded are the ones left.)
Write a fraction of how many pieces of pizza are left.

The amount left over can be shown as a fraction: $\frac{4}{8}$

Example D

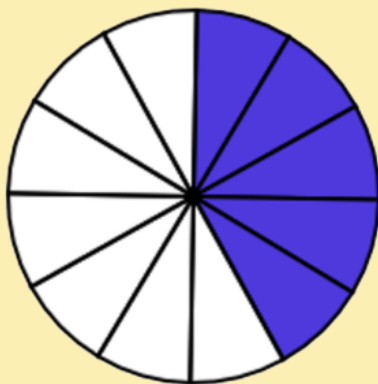


Figure 3.15

It is Peter's 82nd birthday. There were 7 people, including Peter, at the party. Everyone wants a small piece of cake, so

Kathleen cut the cake into 12 equal parts (see Figure 3.1.5). This means the **whole** is 12. There will be some left over.

The amount left over can be shown as a fraction: $\frac{5}{12}$

Example E

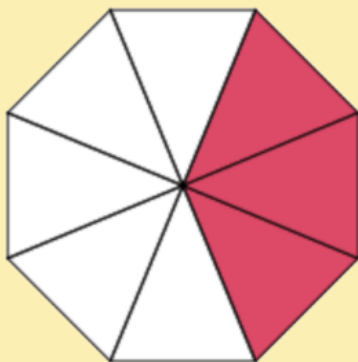


Figure 3.1.6

Sue made a strawberry pie to share with her family of 5. The pie was cut into 8 equal parts. This means the **whole** is 8. The kids are excited because there will be parts left over.

The fraction showing what amount of pie is left is: $\frac{3}{8}$

Exercise 1

Look at the fraction below. Fill in the blank labels in Figure 3.1.7 with the four **terms** you are given:

- numerator
- denominator
- whole
- fraction

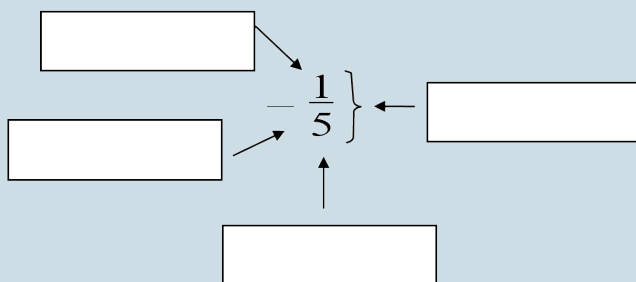


Figure 3.1.7

Exercise 1 Answers

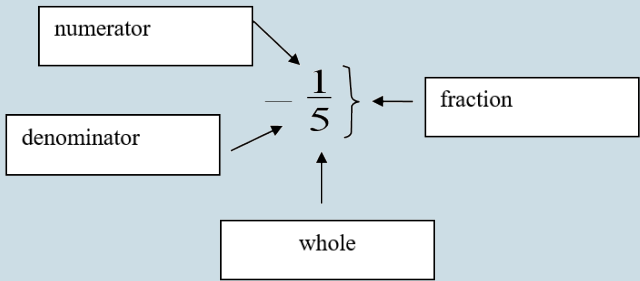


Figure 3.1.8

Writing Common Fractions

Exercise 2

Each shape drawn here is a whole. The shapes have been divided into parts.

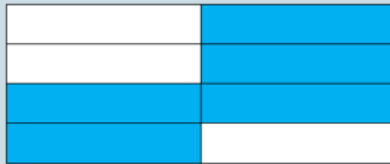
1. Ask yourself, "How many equal parts in the whole?" That number is the denominator.
2. Count the number of parts that are shaded; that is the numerator.
3. Write the common fraction to describe the shaded portion of each shape.

Example:



Figure 3.1.9

- i. How many parts make the whole? **4**
- ii. How many parts are shaded? **3**
- iii. Fraction: $\frac{3}{4}$



a.

Figure 3.1.10

- i. How many parts make the whole?
- ii. How many parts are shaded?
- iii. Fraction:

b.



Figure 3.1.11

- i. How many parts make the whole?
- ii. How many parts are shaded?
- iii. Fraction:

c.

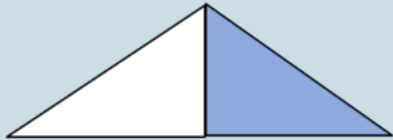


Figure 3.1.12

- i. How many parts make the whole?
- ii. How many parts are shaded?
- iii. Fraction:

Exercise 2 Answers

a. Square.

- i. 4
- ii. 3

iii. $\frac{3}{4}$

b. Rectangle.

i. 8

ii. 5

iii. $\frac{5}{8}$

c. Rectangle.

i. 5

ii. 2

iii. $\frac{2}{5}$

d. Triangle.

i. 2

ii. 1

iii. $\frac{1}{2}$

Exercise 3

Now, let's draw some fractions.

Example: Draw the fraction $\frac{1}{2}$ in a circle (see Figure 3.1.13):

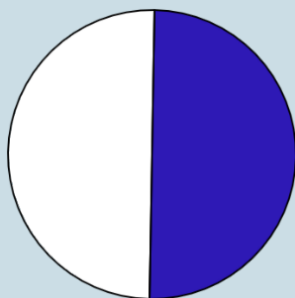
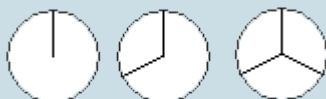


Figure 3.1.13

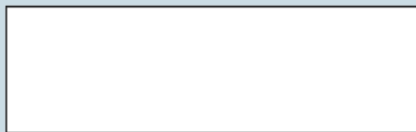
- a. Draw $\frac{1}{4}$ in a circle:
- b. Draw $\frac{1}{3}$ in a circle (here is a hint below):



- c. Draw $\frac{1}{2}$ in the rectangle:



- d. Draw $\frac{2}{4}$ in the rectangle:

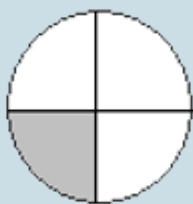


- e. Draw $\frac{4}{8}$ in the rectangle:



- f. What do you see in common with the three last boxes you just drew?

Exercise 3 Answers



a.



b.

- c. 
- d. 
- e. 
- f. All the shaded spaces are equal

Reading Common Fractions

You can read fractions in a few different ways:

- $\frac{1}{2}$ can be called:
 - one over two
 - one half
- $\frac{1}{4}$ can be called:
 - one over four
 - one fourth
 - one quarter
- $\frac{3}{4}$ can be called:
 - three over four
 - three fourths
 - three quarters
- $\frac{1}{3}$ can be called:

- one over three
- one third

Did you notice that the fractions with a denominator of four have an extra name (see Figure 3.1.14)? Do they seem familiar? No other fractions have a third name.

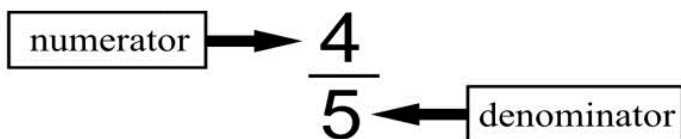


Figure 3.1.14

Reading Numerators & Denominators

Remember: The numerator is read as a number.

The denominator is read with a special ending at the end of the numeral. The ending is usually **th** or **ths** as it was in decimals, but sometimes we use a different word. These are called **ordinal numbers** (see Table 3.1.1).

Table 3.1.1: How to Read Denominators

If the Denominator Is...	Read...
2	half
3	third
4	fourth or quarter
5	fifth
6	sixth
7	seventh
8	eighth
9	ninth
10	tenth
22	twenty-second

Add the “s” if the numerator is 2 or more. So $\frac{2}{3}$ is read “two-thirds.”

Note that the usual practice is to put a hyphen (-) between the words when you write them out:

- $\frac{1}{2}$ is usually read “one-half.”

- $\frac{2}{2}$ is read “two-halves.”
- $\frac{3}{4}$ is read as “three-quarters” or “three-fourths.”

Exercise 4

Look back again at Exercise 2 and write down the word names for your answers.

Example: $\frac{3}{4}$ = three quarters or three-fourths

a. $\frac{3}{4}$

d. $\frac{1}{4}$

b. $\frac{5}{8}$

e. $\frac{2}{4}$

c. $\frac{2}{5}$

f. $\frac{1}{3}$

Exercise 4 Answers

a. Three quarters or three-fourths	c. Two-fifths
b. Five-eighths	d. One quarter or one-fourth
	e. Two quarters or two-fourths

f. One-third

Applying Common Fractions

We make common fractions out of many things in our lives. For example:

- I got 13 out of 15 on my English test. The score is $\frac{13}{15}$.
- The baseball pitcher struck out 2 of the 6 batters in the inning. $\frac{2}{6}$ of the batters were struck out.
- Three of the eggs in that dozen are cracked. $\frac{3}{12}$ of the eggs are cracked.
- Finish your vegetables. I gave you just 8 pieces of carrot, and you have only eaten 4 of them! $\frac{4}{8}$ of the carrots are eaten.

Exercise 5

Answer the questions using a common fraction.

Example: Jill walks for 20 minutes of the 30-minute lunch break. What fraction of her lunch break does Jill walk?

Answer: $\frac{20}{30}$

- a. The test was scored out of 25. Kim got 20 marks. Write his score.
- b. The restaurant has 12 tables. Each waiter looks after 6 of them. What fraction of the tables does each waiter look after?
- c. The new litter of puppies is a big one — 10 pups. Three of the pups have floppy ears. What fraction of the puppies have floppy ears?
- d. Beryl planted 3 dozen tulip bulbs last fall. A mole ate one dozen of them before they flowered. That mole is in trouble!! What fraction of the tulips did the mole eat?
- e. Kay's raisin cookie recipe uses 5 cups of flour altogether. Kay always puts in 2 cups of whole wheat flour and 3 cups of white flour. What fraction of the flour that she uses is whole wheat?

Exercise 5 Answers

a. $\frac{20}{30}$
b. $\frac{20}{25}$

c. $\frac{6}{12}$
d. $\frac{3}{10}$

e. $\frac{1}{3}$

f. $\frac{2}{5}$

These common fractions that you have been writing are called proper fractions. **Proper fractions** are fractions where the numerator is smaller than the denominator.

How Do We Compare Common Fractions?

Example F

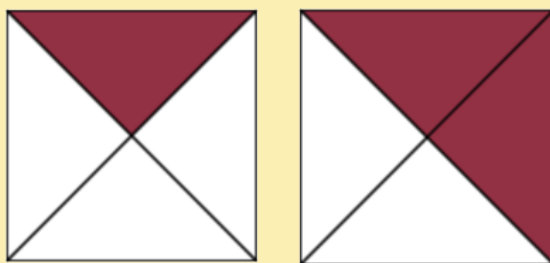


Figure 3.1.15

Step 1: In Figure 3.1.15, which square has more shaded parts?

The second square has more shaded parts.

Step 2: Which is larger: $\frac{1}{4}$ or $\frac{2}{4}$?

$\frac{2}{4}$ is larger because it fills in more parts of the square.

Review:

- Greater than $>$ Less than
- Less than $<$ Greater than

Example G

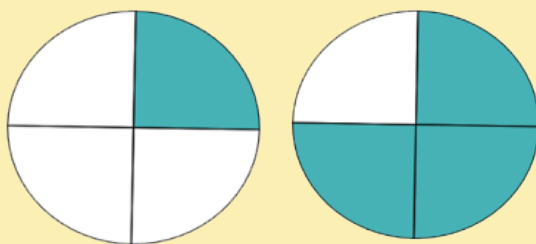


Figure 3.116

Step 1: In Figure 3.116, which circle is shaded more?
(**Hint:** Look at the squares above to help answer this question.)

Step 2: Write the fractions for both drawings: $\frac{1}{4}$ and $\frac{3}{4}$

Step 3: Which fraction is larger? Place a symbol (< or >) in the space above to show your answer.

$$\frac{1}{4} < \frac{3}{4}$$

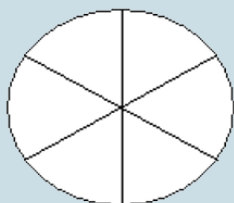
Exercise 6

Compare fractions.

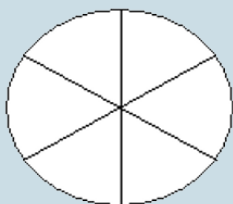
a.

- i. Shade $\frac{2}{6}$ and $\frac{5}{6}$.
- ii. Circle the fraction that is larger.
- iii. Write a mathematical sentence stating which

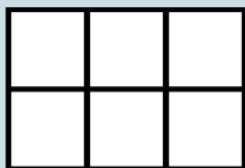
fraction is larger (use $<$ or $>$).



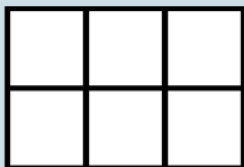
b.



- i. Shade $\frac{4}{6}$ and $\frac{3}{6}$.
- ii. Circle the fraction that is larger.
- iii. Write a mathematical sentence stating which fraction is larger (use $<$ or $>$).



c.



- i. Shade $\frac{2}{6}$ and $\frac{1}{6}$.
- ii. Circle the fraction that is larger.
- iii. Write a mathematical sentence stating which fraction is larger (use $<$ or $>$).

Exercise 6 Answers

- a. $\frac{2}{6} < \frac{5}{6}$
- b. $\frac{4}{6} > \frac{3}{6}$
- c. $\frac{2}{6} > \frac{1}{6}$

When comparing fractions:

As the numerator gets larger and the denominator stays the same, the fraction gets larger. E.g. $\frac{3}{10} < \frac{7}{10}$

Exercise 7

Use the < or > symbols to show which fraction is larger.

a. $\frac{3}{4}$ _____ $\frac{1}{4}$
b. $\frac{5}{6}$ _____ $\frac{1}{6}$

c. $\frac{9}{10}$ _____ $\frac{1}{10}$
d. $\frac{3}{8}$ _____ $\frac{5}{8}$

Exercise 7 Answers

- a. >
b. >

- c. >
d. <

Exercise 8

Show which is larger by using < or >.

- a. One-fourth _____ Three-fourths

- b. Five-sixths _____ Four-ninths
 c. Five-ninths _____ Two-sevenths
 d. Seven-sevenths _____ Three-sevenths

Exercise 8 Answers

- a. <
 b. >
 c. >
 d. >

Exercise 9

Compare fractions with the same denominator. Put the fractions in order from smallest to largest.

- a. $\frac{3}{4}, \frac{1}{4}, \frac{2}{4}$
 b. $\frac{6}{7}, \frac{2}{7}, \frac{3}{7}, \frac{1}{7}, \frac{5}{7}, \frac{4}{7}$
 c. $\frac{50}{361}, \frac{23}{361}, \frac{7}{361}, \frac{360}{361}, \frac{274}{361}, \frac{158}{361}$

Exercise 9 Answers

a. $\frac{1}{4}, \frac{2}{4}, \frac{3}{4}$

b. $\frac{1}{7}, \frac{2}{7}, \frac{3}{7}, \frac{4}{7}, \frac{6}{7}$

c. $\frac{7}{361}, \frac{23}{361}, \frac{50}{361}, \frac{158}{361}, \frac{274}{361}, \frac{360}{361}$

Using a Number Line to Compare Fractions

Using a number line is another way to look at how numbers compare to each other. Fractions can also be plotted on a number line. The number line is numbered 0 to 2. The section between 0 and 1 is split into fractions.

Example H

Draw a line to connect the fractions listed to the fractions on the number line (see Figure 3.1.17).

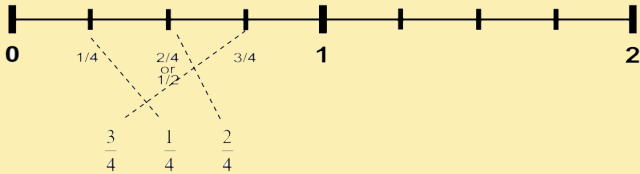
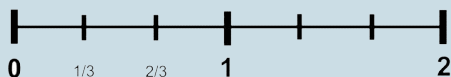


Figure 3.1.17

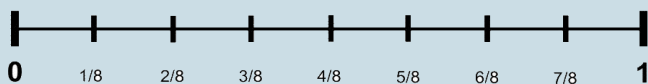
Exercise 10

Draw a line between the following fractions and the fractions on the number line.

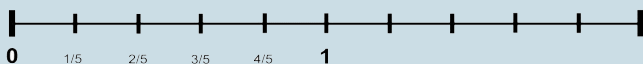
a. $\frac{2}{3}$ $\frac{1}{3}$



b. $\frac{6}{8}$ $\frac{2}{8}$ $\frac{5}{8}$ $\frac{4}{8}$ $\frac{1}{8}$ $\frac{7}{8}$ $\frac{3}{8}$



c. $\frac{2}{5}$ $\frac{1}{5}$



d. Complete the rule for comparing fractions. Circle the correct word.

As the numerator gets **bigger** / **smaller** and the denominator stays the same, the fraction gets **bigger** / **smaller** / **stays the same**.

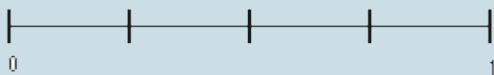
Exercise 10 Answers

Check your answers for Exercise 10 with your instructor.

Exercise 11

Write the fractions on the number lines in order.

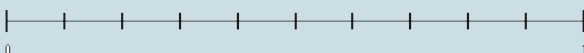
a. $\frac{1}{2}$ $\frac{1}{4}$ $\frac{3}{4}$



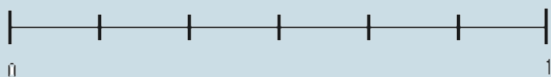
b. $\frac{2}{3}$ $\frac{1}{3}$



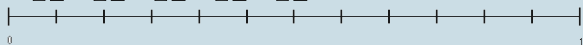
c. $\frac{3}{10}$ $\frac{7}{10}$ $\frac{5}{10}$ $\frac{8}{10}$ $\frac{1}{10}$



d. $\frac{5}{6}, \frac{2}{6}, \frac{3}{6}, \frac{1}{6}$



e. $\frac{7}{12}, \frac{5}{12}, \frac{3}{12}, \frac{10}{12}, \frac{1}{12}$



Exercise II Answers

a. $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}$

b. $\frac{1}{3}, \frac{2}{3}$

c. $\frac{1}{10}, \frac{3}{10}, \frac{5}{10}, \frac{7}{10}, \frac{8}{10}, \frac{7}{10}, \frac{8}{10}$

d. $\frac{1}{6}, \frac{2}{6}, \frac{3}{6}, \frac{5}{6}$

e. $\frac{1}{12}, \frac{3}{12}, \frac{5}{12}, \frac{7}{12}, \frac{10}{12}$

Comparing Fractions With Different Denominators

You now know how to compare fractions with the same denominator, but how do you do it when the denominators of two fractions are different?

Example I

Let's look at some circles:

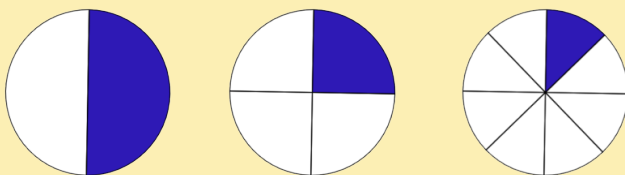


Figure 3.118

Write a fraction for each circle in Figure 3.118 above:

$$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$$

Which is the largest of the three fractions:

$$\frac{1}{2}$$

Look at the following rectangles:

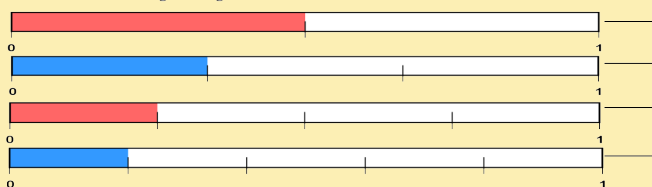


Figure 3.1.19

Write in the fraction for each shaded part in Figure 3.1.19:

$$\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$$

Write the fractions in order from the largest to the smallest:

$$\frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$$

Exercise 12

Look back on the last three fraction drawing sets and compare the following fractions with < or >.

a. $\frac{1}{2}$ _____ $\frac{1}{8}$

b. $\frac{1}{6}$ _____ $\frac{1}{2}$

c. $\frac{1}{4}$ _____ $\frac{1}{3}$

d. $\frac{1}{3}$ _____ $\frac{1}{8}$

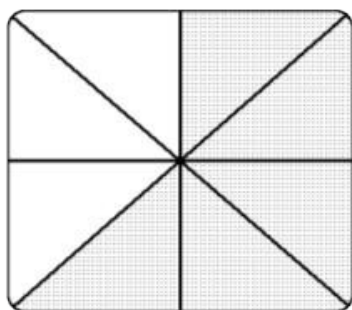
Exercise 12 Answers

a. >
b. <

c. <
d. >

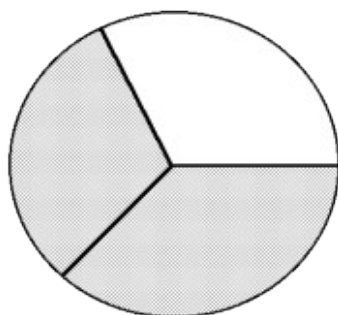
3.1: Practice Questions

1. Write a common fraction to describe:
 - i. the shaded part of each whole thing.
 - ii. the unshaded part of each whole thing.



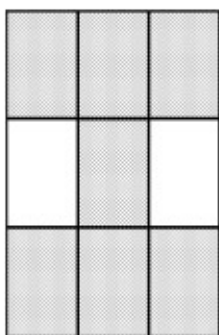
a.

- i. Shaded:
- ii. Unshaded:



b.

- i. Shaded:
- ii. Unshaded:



- c.
- i. Shaded:
 - ii. Unshaded:
2. Draw the following fractions.
- a. $\frac{3}{4}$
 - b. $\frac{1}{5}$
3. Write the word name for the following fractions.
- a. $\frac{3}{4}$
 - b. $\frac{1}{5}$
 - c. $\frac{3}{7}$
4. Answer the question using a common fraction.
- a. The government has ordered the closing of 24 beds at the local hospital. The townspeople are angry because the hospital only has 100 beds in all. What fraction of the hospital beds are being closed?
 - b. The young man ordered six roses for his girlfriend. He asked for five red ones and a special yellow rose. What fraction of the roses are red?

5. Compare the following fractions, use < or >.

a. $\frac{4}{5}$ _____ $\frac{3}{5}$

b. $\frac{12}{23}$ _____ $\frac{20}{23}$

c. $\frac{1}{3}$ _____ $\frac{2}{3}$

3.1: Practice Answers

1. Write a common fraction to describe.

a. Square

1. $\frac{5}{8}$

2. $\frac{3}{8}$

b. Circle

1. $\frac{2}{3}$

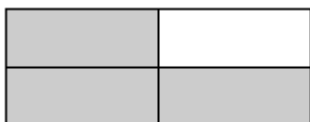
2. $\frac{1}{3}$

c. Rectangle

1. $\frac{7}{9}$

2. $\frac{2}{9}$

2. Draw the following fractions.



a.



b.

3. Write the word name for the following fractions.

- a. Three quarters or Three fourths
- b. One fifth
- c. Three sevenths

4. Answer the question using a common fraction.

- a. $\frac{24}{100}$
- b. $\frac{5}{6}$

5. Compare the following fractions, use < or >.

- a. >
- b. <
- c. <

Attribution

All figures in this chapter are from Topic A: Introducing Common Fractions in Adult Literacy Fundamental Mathematics: Book 5 – 2nd

Edition by Liz Girard, Wendy Tagami, and Leanne Caillier-Smith, via BCcampus.

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3.2: Common Fractions

There are three types of fractions:

- **Proper fractions** are part of the whole thing.
- **Improper fractions** are equal to 1 or are greater than 1.
- **Mixed numbers** are greater than one.

In a **proper fraction**, the numerator is smaller than the denominator. Proper fractions are less than one.

$$\cdot \frac{3}{4} < 1$$

$$\cdot \frac{2}{5} < 1$$

$$\cdot \frac{9}{10} < 1$$

$$\cdot \frac{4}{7} < 1$$

In **improper fractions**, the numerator is the same or larger than the denominator.

$$\cdot \frac{4}{4} = 1$$

$$\cdot \frac{3}{3} = 1$$

$$\cdot \frac{8}{3} > 1$$

$$\cdot \frac{9}{7} > 1$$

In **mixed numbers**, a whole number and a proper fraction are used together.

$$\cdot 1\frac{1}{2} > 1$$

$$\cdot 4\frac{3}{7} > 1$$

$$\cdot 3\frac{2}{5} > 1$$

$$\cdot 1\frac{9}{10} > 1$$

Here are some pictures to visualize mixed numbers:

Example A



Figure 3.2.1

You want to give three small children only $\frac{1}{2}$ an apple each. You need three half apples (see Figure 3.2.1).

You can write that as $\frac{3}{2}$ (three halves). How will you get $\frac{3}{2}$ of an apple?

You must use **more than one apple**.

$$\frac{3}{2} = 1\frac{1}{2} \text{ apples}$$

This is one whole apple and $\frac{1}{2}$ of another one.

Example B



Figure 3.2.2

In Figure 3.2.2, 10 pieces of pizza are shown. Each pizza was cut into 8 pieces.

The fraction can be written as $\frac{10}{8}$, which is an **improper fraction**.

It can also be written as a **mixed number**: $1\frac{2}{8}$. This is 1 whole pizza and $\frac{2}{8}$ of another one.

Exercise 1

Answer these questions.



**Figure
3.2.3**

- a.
 - i. How many pieces of apple are shown in Figure 3.2.3?
 - ii. Each apple was cut into 4 pieces, so the denominator is .
 - iii. Write the improper fraction that describes the photo.

- iv. The photo shows whole apple and of a second apple.
- v. Write the mixed number that describes the apple.

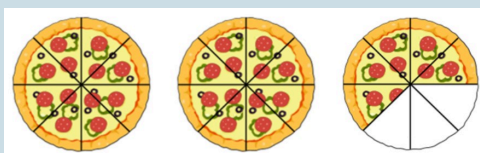


**Figure
3.2.4**

- b.
 - i. How many pieces of pizza are shown in Figure 3.2.4?
 - ii. Each pizza was cut into 8 pieces, so the denominator is .
 - iii. Write the improper fraction that describes the photo.
 - iv. The photo shows whole pizzas and of a third pizza.
 - v. Write the mixed number that describes the pizza.

Exercise 1 Answers

a. Apples.

- i. $\frac{7}{4}$
- ii. $\frac{4}{7}$
- iii. $\frac{7}{4}$

iv. $1 \text{ and } \frac{3}{4}$

v. $1 \frac{3}{4}$

b. Pizza.

$$\begin{array}{l} \text{i. } \frac{21}{8} \\ \text{ii. } \frac{21}{8} \\ \text{iii. } \frac{21}{8} \end{array}$$

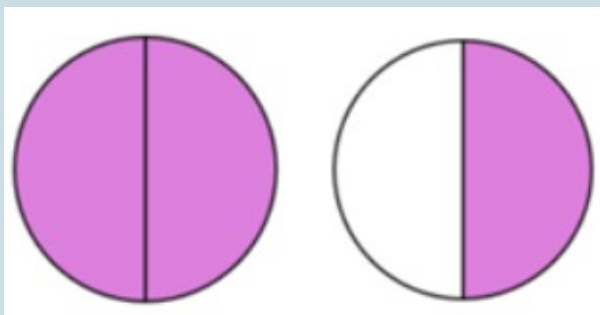
$$\begin{array}{l} \text{iv. } 2 \text{ and } \frac{5}{8} \\ \text{v. } 2\frac{5}{8} \end{array}$$

Exercise 2

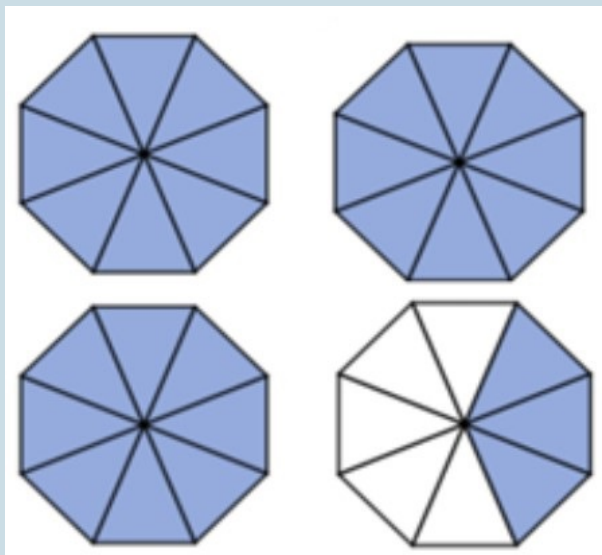
Write the **improper fraction** and the **mixed number** that describe the shaded part in each drawing.

First, decide on the denominator. The denominator is what one whole thing has been divided into.

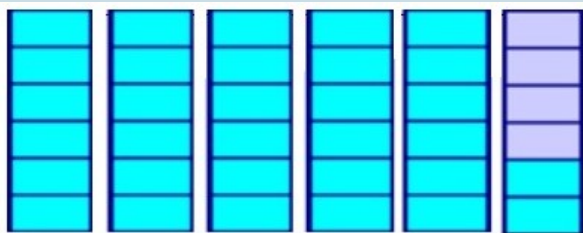
Example:



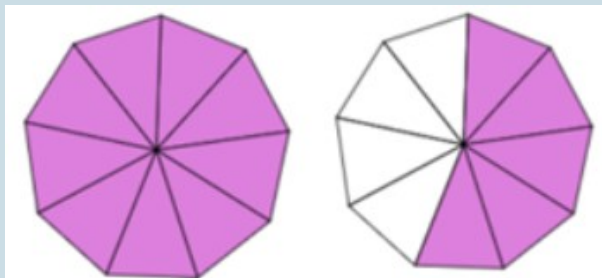
Answer: $\frac{3}{2} = 1\frac{1}{2}$



a.



b.



c.

d.



Exercise 2 Answers

a. $\frac{27}{8} = 3\frac{3}{8}$

b. $\frac{32}{6} = 5\frac{2}{6}$

c. $\frac{14}{9} = 1\frac{5}{9}$

d. $\frac{11}{4} = 2\frac{3}{4}$

Now, it is your turn to draw some **mixed numbers**. The following is an example of how to do that.

Example C

Shade in this fraction: $4\frac{3}{4}$

(Note: you will not need to use all the squares drawn below.)

Answer: see Figure 3.2.5.

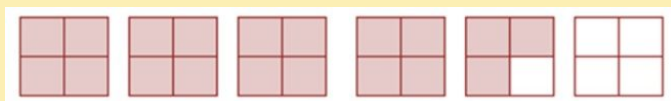


Figure 3.2.5

Exercise 3

Shade the following mixed fractions in the given shapes.

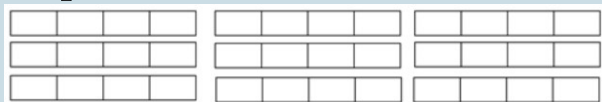
a. $2\frac{3}{5}$



b. $3\frac{2}{3}$



c. $5\frac{1}{4}$



d. $1\frac{1}{2}$



Draw the following mixed fractions:

e. $4\frac{1}{2}$

g. $2\frac{2}{3}$

f. $3\frac{4}{5}$

h. $5\frac{3}{4}$

Exercise 3 Answers

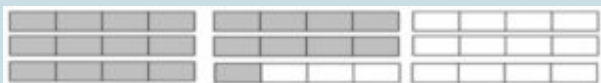
a.



b.

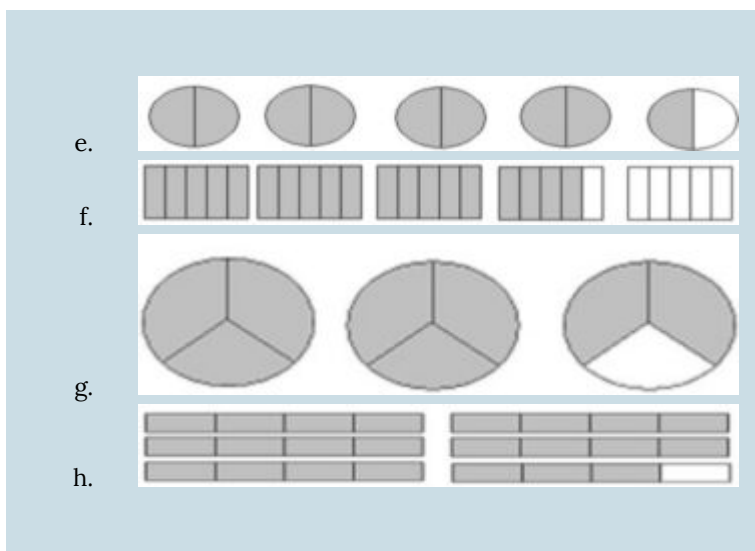


c.



d.





Writing Improper Fractions as Mixed Numbers

In the last exercise, you were able to write (rename) an improper fraction as a mixed number by looking at the drawing — you could see how many whole things were represented.

You could see that:

- If the denominator was 3 — Every time you had $\frac{3}{3}$  that was one whole.
- If the denominator was 6 — Every time you had $\frac{6}{6}$  that was one whole
- And so on.

An improper fraction is written (renamed) as a mixed number by

dividing the **numerator** by the **number of parts in the whole** (the **denominator**).

Example D

Write $\frac{8}{5}$ as a mixed number.

Step 1: Write as a long division problem or as a short division question.

$$\text{denominator} \overline{) \text{numerator}} = 5 \overline{) 8}$$

$$\text{numerator} \div \text{denominator} = 8 \div 5$$

Step 2: Divide. Write the remainder as a fraction, using the same denominator.

1. The 1 becomes the whole number.
2. Use the **remainder** (3) as the numerator
3. Use the divisor (5) as the denominator.

$$\begin{array}{r} 1 \\ 5 \overline{) 8} \\ \underline{-5} \\ \text{R } 3 \end{array}$$

or

$$8 \div 5 = 1 \text{ R } 3$$

$$\frac{8}{5} = 1\frac{3}{5}$$

Example E

Write $\frac{16}{4}$ as a mixed number.

$$4 \overline{)16} \text{ or } 16 \div 4$$

$$\begin{array}{r} 4 \\ 4 \overline{)16} \\ -16 \\ \hline 0 \end{array}$$

or $16 \div 4 = 4$

$$\frac{16}{4} = 4$$

Exercise 4

Rewrite each improper fraction as an equivalent mixed number or whole number.

$$\text{a. } \frac{9}{2} =$$

$$\text{d. } \frac{17}{6} =$$

$$\text{g. } \frac{7}{4} =$$

$$\text{b. } \frac{11}{10} =$$

$$\text{e. } \frac{13}{3} =$$

$$\text{c. } \frac{20}{4} =$$

$$\text{f. } \frac{20}{10} =$$

Exercise 4 Answers

$$\text{a. } 4\frac{1}{2}$$

$$\text{d. } 2\frac{5}{6}$$

$$\text{g. } 1\frac{3}{4}$$

$$\text{b. } 1\frac{1}{10}$$

$$\text{e. } 4\frac{1}{3}$$

$$\text{c. } 5$$

$$\text{f. } 2$$

Renaming Mixed Numbers as Improper Fractions

This process will be used when you multiply and divide common fractions and “borrow” in subtraction.

Example F

Take the whole number 2.

Here are 2 equal shapes (see Figure 3.2.6).

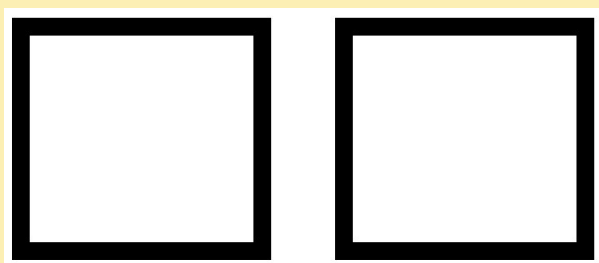


Figure 3.2.6

The shapes are each divided into 3 parts (thirds) (see Figure 3.2.7).

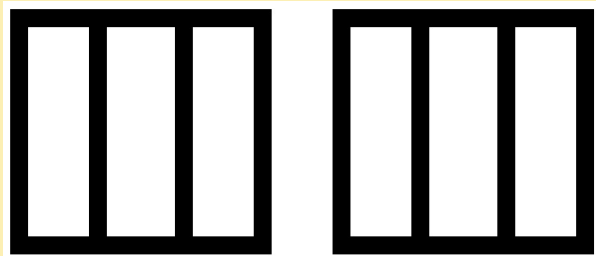


Figure 3.2.7

How many thirds are there? 6 thirds.

$$2 = \frac{6}{3}$$

Example G

Take the whole number 1.

Draw one shape (a circle or a box) (see Figure 3.2.8).

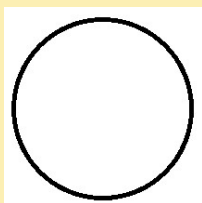


Figure 3.2.8

Now, divide the shape into half (see Figure 3.2.9).

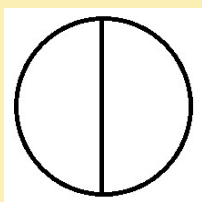


Figure 3.2.9

How many halves are there?

$$1 = \frac{2}{2}$$

Have You Found the Shortcut?

The shortcut to rename a whole number as an improper fraction is to **multiply the whole number by the denominator**.

$$\bullet \quad 2 = \frac{\quad}{4}$$

- Each whole number has 4 equal parts (4 quarters) in it,

- So

$$2 \text{ (whole numbers)} \times 4 \text{ (parts)} = 8 \text{ parts}$$

- So $2 = \frac{8}{4}$

$$\bullet \quad 6 = \frac{\quad}{2}$$

- Each whole number has 2 equal parts (2 halves),

- So

$$6 \text{ (whole numbers)} \times 2 \text{ (parts)} = 12 \text{ parts}$$

- So $6 = \frac{12}{2}$

$$\bullet \quad 8 = \frac{\quad}{3}$$

- $8 \times 3 = 24$

- So $8 = \frac{24}{3}$

Until you are comfortable, draw a little sketch like you did in the example.

Exercise 5

Rename each whole number as an improper fraction using the denominator shown.

a. $6 = \frac{24}{4}$, $2 = \frac{3}{3}$, $4 = \frac{3}{3}$, $8 = \frac{80}{10}$

b. $7 = \frac{7}{3}$, $1 = \frac{4}{4}$, $6 = \frac{60}{10}$, $5 = \frac{5}{6}$

Exercise 5 Answers

a. $\frac{24}{4}$, $\frac{6}{3}$, $\frac{12}{3}$, $\frac{80}{10}$

b. $\frac{21}{3}$, $\frac{4}{4}$, $\frac{60}{10}$, $\frac{30}{6}$

Now, let's take this idea further. How can you rename a **mixed number** as an improper fraction?

Example H

Take the mixed number $2\frac{1}{4}$.

This is two whole things and **part of a third** whole thing.

Figure 3.2.10 shows three equal shapes divided into fourths.

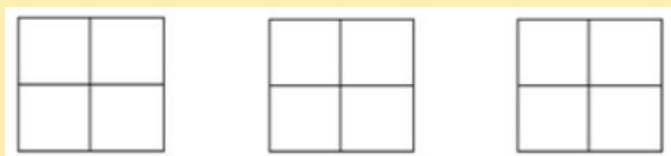


Figure 3.2.10

Shade in two whole shapes $\frac{9}{4}$ and $\frac{1}{4}$ of the third shape.

How many fourths have you shaded in all? 9 fourths

$$2\frac{1}{4} = \frac{9}{4}$$

Example I

Take the mixed number $1\frac{2}{3}$. This is one whole thing and part of a second whole thing.

Draw two equal shapes and divide them into thirds (see Figure 3.2.11).

Shade in one whole shape $\frac{3}{3}$ and $\frac{2}{3}$ of the second shape.

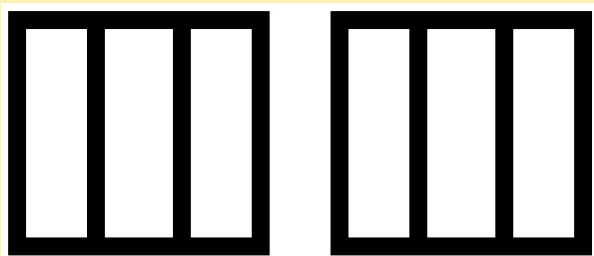


Figure 3.2.11

How many thirds have you shaded in all? _ thirds

$$1\frac{2}{3} = \frac{\quad}{3}$$

To rename a mixed number as an improper fraction, multiply the

whole number by the denominator of the fraction and then add this to the numerator. Write the **total** as the new numerator over the denominator.

$$3\frac{1}{4} = \frac{13}{4}$$

$$3 = \frac{12}{4}, \text{ so } 3\frac{1}{4} = \frac{12}{4} + \frac{1}{4} = \frac{13}{4}$$

$$2\frac{1}{2} \quad 2 \times 2 = \frac{4}{2} + \frac{1}{2} = \frac{5}{2}$$

Exercise 6

Rename each mixed number as an improper fraction.

a. $2\frac{4}{5} = \frac{14}{5}$	e. $8\frac{4}{5} =$	i. $1\frac{8}{10} =$
b. $8\frac{3}{8} = \frac{67}{8}$	f. $6\frac{1}{8} =$	j. $3\frac{6}{8} =$
c. $6\frac{5}{8} = \frac{5}{8}$	g. $5\frac{3}{6} =$	
d. $5\frac{1}{2} = \frac{1}{2}$	h. $2\frac{7}{8} =$	

Exercise 6 Answers

a. $\frac{14}{5}$

b. $\frac{67}{8}$

c. $\frac{53}{8}$

d. $\frac{11}{2}$

e. $\frac{44}{5}$

f. $\frac{49}{8}$

g. $\frac{33}{6}$

h. $\frac{23}{8}$

i. $\frac{18}{10}$

j. $\frac{30}{8}$

To Say or Write a Mixed Number

If you want to say these strange looking fractions out loud, do as follows.

Write or say: the whole number + and + the fraction

- $3\frac{2}{5}$ = three and two fifths
- $5\frac{1}{6}$ = five and one sixth

Exercise 7

Write the following fractions as words.

a. $2\frac{1}{8}$

b. $4\frac{3}{4}$

c. $1\frac{1}{2}$

d. $3\frac{3}{4}$

Exercise 7 Answers

- a. two and one eighth c. one and one half
b. four and three fourths (or d. three and three fourths (or
four and three quarters) three and three quarters)

3.2: Practice Questions

1. Write the improper fraction and the mixed number that describe the shaded part of the drawings in each question.

a.





2. Rename each improper fraction as an equivalent mixed number or whole number.

a. $\frac{9}{4} =$

c. $\frac{5}{3} =$

b. $\frac{12}{3} =$

d. $\frac{7}{2} =$

3. Rename as improper fractions.

a. $3\frac{1}{2} =$

c. $1\frac{7}{8} =$

b. $4\frac{3}{8} =$

d. $2\frac{5}{6} =$

3.2: Practice Answers

1. Write the improper fraction and the mixed number that describe the shaded part of the drawings in each question.

a. $\frac{10}{3} = 3\frac{1}{3}$

b. $\frac{7}{4} = 1\frac{3}{4}$

2. Rename each improper fraction as an equivalent mixed number or whole number.

a. $2\frac{1}{4}$

b. 4

c. $1\frac{2}{3}$

d. $3\frac{1}{2}$

3. Rename as improper fractions

a. $\frac{7}{2}$

b. $\frac{35}{8}$

c. $\frac{15}{8}$

d. $\frac{17}{6}$

Attribution

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3.3: When to Use a Fraction or a Decimal

Sometimes, you need to decide to write or say a number as a fraction or a decimal. We usually choose the most common method which is usually the easiest way to say something. Right now, it may seem to you that nothing is easy, so here are some tips.

Money

We almost always talk about a part of a dollar when talking about money,.

Example: Two dollars and fifteen cents is two whole dollars and fifteen parts of one more dollar.

\$2.15 or $\$2\frac{15}{100}$ — which of these ways of writing money is more common or easier to you?

We usually write or talk about money in decimals:

- **Example 1:** \$2.50 (two dollars and fifty cents) instead of $\$2\frac{1}{2}$ (two and a half dollars)
- **Example 2:** \$0.50 (fifty cents) instead of $\$\frac{1}{2}$ (half a dollar)
- **Example 3:** \$67.30 (sixty-seven dollars and thirty cents) instead of \$67 (sixty-seven dollars and thirty-hundredths cents)

However, there is one place where we talk about money as a fraction: The quarter!

A quarter equals \$0.25, but we often say, “It costs a quarter,” as much as we say, “It costs 25 cents.”

Really, if we were speaking correctly, it would be a quarter of a dollar, but it gets shortened. Also, we still write \$0.25 or 25¢, not $\$\frac{25}{100}$ or $\$\frac{1}{4}$.

For Other Things: What Is Easiest

In most other ways of talking and writing, fractions and decimals are expressed in what seems easiest. This means that you get to say the number in the way you like best.

Example: Saying, “Six point four grams,” may be faster and easier than saying, “Six and two-fifths grams.”

But, saying $\frac{3}{4}$ of a tank of gas makes more sense than saying 0.75 of a tank of gas.

Exercise 1

Circle the way that you think a number should be said out loud.

Read the numbers out loud with a friend to help hear them. Remember that sometimes your answers will be different from the ones in the answer key because you have a different opinion, and that is OK.

- a. \$12.25 or $12\frac{1}{4}$
- b. 51.4 cm or $51\frac{2}{5}$ cm
- c. One million two hundred thousand dollars or one point two million dollars

- d. 563.56 km or $563\frac{56}{100}$ km
- e. $5\frac{7}{10}$ L or 5.7 L
- f. I ran a tenth of a kilometre or I ran zero point one kilometres
- g. It weighs five and a half grams or It weighs five point five grams
- h. $\$39\frac{99}{100}$ or \$39.99

Exercise 1 Answers

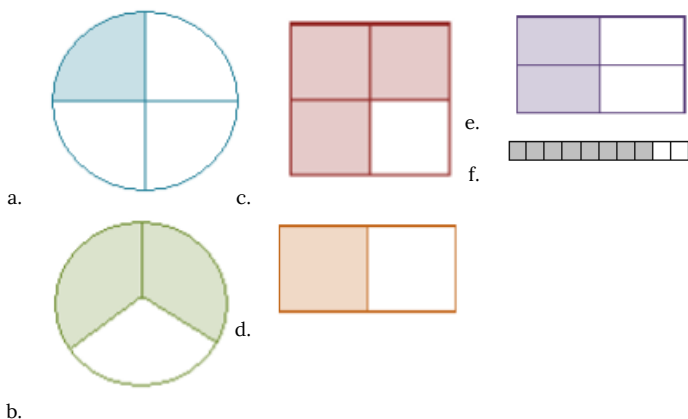
a. \$12.25	d. 563.56 km	g. Both
b. 51.4 cm	e. 5.7 L	h. \$39.99
c. One point million dollars	twof. I ran a tenth of a kilometre	

Attribution

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3.4: Common Fractions Review

1. Write fractions from the pictures.



2. Draw your own fractions.

a. $\frac{2}{5}$

b. $\frac{3}{4}$

c. $\frac{4}{9}$

d. $\frac{1}{6}$

e. $\frac{1}{3}$

f. $\frac{7}{10}$

3. Write the following fractions in words.

a. $\frac{1}{2}$

b. $\frac{1}{4}$

c. $\frac{3}{4}$

d. $\frac{21}{25}$

e. $\frac{1}{3}$

f. $\frac{5}{6}$

4. Answer the questions using a common fraction.

- a. Suzie jogged 20 minutes out of 1 hour. What fraction of the

hour did she jog? (Remember, 1 hour = 60 minutes.)

- b. Oliver planted 30 garlic cloves in September. 25 shoots have come up in the spring. What fractions of garlic bulbs did not grow a shoot?
 - c. The class usually had 8 students, but 6 did not come on Monday. What fraction of students did not come?
 - d. Stephen made 60 Easter cookies. His brother ate 3, and his mom ate 3 more. What fraction of cookies were eaten by Stephen's family?
 - e. Save On Foods grocery store sold 300 dozen eggs in one week. They had 450 dozen in stock. What fraction of the stock was sold?
 - f. The test was out of 32. Sasha got 30 marks. What was her score?
5. Compare the following fractions.

a. $\frac{3}{4} - \frac{1}{4}$

b. $\frac{9}{10} - \frac{3}{10}$

c. $\frac{1}{5} - \frac{3}{5}$

d. $\frac{3}{8} - \frac{7}{8}$

e. seven tenths $\frac{\quad}{10}$ three tenths $\frac{\quad}{10}$

f. one quarter $\frac{\quad}{4}$ three quarters $\frac{\quad}{4}$

g. four fifths $\frac{\quad}{5}$ three fifths $\frac{\quad}{5}$

h. one twelfth $\frac{\quad}{12}$ eleven twelfths $\frac{\quad}{12}$

6. Identify each fraction by writing: **proper fraction**, **improper fraction**, or **mixed number** to each fraction.

a. $\frac{1}{2}$

b. $\frac{100}{47}$

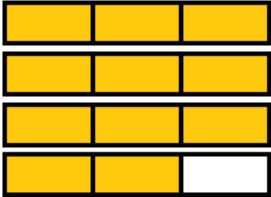
c. $\frac{5}{6}$

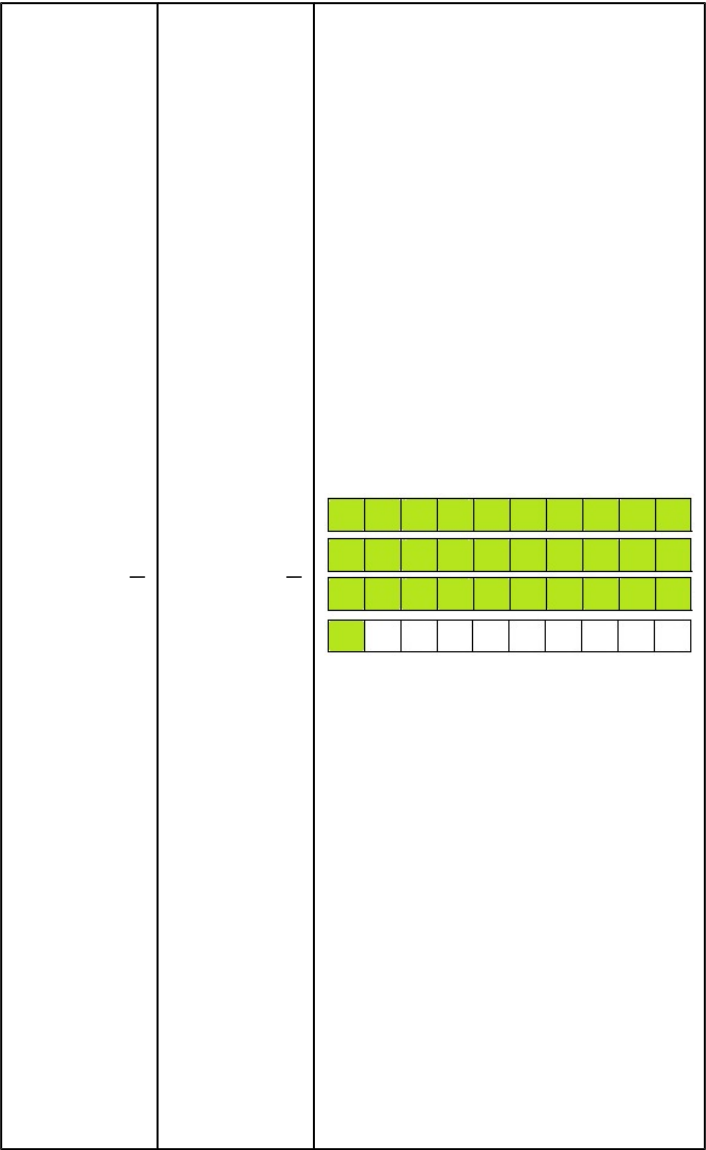
d. $\frac{51}{2}$

e. $42\frac{1}{4}$

7. Fill in the missing parts of the chart (see Table 3.4.1).

Table 3.4.1: Question 7 Fill in the Table

Improper Fraction	Mixed Number	Drawing
—	—	
$\frac{29}{6}$	—	—
—	$2\frac{3}{4}$	—



$\frac{9}{2}$	—	—
—	$3\frac{1}{3}$	—

8. Convert each improper fraction to a mixed number or a whole number.

a. $\frac{9}{5} =$

d. $\frac{14}{5} =$

g. $\frac{7}{3} =$

b. $\frac{7}{2} =$

e. $\frac{4}{3} =$

h. $\frac{8}{4} =$

c. $\frac{11}{2} =$

f. $\frac{11}{6} =$

9. Convert each whole number to an improper fraction. Use the denominator given to you.

a. $6 = \frac{\quad}{3}$

d. $7 = \frac{\quad}{3}$

g. $2 = \frac{\quad}{10}$

b. $5 = \frac{\quad}{2}$

e. $8 = \frac{\quad}{7}$

c. $3 = \frac{\quad}{7}$

f. $1 = \frac{\quad}{3}$

10. Convert each mixed number to an improper fraction.

$$\text{a. } 6\frac{7}{8} =$$

$$\text{c. } 15\frac{4}{5} =$$

$$\text{e. } 18\frac{1}{2} =$$

$$\text{b. } 2\frac{1}{2} =$$

$$\text{d. } 20\frac{3}{7} =$$

$$\text{f. } 5\frac{9}{11} =$$

3.4: Review Answers

1. Write fractions from the pictures.

$$\text{a. } \frac{1}{4}$$

$$\text{d. } \frac{1}{2}$$

$$\text{g. } \frac{8}{10}$$

$$\text{b. } \frac{2}{3}$$

$$\text{e. } \frac{2}{4}$$

$$\text{c. } \frac{3}{4}$$

$$\text{f. } \frac{4}{6}$$

2. Draw your own fractions.

3. Write the following fractions in words.

a. one half

d. twenty-one twenty fifths

b. one fourth or one quarter

e. one third

c. three fourths or three quarters

f. five sixths

4. Answer the questions using a common fraction.

$$\text{a. } \frac{20}{60}$$

$$\text{c. } \frac{6}{8}$$

$$\text{e. } \frac{300}{450}$$

$$\text{b. } \frac{5}{30}$$

$$\text{d. } \frac{6}{60}$$

$$\text{f. } \frac{30}{32}$$

5. Compare the following fractions:

a. >

d. <

g. >

b. >

e. >

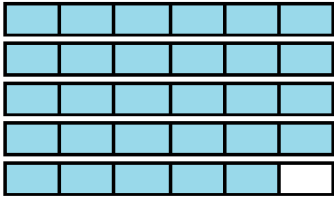
h. <

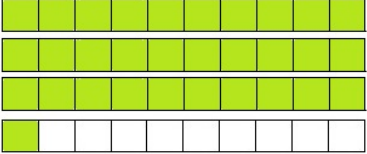
c. <

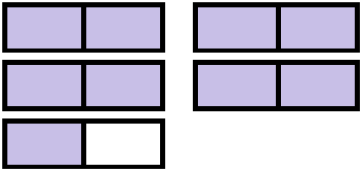
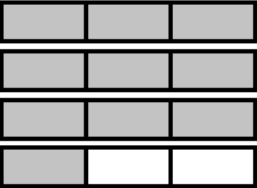
f. <

6. Identify each fraction by writing: **proper fraction**, **improper fraction**, or **mixed number** to each fraction.
- | | |
|----------------------|----------------------|
| a. proper fraction | d. improper fraction |
| b. improper fraction | e. mixed number |
| c. proper fraction | |
7. Fill in the missing parts of the chart (see Table 3.4.2).

Table 3.4.2: Questions 7 Answers

Improper Fraction	Mixed Number	Drawing
$\frac{11}{3}$	$3\frac{2}{3}$	
$\frac{29}{6}$	$4\frac{5}{6}$	
$\frac{11}{4}$	$2\frac{3}{4}$	

$\frac{31}{10}$	$3\frac{1}{10}$	

$\frac{9}{2}$	$4\frac{1}{2}$	
$\frac{10}{3}$	$3\frac{1}{3}$	

8. Convert each improper fraction to a mixed number or a whole number.

a. $1\frac{4}{5}$

d. $2\frac{4}{5}$

g. $2\frac{1}{3}$

b. $3\frac{1}{2}$

e. $1\frac{1}{3}$

h. 2

c. $5\frac{1}{2}$

f. $1\frac{5}{6}$

9. Convert each whole number to an improper fraction. Use the denominator given to you.

a. $\frac{28}{3}$

d. $\frac{21}{3}$

g. $\frac{20}{10}$

b. $\frac{10}{2}$

e. $\frac{56}{7}$

c. $\frac{21}{7}$

f. $\frac{3}{3}$

10. Convert each mixed number to an improper fraction.

a. $\frac{55}{8}$

c. $\frac{79}{5}$

e. $\frac{37}{2}$

b. $\frac{5}{2}$

d. $\frac{143}{7}$

f. $\frac{64}{11}$

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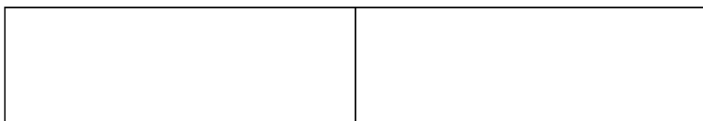
3.5: Equivalent Fractions

Start from the left side of each drawing and shade in the fraction shown.

This shape is the whole thing:



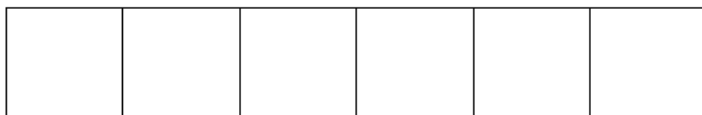
Shade $\frac{1}{2}$:



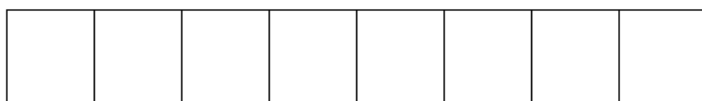
Shade $\frac{2}{4}$:



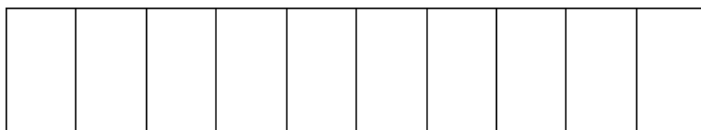
Shade $\frac{3}{6}$:



Shade $\frac{4}{8}$:



Shade $\frac{5}{10}$:



Did you notice that the amount you shaded was the same in each drawing?

The fractions that you were asked to shade are **equivalent fractions**. Equivalent fractions **are fractions that are equal**.

Now shade the fractions asked for in these drawings the same way.

This shape is the whole thing:

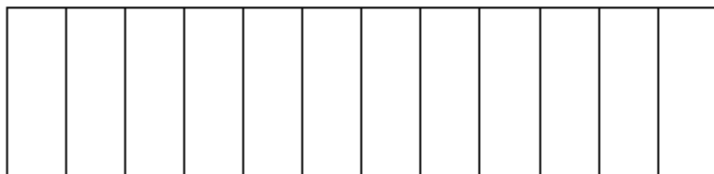


Shade $\frac{1}{3}$:

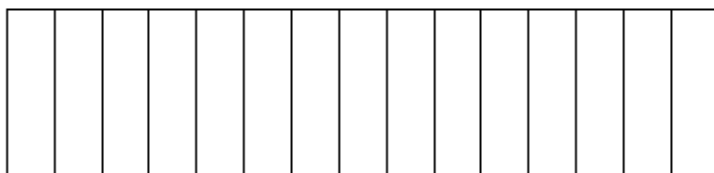
Shade $\frac{2}{6}$

Shade $\frac{3}{9}$:

Shade $\frac{4}{12}$:



Shade $\frac{5}{15}$:



The above examples are all equivalent fractions.

$$\frac{1}{3} = \frac{2}{6} = \frac{3}{9} = \frac{4}{12} = \frac{5}{15}$$

Working with common fractions often requires using an equivalent fraction in place of the given fraction. You will learn about several processes to help you to find equivalent fractions.

Factors

Factors are the numbers which are multiplied together to make a

product. An understanding of factors is needed to express fractions in the **lowest terms**.

Example A

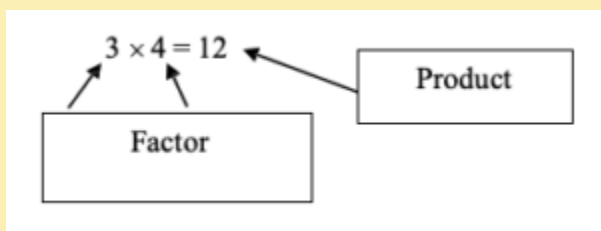


Figure 3.5.1

We say, “The factors of 12 are 3 and 4” (see Figure 3.5.1).

Does 12 have any other factors?

What other numbers can be multiplied together to equal 12?

- $1 \times 12 = 12$ or $12 \times 1 = 12$
- $2 \times 6 = 12$ or $6 \times 2 = 12$
- $3 \times 4 = 12$ or $4 \times 3 = 12$

The factors of 12 are 1, 2, 3, 4, 6, 12.

Example B

Find the factors of 10.

- $1 \times 10 = 10$
- $2 \times 5 = 10$

The factors of 10 are 1, 2, 5, 10.

Example C

Find the factors of 9.

- $1 \times 9 = 9$
- $3 \times 3 = 9$

The factors of 9 are 1, 3, 9.

Exercise 1

Find all the factors.

Examples:

The factors of 16:
 $1 \times 16 = 16$; $2 \times 8 = 16$; $4 \times 4 = 16$
The factors of 16 are 1, 2, 4, 8, 16.

The factors of 4: 1
 $\times 4 = 4$; $2 \times 2 = 4$
The factors of 16
are 1, 2, 4, 8, 16.

a. The factors of 8: _____	e. The factors of 21: _____
b. The factors of 20: _____	f. The factors of 6: _____
c. The factors of 5: _____	g. The factors of 25: _____
d. The factors of 15: _____	

Exercise 1 Answers

a. 1, 2, 4, 8	d. 1, 3, 5, 15	g. 1, 5, 25
b. 1, 2, 4, 5, 10, 20	e. 1, 3, 7, 21	
c. 1, 5	f. 1, 2, 3, 6	

Some numbers **only have two factors: 1 and the number itself**.
These numbers are called **prime numbers**.

Look at Table 3.5.1 for some prime numbers.

Table 3.5.1: Factors of Prime Numbers

Prime Numbers	Factors
1	1, 1
2	1, 2
3	1, 3
5	1, 5
7	1, 7
11	1, 11
13	1, 13
17	1, 17
19	1, 19
23	1, 23
29	1, 29

Add other prime numbers to the chart as you find them.

Reminder: Prime numbers only have two prime factors.

Finding Common Factors

A **common factor** is a number used to reduce the numerator and denominator.

Example D

What are the common factors for $\frac{4}{6}$?

Find the factors of 4 and 6:

- The factors of 4 are 1, 2, 4.
- The factors of 6 are 1, 2, 3, 6.

What factors do 4 and 6 have in common?

- 4: 1, 2, 4
- 6: 1, 2, 3, 6

The common factors of 4 and 6 are **1 and 2**.

For the above equation, the factors are 1 and 2; however, 1 is not used as a **common factor**. This is because 1 is a factor of all **whole numbers**.

Example E

What are the common factors for $\frac{6}{15}$?

Find the factors of 6 and 15:

- The factors of 6 are 1, 2, 3, 6
- The factors of 15 are 1, 3, 5, 15.

What factors do 6 and 15 have in common?

- 6: 1, 2, 3, 6
- 15: 1, 3, 5, 15

The common factor of 6 and 15 is 3

Example F

Find the common factors of $\frac{16}{24}$?

Find the factors of 16 and 24:

- The factors of 16 are 1, 2, 4, 8, 16.
- The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.

The common factors of 16 and 24 are: 2, 4, 8.

8 is called the **greatest common factor (GCF)** of 16 and 24 because it is the largest of all the common factors.

Exercise 2

Find the **common factors** for each set of numbers. Then, identify the **greatest common factor (GCF)**.

Examples:

10, 15

Factors of 10: 1, 2, 5, 10

Factors of 15: 1, 3, 5, 15

Common factors: 5

Greatest common factor: 5

4, 16

Factors of 4: 1, 2, 4

Factors of 16: 1, 2, 4, 8, 16

Common factors: 2, 4

Greatest common factor: 4

a. 9, 12

- Factors of 9:
- Factors of 12:
- Common factors:
- Greatest common factor:

- Factors of 20:
- Factors of 30:
- Common factors:
- Greatest common factor:

c. 18, 12

b. 20, 30

- Factors of 18:

- Factors of 12:
 - Common factors:
 - Greatest common factor: f. 6, 9
- d. 24, 32
- Factors of 24:
 - Factors of 32:
 - Common factors:
 - Greatest common factor: g. 9, 15
- e. 8, 12
- Factors of 8:
 - Factors of 12:
 - Common factors:
 - Factors of 9:
 - Factors of 15:
 - Common factors:
 - Greatest common factor:

Exercise 2 Answers

- a. 9, 12
- Factors of 9: 1, 3, 9
 - Factors of 12: 1, 2, 3, 4, 6, 12
 - Common factors: 3
 - Greatest common factor: 3
- d. 24, 32
- Common factors: 2, 3, 6
 - Greatest common factor: 6
 - Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
 - Factors of 32: 1, 2, 4, 8, 16, 32
 - Common factors: 2, 4, 8
 - Greatest common factor: 8
- b. 20, 30
- Factors of 20: 1, 2, 4, 5, 10, 20
 - Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30
 - Common factors: 2, 5, 10
 - Greatest common factor: 10
- e. 8, 12
- Factors of 8: 1, 2, 4, 8
 - Factors of 12: 1, 2, 3, 4, 6, 12
 - Common factors: 2, 4
 - Greatest common factor: 4
- c. 18, 12
- Factors of 18: 1, 2, 3, 6, 9, 18
 - Factors of 12: 1, 2, 3, 4, 6, 12
 - Factors of 6: 1, 2, 3, 6
- f. 6, 9

◦ Factors of 9: 1, 3, 9 ◦ Common factors: 3 ◦ Greatest common factor: 3	◦ Factors of 9: 1, 3, 9 ◦ Factors of 15: 1, 3, 5, 15 ◦ Common factors: 3 ◦ Greatest common factor: 3
- g. 9, 15

Expressing Fractions in Lower Terms

Express means to say it or write it.

Lower terms mean to express equivalent fractions with smaller (lower) denominators.

The words **simplify** and **reduce** are another way to say “express fractions in lower (or lowest) terms.”

To express a fraction in the lowest terms, do this:

Step 1: Find the greatest common factor (GCF) of the numerator and denominator.

- $\frac{4}{12}$ The factors of 4 are 1, 2, 4
- The factors of 12 are 1, 2, 3, 4, 6, 12

The GCF is 4.

Step 2: Divide the numerator and the denominator by the greatest common factor.

- $\frac{4}{12} \div 4 = \frac{1}{3}$
- $\frac{4}{12} = \frac{1}{3}$

Example G

$$\frac{6}{9}$$

Step 1:

- The factors of 6 are 1, 2, 3, 6.
- The factors of 9 are 1, 3, 9.

The GCF is 3.

Step 2:

$$\frac{6 \div 3}{9 \div 3} = \frac{2}{3}$$
$$\frac{6}{9} = \frac{2}{3}$$

Example H

$$\frac{15}{24}$$

Step 1:

- The factors of 15 are 1, 3, 5, 15
- The factors of 24 are 1, 3, 4, 6, 8, 24.

The GCF is 3.

Step 2:

$$\frac{15 \div 3}{24 \div 3} = \frac{5}{8}$$
$$\frac{15}{24} = \frac{5}{8}$$

There are several reasons lower terms are used:

- The math is usually easier with lower numbers.
- Is it easier to think of $\frac{1}{2}$ an apple or $\frac{15}{30}$ of an apple? ($\frac{1}{2} = \frac{15}{30}$)
- Do you want to think about $\frac{155}{620}$ of your pay cheque or $\frac{1}{4}$ of your pay cheque? ($\frac{1}{4} = \frac{155}{620}$)
- Always express fractions in lowest terms!

Dividing the numerator and denominator by the GCF will give an equivalent fraction in lower terms.

Exercise 3

Express each fraction in the **lowest terms**. (The directions could also say, “Simplify each fraction,” or, “Reduce these fractions”).

Make sure that you write in the GCF you are dividing with. Do not skip this step until you are absolutely sure you can do it correctly in your head each time.

(Good mathematicians know when to skip steps and when not to. Sometimes easy steps are never skipped by good mathematicians).

a. $\frac{2 \div 2}{4 \div 2} = \frac{1}{2},$

$\frac{3 \div}{9 \div} = \underline{\hspace{2cm}}$

b. $\frac{2 \div 2}{12 \div 2} = \frac{1}{6},$

$\frac{3 \div}{15 \div} = \underline{\hspace{2cm}}$

c. $\frac{5 \div}{10 \div} = \underline{\hspace{2cm}},$

$\frac{4 \div}{24 \div} = \underline{\hspace{2cm}}$

d. $\frac{10 \div}{25 \div} = \underline{\hspace{2cm}},$

$\frac{9 \div}{12 \div} = \underline{\hspace{2cm}}$

e. $\frac{3}{30} = \underline{\hspace{2cm}},$

$\frac{6}{10} = \underline{\hspace{2cm}}$

f.	$\frac{9}{\frac{24}{4}} = \underline{\hspace{2cm}}$	$\frac{18}{\frac{27}{3}} = \underline{\hspace{2cm}}$
g.	$\frac{16}{15} = \underline{\hspace{2cm}}$	$\frac{12}{15} = \underline{\hspace{2cm}}$
h.	$\frac{24}{2} = \underline{\hspace{2cm}}$	$\frac{25}{6} = \underline{\hspace{2cm}}$
i.	$\frac{2}{32} = \underline{\hspace{2cm}}$	$\frac{6}{20} = \underline{\hspace{2cm}}$

Exercise 3 Answers

a.	$\frac{1}{2}, \frac{1}{3}$	f.	$\frac{3}{8}, \frac{2}{3}$
b.	$\frac{1}{6}, \frac{1}{5}$	g.	$\frac{1}{4}, \frac{1}{4}$
c.	$\frac{1}{2}, \frac{1}{6}$	h.	$\frac{5}{8}, \frac{3}{8}$
d.	$\frac{2}{5}, \frac{3}{4}$	i.	$\frac{1}{16}, \frac{3}{10}$
e.	$\frac{1}{10}, \frac{3}{5}$		

Expressing Fractions in Higher Terms

Higher Terms are needed when you add and subtract fractions with different denominators.

You have learned that dividing the numerator and denominator of a fraction by a common factor gives an equivalent fraction in lower terms. You know that dividing and multiplying are opposite operations, so this next rule will match the one you just learned for reducing:

Multiplying the numerator and denominator of a fraction by the same number (a common factor) will give an **equivalent fraction** in **higher terms**.

Example I

$$\frac{3}{5} \left(\frac{\times 2}{\times 2} \right) = \frac{6}{10}$$
$$\frac{3}{5} = \frac{6}{10}$$

Example J

$$\frac{1}{2} \left(\frac{\times 8}{\times 8} \right) = \frac{8}{16}$$
$$\frac{1}{2} = \frac{8}{16}$$

Example K

$$\frac{2}{3} \left(\frac{\times 3}{\times 3} \right) = \frac{6}{9}$$
$$\frac{2}{3} = \frac{6}{9}$$

Are the Fractions Equivalent?

If the denominators are the same, you can easily judge if the fractions are equivalent by comparing the numerators.

Compare $\frac{4}{5}$ and $\frac{3}{5}$: $\frac{4}{5} \neq \frac{3}{5}$ ($\neq$ means 'not equal')

Compare $\frac{12}{20}$ and $\frac{12}{20}$: $\frac{12}{20} = \frac{12}{20}$

If the denominators are different, you might be able to rewrite one or more of the fractions so they have the same denominator.

Compare $\frac{4}{5}$ and $\frac{6}{10}$: $\frac{6 \div 2}{10 \div 2} = \frac{3}{5}$ So:

$$\frac{4}{5} \neq \frac{3}{5}$$

Compare $\frac{12}{16}$ and $\frac{5}{8}$: $\frac{5 \times 2}{8 \times 2} = \frac{10}{16}$ So:

$$\frac{12}{16} \neq \frac{10}{16}$$

or you could do this: $\frac{12 \div 2}{16 \div 2} = \frac{6}{8}$ So:

$$\frac{6}{8} \neq \frac{5}{8}$$

A quick method is to **cross-multiply**:

1. Multiply the numerator of one fraction by the denominator of the second fraction
2. Multiply the numerator of the other fraction by the denominator of the first fraction These are called the cross-products.

If the cross products are the same, then the fraction is equivalent.

Example L

Compare $\frac{4}{7}$ and $\frac{5}{9}$ (see Figure 3.5.2).



Figure 3.5.2

Step 1: Multiply the numerator 4 by the denominator 9.

$$4 \times 9 = 36$$

Step 2: Multiply the denominator 7 by the numerator 5.

$$7 \times 5 = 35$$

Step 3: The products 36 and 35 **are not** the same.

Therefore, $\frac{4}{7} \neq \frac{5}{9}$

Example M

Compare $\frac{2}{3}$ and $\frac{12}{18}$ (see Figure 3.5.3).

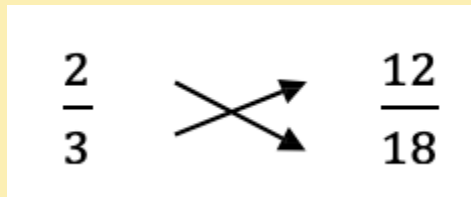


Figure 3.5.3

$$2 \times 18 = 36$$

$$3 \times 12 = 36$$

The products 36 and 36 **are** the same.

Therefore, $\frac{2}{3} \neq \frac{12}{18}$

Example N

Compare $\frac{24}{40}$ and $\frac{4}{10}$

$$24 \times 10 = 240$$

$$40 \times 4 = 160$$

The products 240 and 160 **are not** the same.

Therefore, $\frac{24}{40} \neq \frac{4}{10}$

Exercise 4

State if each pair is **equivalent** (=) or **not equivalent** (≠).
Use whichever method you wish to find the answer.

a. $\frac{5}{6} =$ _____ $\frac{30}{60}$

b. $\frac{12}{24} =$ _____

$$\begin{array}{lll}
 \frac{1}{2} & \frac{24}{72} & \text{h. } \frac{4}{10} = \underline{\hspace{2cm}} \\
 \text{c. } \frac{6}{7} = \underline{\hspace{2cm}} & \frac{3}{4} = \underline{\hspace{2cm}} & \frac{50}{5} = \underline{\hspace{2cm}} \\
 \text{d. } \frac{2}{3} = \underline{\hspace{2cm}} & \frac{20}{12} = \underline{\hspace{2cm}} & \text{i. } \frac{10}{7} = \underline{\hspace{2cm}} \\
 \frac{12}{18} & \text{g. } \frac{14}{6} = \underline{\hspace{2cm}} & \\
 \text{e. } \frac{1}{3} = \underline{\hspace{2cm}} & \frac{7}{7} &
 \end{array}$$

Exercise 4 Answers

$$\begin{array}{lll}
 \text{a. } \neq & \text{d. } = & \text{g. } = \\
 \text{b. } = & \text{e. } = & \text{h. } = \\
 \text{c. } \neq & \text{f. } = & \text{i. } =
 \end{array}$$

Rounding Common Fractions to Whole Numbers

When rounding to a whole number, if a fraction is less than do not change the whole number.

Examples:

$$\bullet \ 2\frac{4}{7} \approx 2$$

$$\bullet \ 23\frac{1}{3} \approx 23$$

$$\cdot \frac{1}{4} \approx 0$$

$$\cdot 5\frac{3}{8} \approx 5$$

If the fraction is $\frac{1}{2}$ or more, consider the fraction as another one that must be added to the whole number.

Examples:

$$\cdot 2\frac{1}{2} \approx 3$$

$$\cdot 6\frac{7}{8} \approx 7$$

$$\cdot 15\frac{4}{5} \approx 16$$

$$\cdot \frac{3}{4} \approx 1$$

If you are not sure if a fraction is more or less than $\frac{1}{2}$, you can compare it to $\frac{1}{2}$, by making equivalent fractions with a common denominator.

Reminder: greater > smaller

Example O

Round $\frac{2}{3}$ to a whole number.

$$\text{Is } \frac{2}{3} > \frac{1}{2} ? \longrightarrow \frac{2}{3} = \frac{4}{6} \text{ and } \frac{1}{2} = \frac{3}{6}$$

$$\text{YES! } \frac{2}{3} > \frac{1}{2} \text{ so } \frac{2}{3} \approx 1$$

Example P

Round $2\frac{4}{7}$ to a whole number.

$$\text{Is } \frac{4}{7} > \frac{1}{2} ? \longrightarrow \frac{4}{7} = \frac{8}{14} \text{ and } \frac{1}{2} = \frac{7}{14}$$

$$\text{YES! } \frac{4}{7} > \frac{1}{2} \text{ so } 2\frac{4}{7} \approx 1$$

Exercise 5

Round to the nearest whole number.

Examples:

$$\frac{4}{5} \approx 1$$

$$2\frac{1}{3} \approx 2$$

- a. $18\frac{1}{2} \approx \underline{\hspace{1cm}}$ d. $\frac{1}{8} \approx \underline{\hspace{1cm}}$ h. $20\frac{3}{7} \approx \underline{\hspace{1cm}}$
b. $3\frac{7}{8} \approx \underline{\hspace{1cm}}$ e. $4\frac{1}{6} \approx \underline{\hspace{1cm}}$ i. $\frac{13}{15} \approx \underline{\hspace{1cm}}$
c. $9\frac{9}{10} \approx \underline{\hspace{1cm}}$ f. $12\frac{7}{9} \approx \underline{\hspace{1cm}}$ j. $99\frac{2}{3} \approx \underline{\hspace{1cm}}$
g. $6\frac{3}{5} \approx \underline{\hspace{1cm}}$

Exercise 5 Answers

a. 19	e. 4	i. 1
b. 4	f. 13	j. 100
c. 10	g. 7	
d. 0	h. 20	

3.5: Practice Questions

1. Define.

- Equivalent _____
- Prime number _____
- Greatest common factor (GCF) _____

2. Complete the chart.

Table 3.5.2: Question 2 Complete the Chart

Numbers	Factors	Common Factors	Greatest Common Factor
12 18	—	—	—
15 30	—	—	—
7 28	—	—	—
6 16	—	—	—
18 27	—	—	—

3. Express in lowest terms.

a. $\frac{10}{15} = \underline{\hspace{2cm}}$.

c. $\frac{8}{12} = \underline{\hspace{2cm}}$.

b. $\frac{14}{16} = \underline{\hspace{2cm}}$.

4. State if each pair of fractions is equivalent (=) or not equivalent (≠).

a. $\frac{5}{9}$ _____ $\frac{15}{27}$

b. $\frac{3}{7}$ _____ $\frac{15}{35}$

5. Round to the nearest whole number.

a. $4\frac{5}{8} \approx$ _____

d. $6\frac{3}{4} \approx$ _____

b. $19\frac{4}{10} \approx$ _____

e. $\frac{1}{3} \approx$ _____

c. $\frac{1}{2} \approx$ _____

3.5: Practice Answers

1. Define.

1. Check your definitions in the glossary

2. Complete the chart.

Table 3.5.3: Question 2 Answers

Numbers	Factors	Common Factors	Greatest Common Factor
12 18	... of 12 are 1, 2, 3, 4, 6, 12 ... of 18 are 1, 2, 3, 6, 9, 18	2, 3, 6	6
15 30	... of 15 are 1, 3, 5, 15 ... of 30 are 1, 2, 3, 5, 6, 10, 15, 30	3, 5, 15	15
7 28	... of 7 are 1, 7 ... of 28 are 1, 2, 4, 7, 14, 28	7	7
6 16	... of 6 are 1, 2, 3, 6 ... of 16 are 1, 2, 4, 8, 16	2	2
18 27	... of 18 are 1, 2, 3, 6, 9, 18 ... of 27 are 1, 3, 9, 27	3, 9	9

3. Express in lowest terms.

a. $\frac{2}{5}$
b. $\frac{7}{8}$

c. $\frac{2}{3}$

4. State if each pair of fractions is equivalent (=) or not equivalent (≠).

a. =
b. =

c. ≠
d. ≠

5. Round to the nearest whole number.

a. 5

b. 19

c. 1
d. 7

e. 0

Attribution

All figures in this chapter are from Topic A: Equivalent Fractions in Adult Literacy Fundamental Mathematics: Book 5 – 2nd Edition by Liz Girard, Wendy Tagami, and Leanne Caillier-Smith, via BCcampus.

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3.6: Equivalent Fractions Review

1. Find all the factors for each number. If a number is a prime number, write “prime” next to it.
 - a. 4 _____
 - b. 10 _____
 - c. 21 _____
 - d. 6 _____
 - e. 2 _____
 - f. 16 _____

2. Find the factors, common factors and the **Greatest Common Factor (GCF)** (fill in Table 3.6.1).

Table 3.6.1: Question 2 Fill in the Table

Fraction	Factors	Common Factors	GCF
$\frac{2}{8}$			
$\frac{8}{16}$			
$\frac{24}{32}$			
$\frac{9}{12}$			
$\frac{5}{15}$			
$\frac{25}{30}$			
$\frac{4}{12}$			

3. Express each fraction **in lowest terms**. Remember: be sure to write the greatest common factor (GCF) you are dividing with.

a. $\frac{6}{9} =$

c. $\frac{12}{28} =$

e. $\frac{4}{24} =$

b. $\frac{6}{18} =$

d. $\frac{15}{30} =$

f. $\frac{10}{18} =$

4. Circle the fractions that are in lowest terms.

$$\text{a. } \frac{1}{2} \\ \frac{3}{6}$$

$$\text{c. } \frac{4}{5} \\ \frac{3}{9}$$

$$\text{e. } \frac{4}{8} \\ \frac{5}{10}$$

5. Find all the fractions that are **not already** in lowest terms and reduce them. Write “lowest terms” next to those already reduced.

$$\text{a. } \frac{4}{8} = \\ \frac{2}{5} = \\ \frac{8}{12} =$$

$$\text{d. } \frac{15}{35} = \\ \frac{42}{80} = \\ \frac{6}{36} =$$

$$\text{g. } \frac{9}{15} =$$

6. State if each pair of fractions is equivalent (=) or not equivalent (≠).

$$\text{a. } \frac{4}{5} \text{ ————— } \frac{7}{8} \\ \frac{10}{12} \text{ ————— } \frac{5}{6} \\ \frac{5}{15} \text{ ————— } \frac{1}{3}$$

$$\text{d. } \frac{6}{7} \text{ ————— } \frac{36}{41} \\ \frac{3}{5} \text{ ————— } \frac{15}{25}$$

7. Round to the nearest whole number.

$$\text{a. } 1\frac{1}{4} = \\ 4\frac{3}{4} =$$

$$\text{c. } 6\frac{4}{5} = \\ 3\frac{1}{4} =$$

e. $12\frac{8}{9} =$

3.6: Review Answers

1. Find all the factors for each number.

a. 1,2,4

b. 1,2,5,10

c. 1,3,7,21

d. 1,2,3,6

e. 1,2, prime

f. 1,2,4,8,16

2. Find the factors, common factors and the GCF (see Table 3.6.2).

Table 3.6.2: Question 2 Answers

Fraction	Factors	Common Factors	GCF
$\frac{2}{8}$	1,2 1,2,4,8	1,2	2
$\frac{8}{16}$	1,2,4,8 1,2,4,8,16	2,4,8	8
$\frac{24}{32}$	1,2,3,4,6,8,12,24 1,2,4,8,16,32	2,4,8	8
$\frac{9}{12}$	1,3,9 1,2,3,4,6,12	3	3
$\frac{5}{15}$	1,5 1,3,5,15	5	5
$\frac{25}{30}$	1,5,25 1,2,3,5,6,10,15,30	5	5
$\frac{4}{12}$	1,2,4 1,2,3,4,6,12	24	4

3. Express each fraction in lowest terms.

a. $\frac{2}{3}$

b. $\frac{1}{3}$

c. $\frac{3}{3}$

d. $\frac{1}{2}$

e. $\frac{1}{6}$

f. $\frac{7}{8}$

g. $\frac{5}{9}$

4. Circle the fractions that are in lowest terms.

1. $\frac{1}{2}$

2. $\frac{4}{5}$

5. Find all the fractions that are not already in lowest terms and reduce them.

a. $\frac{1}{2}$

b. lowest terms

c. $\frac{2}{3}$

d. lowest terms

e. $\frac{(3)}{(7)}$

f. $\frac{21}{40}$

g. $\frac{1}{6}$

h. $\frac{3}{5}$

6. State if each pair of fractions is equivalent (=) or not equivalent ($\neq$).

1. $\neq$

2. $=$

3. $=$

4. $\neq$

5. $=$

7. Round to the nearest whole number.

1. 1

2. 5

3. 7

4. 3

5. 13

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3.7: Multiplying Fractions

Example A

What is $\frac{1}{4}$ of 4? (**Hint:** Replace the word **of** with a $\times$ sign)

Figure 3.7.1 shows four equal shapes:

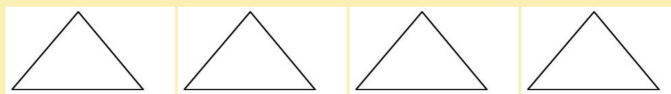


Figure 3.7.1

Shade in $\frac{1}{4}$ of the shapes. **You should have one shape shaded.**

You have just done this multiplication question:

$$\frac{1}{4} \times 4 = 1 \text{ or } \frac{1}{4} \text{ of } 4 = 1$$

Example B

What is $\frac{2}{5}$ of 5?

Step 1: Draw 5 equal shapes.

Step 2: Shade in $\frac{2}{5}$ of the shapes.

You should have shaded in two shapes.

$$\frac{2}{5} \times 5 = 2 \text{ or } \frac{2}{5} \text{ of } 5 = 2$$

Example C

What is $\frac{1}{2}$ of 10?

Step 1: Figure 3.7.2 shows 10 equal shapes:



Figure 3.7.2

Step 2: Shade in $\frac{1}{2}$ of the shapes.

Did you shade 5?

$$\frac{1}{2} \times 10 = 5 \text{ or } \frac{1}{2} \text{ of } 10 = 5 \text{ of } 10 = 5$$

Example D

What is $\frac{1}{4}$ of 8?



Figure 3.7.3

Step 1: Shade in $\frac{1}{4}$ of the shapes in Figure 3.7.3.

Step 2: Divide the 8 shapes into 4 equal groups.

Step 3: Shade 1 group.

$$\frac{1}{4} \times 8 = 2 \text{ or } \frac{1}{4} \text{ of } 8 = 2$$

Example E

What is $\frac{2}{3}$ of 6?



Figure 3.7.4

Step 1: Shade in $\frac{2}{3}$ of the shapes in Figure 3.7.4.

Step 2: Divide the 6 shapes into 3 equal groups.

Step 3: Shade 2 group.

$$\frac{2}{3} \times 6 = 4$$

These examples calculate a **fraction of a whole number**. Some everyday examples, with the answers, are listed:

- I burned $\frac{1}{2}$ of the hamburger patties. There were 8 patties.

How many patties were burned?

- $\frac{1}{2}$ of 8 patties = $\frac{1}{2} \times 8 = 4$ burned patties.

- Mary only finished $\frac{3}{4}$ of the test. The test had 20 questions.

How many questions did Mary finish?

- $\frac{3}{4}$ of test = $\frac{3}{4}$ of 20 questions = $\frac{3}{4} \times 20 = 15$ questions.

- $\frac{1}{5}$ of the employees have been laid off. There are 50 employees. How many have been laid off?

- $\frac{1}{5}$ of 50 employees = $\frac{1}{5} \times 50 = 10$ employees laid off.

- We spend $\frac{1}{4}$ of our monthly take-home pay on rent. Our take-home pay is \$1600. How much is the rent?

- $\frac{1}{4}$ of pay = $\frac{1}{4}$ of \$1600 = $\frac{1}{4} \times \$1600 = \$400$ on rent

Exercise 1

Write the multiplication equation you would use to find the fraction of the whole number. You do not have to calculate the answers.

- a. More than $\frac{1}{3}$ of the students are single parents.

There are 27 students. How many students are single parents?

- b. We have ten houses on our street $\frac{2}{5}$ of the houses have cedar shake roofs. How many houses have cedar shake roofs?

- c. The guinea hen hatched 16 chicks. The ravens snatched $\frac{3}{8}$ of the chicks. How many chicks did the ravens take?

Exercise 1 Answers

a. $\frac{1}{3} \times 27 = 9$

b. $\frac{2}{5} \times 10 = 4$

c. $\frac{3}{8} \times 16 = 6$

Now, let's look at multiplying a fraction by a whole number:

• $4 \times \frac{1}{3} =$

• $3 \times \frac{4}{5} =$

$$\bullet \quad 22 \times \frac{1}{4} =$$

The order of writing the multiplication equation will not change the product, but it does change how we understand what the numbers mean. Again, look at the examples:

Example F

$4 \times \frac{1}{2}$ means you have four halves.

Imagine tomatoes cut in half and you have 4 halves (see Figure 3.7.5).

How many tomatoes would you have altogether?



Figure 3.7.5

4 halves = 2 tomatoes

$$4 \times \frac{1}{2} = 2$$

Example G

$3 \times \frac{1}{4}$ means that you have $\frac{1}{4}$ of something three times.

Imagine that you spent $\frac{1}{4}$ of an hour exercising in the morning, $\frac{1}{4}$ of an hour exercising after lunch, and $\frac{1}{4}$ of an hour exercising in the evening.

How long did you exercise?

$$3 \times \frac{1}{4} \text{ hour} = \frac{3}{4} \text{ hour} = \text{three quarters of an hour}$$

Here are some everyday examples of multiplying a fraction by a whole number:

- There are six boxes of cereal open in the cupboard, and each one is $\frac{1}{3}$ full. It is the same as having _____ full boxes of cereal.
 - $6 \times \frac{1}{3} \text{ box of cereal} = \frac{6}{3} = 2 \text{ boxes of cereal}$
- We have three packs of ground beef that are $\frac{1}{2}$ full. How much meat is there altogether?
 - $3 \times \frac{1}{2} \text{ pack of meat} = \frac{3}{2} = 1\frac{1}{2} \text{ packs of meat.}$

- How much gas do we have for the motorboat? There are 4 jerry cans (cans for carrying gas), each about $\frac{1}{4}$ full.
 - $4 \times \frac{1}{4}$ cans of gas = $\frac{4}{4} = 1$ can of gas.

Exercise 2

Write the multiplication equation you would use to multiply a fraction by a whole number. You do not have to calculate the answer.

- I should buy more shampoo, but this bathroom has five bottles of the stuff lying around! Each bottle is about $\frac{1}{8}$ full. How much shampoo is there altogether?
- When we double a recipe, we multiply each ingredient by 2. Double a recipe that uses $\frac{1}{4}$ teaspoon of nutmeg. How much nutmeg is needed?

Exercise 2 Answers

- $5 \times \frac{1}{8} = \frac{5}{8}$

$$\text{b. } \frac{1}{4} \times 2 = \frac{1}{2}$$

Multiplying a Whole Number and a Proper Fraction

Any whole number can be written with a denominator of 1. (This does not change the value of the whole number because a number divided by one is still the same whole number in the end.)

$$\bullet \ 1 = \frac{1}{1}$$

$$\bullet \ 2 = \frac{2}{1}$$

$$\bullet \ 3 = \frac{3}{1}$$

$$\bullet \ 4 = \frac{4}{1}$$

$$\bullet \ 100 = \frac{100}{1}$$

• and so on.

To multiply a whole number and a fraction, do the following:

1. Write the whole number as a fraction with a denominator of 1.
2. Multiply the numerator by the numerator.
3. Multiply the denominator by the denominator.
4. Simplify the product.

Example H

$$\frac{4}{6} \times 6 =$$

Step 1: Write the whole number with a denominator of 1.

$$\frac{4}{5} \times \frac{6}{1} =$$

Step 2: Multiply the numerators.

$$\frac{4}{5} \times \frac{6}{1} = \frac{24}{5}$$

Step 3: Multiply the denominators.

$$\frac{4}{5} \times \frac{6}{1} = \frac{24}{5}$$

Step 4: Write your answer in lowest terms.

$$\begin{array}{r} 4 \\ 5 \overline{)24} \\ \underline{20} \\ 04 \end{array} \quad \frac{24}{5} = 4\frac{4}{5} \quad \text{Then,}$$
$$\frac{4}{6} \times 6 = 4\frac{4}{5}$$

Example I

$$\frac{1}{2} \times 3 =$$

Step 1:

$$\frac{1}{2} \times \frac{3}{1} =$$

Step 2 & 3:

$$\frac{1}{2} \times \frac{3}{1} = \frac{3}{2}$$

Step 4: Write your answer in lowest terms.

$$\text{Since } \begin{array}{r} 1 \\ 2 \overline{)3} \\ -2 \\ \hline 1 \end{array} \quad \frac{3}{2} = 1\frac{1}{2} \quad \text{Then, } \frac{1}{2} \times 3 = 1\frac{1}{2}$$

Example J

$$\frac{2}{3} \times 4 =$$

$$\frac{2}{3} \times \frac{4}{1} = \frac{8}{3} = 2\frac{2}{3}$$

$$\begin{array}{r} 2 \\ 3 \overline{)8} \\ \underline{-6} \\ 2 \end{array}$$

Example K

$$7 \times \frac{3}{4} =$$

$$\frac{7}{1} \times \frac{3}{4} = \frac{21}{4} = 5\frac{1}{4}$$

$$\begin{array}{r} 5 \\ 4 \overline{)21} \\ \underline{-20} \\ 01 \end{array}$$

Exercise 3

Multiply these fractions. Write your answers in the lowest terms.

Examples:

$$\frac{\frac{3}{4} \times 10}{\frac{2}{3} \times \frac{10}{4} = \frac{20}{4} = 5}$$

$$\frac{8 \times \frac{1}{8}}{\frac{4}{1} \times \frac{1}{10} = \frac{4 \times 1}{10 \times 1} = \frac{4}{10} = \frac{2}{5}}$$

- a. $\frac{2}{3} \times 9 =$ c. $1 \times \frac{3}{8} =$ e. $5 \times \frac{2}{3} =$
 b. $4 \times \frac{1}{6} =$ d. $\frac{1}{2} \times 5 =$ f. $\frac{1}{2} \times 8 =$

$$\text{g. } 6 \times \frac{1}{5} = \quad \text{i. } \frac{3}{8} \text{ of } 4 =$$

$$\text{h. } \frac{3}{2} \times 12 = \quad \text{j. } \frac{7}{8} \text{ of } 3 =$$

Exercise 3 Answers

$$\text{a. } \frac{6}{2}$$

$$\text{b. } \frac{3}{3}$$

$$\text{c. } \frac{3}{8}$$

$$\text{d. } 2\frac{1}{2}$$

$$\text{e. } 3\frac{1}{3}$$

$$\text{f. } 4\frac{1}{5}$$

$$\text{g. } 1\frac{1}{5}$$

$$\text{h. } 18$$

$$\text{i. } 1\frac{1}{2}$$

$$\text{j. } 2\frac{5}{8}$$

Multiplying Common Fractions Together

To multiply common fractions, multiply the numerator times the numerator and then the denominator times the denominator and simplify the answer. (Write the answer in the lowest terms.)

The method is easy, but let's look at what you're doing.

Example L

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{2} \text{ of } \frac{1}{2} =$$

Take an apple and cut it in half (see Figure 3.7.6).

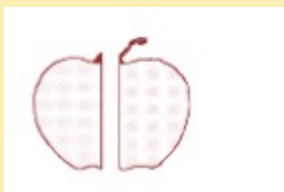


Figure 3.7.6

Now, cut one of the halves in half (see Figure 3.7.7). What fraction of the whole apple do you get?



Figure 3.7.7

You get $\frac{1}{4}$ of the apple.

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Example M

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{2} \text{ of } \frac{1}{2} =$$

You borrowed $\frac{3}{4}$ of a bag of cement from your neighbour. You used $\frac{1}{3}$ of the cement and gave the bag back to him. How much of your neighbour's bag of cement did you use?

$$\frac{3}{4} \text{ of a bag (see Figure 3.7.8)}$$

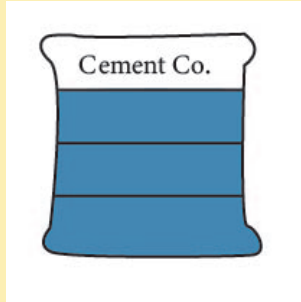


Figure 3.7.8

$\frac{2}{4}$ of a bag (see Figure 3.7.9)

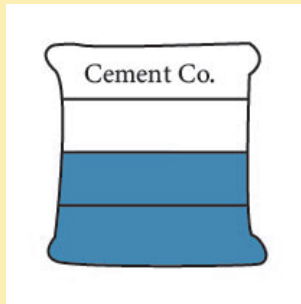


Figure 3.7.9

Use $\frac{1}{3}$ of this.

$$\frac{1}{3} \text{ of } \frac{3}{4} = \frac{1}{4} \text{ of the bag used.}$$

You used $\frac{1}{4}$ of a bag.

Example N

You are making a marinade to tenderize that cheap steak you bought. It calls for $\frac{2}{3}$ cup of beer. You only need $\frac{1}{2}$ of the amount the recipe makes, and it would be a shame to waste the beer. How much beer is needed?

$$\frac{1}{2} \text{ of } \frac{2}{3} \text{ (see Figure 3.7.10)}$$



Figure 3.7.10

$\frac{1}{3}$ of a cup (see Figure 3.7.11)



Figure 3.7.11

$\frac{1}{3}$ of a cup

$$\frac{1}{2} \text{ of } \frac{2}{3} \text{ cup of beer} = \frac{1}{2} \text{ of } \frac{2}{3} \text{ cup of beer} = \frac{1}{2} \times \frac{2}{3} =$$
$$\frac{2}{6} = \frac{2}{6} \div 2 = \frac{1}{3} \text{ cup of beer.}$$

Exercise 4

Multiply these fractions. Write the answers in the lowest terms.

Example: $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

a. $\frac{1}{2} \times \frac{4}{2} =$

d. $\frac{5}{6} \times \frac{1}{4} =$

g. $\frac{8}{10} \times \frac{2}{3} =$

b. $\frac{3}{5} \times \frac{2}{4} =$

e. $\frac{5}{10} \times \frac{1}{4} =$

c. $\frac{2}{5} \times \frac{3}{5} =$

f. $\frac{1}{4} \times \frac{1}{8} =$

Exercise 4 Answers

a. $\frac{1}{3}$

d. $\frac{5}{24}$

f. $\frac{1}{32}$

b. $\frac{10}{6}$

e. $\frac{1}{8}$

g. $\frac{8}{15}$

c. $\frac{6}{25}$

Simplify Before Multiplying

Multiplying common fractions can be much easier if you simplify before you multiply. In a multiplication question, any numerator and any denominator may be divided by a common factor. This is sometimes called cancelling.

For example:

$$\frac{2}{3} \times \frac{9}{10} = \frac{5}{8} = \frac{3}{8}$$

Cancel any numerator with any denominator. Multiply all numerators together and then all denominators. Study this worked example:

Numerator 2 and denominator 8 have a common factor of 2, numerator 9 and denominator 3 have a common factor of 3, AND numerator 5 and denominator 10 have a common factor of 5:

$$\frac{\cancel{2}^1}{\cancel{3}_1} \times \frac{\cancel{9}^3}{\cancel{10}_2} \times \frac{\cancel{5}^1}{\cancel{4}_4} = \frac{3}{8}$$

To simplify before multiplying:

1. Look to see if any numerator and any denominator have **Common Factors**. Choose the **Greatest Common Factor**.
2. Divide that numerator and that denominator by the **Greatest Common Factor (GCF)**. Be sure to cross out the old numerals and put in the **lower terms**.
3. Multiply the numerators (Be sure to use the lower term!) and then the denominators.
4. Simplify.

Example O

$$\frac{3}{4} \times \frac{1}{6}$$

Step 1: Numerator 3 and denominator 6 have a common factor of 3.

Step 2:

$$\frac{\cancel{3}^1}{4} \times \frac{1}{\cancel{6}_2}$$

$$3 \div 3 = 1 \text{ and } 6 \div 3 = 2$$

Step 3:

$$\frac{\cancel{3}^1}{4} \times \frac{1}{\cancel{6}_2} = \frac{1}{8}$$

Step 4: The answer is already in the lowest terms.

Example P

$$\frac{3}{4} \times \frac{8}{9} =$$

Step 1: Numerator 3 and denominator 9 have a common factor of 3, AND numerator 8 and denominator 4 have common factors of 2 and 4. The GCF is 4.

Step 2:

$$\frac{\cancel{3}1}{\cancel{4}1} \times \frac{\cancel{8}2}{\cancel{9}3}$$

$3 \div 3 = 1$, $9 \div 3 = 3$, $8 \div 4 = 2$, and $4 \div 4 = 1$

Step 3:

$$\frac{\cancel{3}1}{\cancel{4}1} \times \frac{\cancel{8}2}{\cancel{9}3} = \frac{2}{3}$$

Step 4: The answer is already in the lowest terms.

Example Q

$$\frac{3}{8} \times 12 =$$

$$\frac{3}{8} \times \frac{12}{1}$$

Numerator 12 and denominator 8 have a GCF. of 4.

$$\frac{3}{\cancel{8}2} \times \frac{\cancel{12}3}{1} = \frac{9}{2} = 4\frac{1}{2}$$

It is easier to simplify (or cancel) before you multiply because the numbers are smaller, and the factors are easier to find.

You may make fewer multiplying mistakes, too.

Exercise 5

Find the products. Simplify before multiplying when possible.

Examples:

$$\frac{\frac{1}{8} \cdot \frac{4}{1}}{\frac{2}{3} \cdot \frac{2}{3}} = \frac{2}{3}$$

$$\frac{\frac{1}{2} \cdot \frac{1}{3}}{\frac{2}{3} \cdot \frac{1}{3}} = \frac{1}{2}$$

a. $\frac{3}{16} \times \frac{8}{9} =$	e. $\frac{2}{5} \times \frac{10}{7} =$	i. $\frac{3}{5} \times \frac{2}{3} \times \frac{1}{2} =$
b. $\frac{2}{5} \times \frac{5}{8} =$	f. $\frac{3}{4} \times \frac{1}{12} =$	j. $2 \times \frac{3}{4} \times \frac{5}{12} =$
c. $\frac{4}{21} \times \frac{7}{8} =$	g. $\frac{8}{9} \times \frac{3}{4} =$	k. $\frac{4}{9} \times \frac{3}{5} \times \frac{15}{16} =$
d. $\frac{9}{10} \times \frac{2}{3} =$	h. $\frac{1}{2} \times \frac{2}{5} \times \frac{5}{7} =$	l. $\frac{2}{3} \times \frac{3}{4} \times \frac{8}{9} =$

Exercise 5 Answers

a. $\frac{1}{6}$

b. $\frac{1}{4}$

c. $\frac{1}{6}$

d. $\frac{3}{5}$

e. $\frac{4}{7}$

f. $\frac{1}{12}$

g. $\frac{2}{3}$

h. $\frac{1}{7}$

i. $\frac{1}{5}$

j. $\frac{5}{8}$

k. $\frac{1}{4}$

l. $\frac{4}{9}$

Multiplying Mixed Numbers

To multiply with a mixed number:

1. Rename any mixed numbers as improper fractions.
2. Write any whole number by itself as an improper fraction with a denominator of 1.
3. Rewrite the question with the new improper fraction(s).
4. Simplify (cancel).
5. Multiply the numerator by the numerator. Multiply the denominator by the denominator.
6. The answer will often be an improper fraction. Rename improper fractions as mixed numbers, and be sure any fraction is in the lowest terms.

Example R

$$2\frac{3}{4} \times \frac{1}{3} =$$

Step 1:

$$2\frac{3}{4} = \frac{11}{4}$$

Step 2: No whole numbers by themselves.

Step 3: Question is rewritten as $\frac{11}{4} \times \frac{1}{3}$.

Step 4: Simplify.

The fraction has no common factors, so it cannot be simplified.

Step 5:

$$\frac{11}{4} \times \frac{1}{3} = \frac{11}{12}$$

Step 6: It is already in the lowest terms.

Example S

$$1\frac{1}{5} \times 2\frac{2}{3}$$

Step 1:

$$1\frac{1}{5} = \frac{6}{5} \text{ and } 2\frac{2}{3} = \frac{8}{3}$$

Step 2: No whole numbers by themselves.

Step 3: Question is rewritten as $\frac{6}{5} \times \frac{8}{3}$.

Step 4 & 5:

$$\frac{\cancel{6}^2}{5} \times \frac{8}{\cancel{3}_1} = \frac{16}{5} \text{ (an improper fraction)}$$

Step 6:

$$\frac{16}{5} = 3\frac{1}{5}$$

$$\begin{array}{r} 3 \\ 5 \overline{)16} \\ \underline{15} \\ 01 \end{array}$$

$$1\frac{1}{5} \times 2\frac{2}{3} = 3\frac{1}{5}$$

Example T

$$4 \times 2\frac{5}{6}$$

Step 1 and 2:

$$4 = \frac{4}{1} \text{ and } 2\frac{5}{6} = \frac{17}{6}$$

Step 3: The question is rewritten as $\frac{4}{1} \times \frac{17}{6}$.

Step 4, 5 and 6:

$$4 \times 2\frac{5}{6} = \frac{\cancel{4}^2}{1} \times \frac{17}{\cancel{6}_3} = \frac{34}{3} = 11\frac{1}{3}$$

Remember: Only skip steps when you are totally confident in your method. Writing out the steps will help you to get the answer right more often.

Exercise 6

Find the products.

Examples:

$$2\frac{1}{2} \cdot 4 =$$

$$\frac{5}{2} \cdot \frac{4}{1} = 21$$

$$3 \cdot \frac{1}{2} =$$

$$\frac{3}{1} \cdot \frac{1}{2} = \frac{3 \cdot 1}{1 \cdot 2} = \frac{3}{2}$$

a. $\frac{3}{14} \times 2\frac{1}{6} \times 12 =$ c. $1\frac{2}{5} \times 15 =$ e. $4 \times 2\frac{1}{2} =$

b. $3 \times 4\frac{1}{3} =$ d. $3\frac{3}{8} \times 8 =$ f. $2\frac{1}{4} \times 8 =$

Exercise 6 Answers

a. $5\frac{4}{7}$

b. 13
c. 21
d. 27

e. 10
f. 18

Exercise 7

This is extra practice if you feel you need it.

$$\begin{array}{lll} \text{a. } 1\frac{1}{2} \times \frac{2}{3} = & \text{d. } 1\frac{1}{6} \times 1\frac{5}{7} = & \text{g. } 7\frac{1}{3} \times \frac{3}{8} = \\ \text{b. } 1\frac{1}{4} \times 3\frac{1}{2} = & \text{e. } \frac{1}{2} \times 2\frac{1}{2} = & \text{h. } 2\frac{1}{4} \times 3\frac{1}{2} = \\ \text{c. } \frac{1}{3} \times 5\frac{1}{2} = & \text{f. } 7\frac{1}{4} \times 3\frac{1}{3} = & \end{array}$$

Exercise 7 Answers

$$\begin{array}{lll} \text{a. } 1\frac{3}{8} & \text{d. } 2\frac{1}{4} & \text{g. } 2\frac{3}{4} \\ \text{b. } 4\frac{5}{8} & \text{e. } 1\frac{1}{4} & \text{h. } 7\frac{7}{8} \\ \text{c. } 1\frac{5}{6} & \text{f. } 24\frac{1}{6} & \end{array}$$

Problems Using Multiplication of Common Fractions

The following exercise gives some typical word problems for the multiplication of fractions. The type of wording is similar for decimals and common fractions.

Remember this important word:

A fraction **of** some number means to **multiply**.

Example:

- $\frac{2}{3}$ of her money means $\frac{2}{3} \times$ amount of the money
- $\frac{1}{4}$ of the children means $\frac{1}{4} \times$ number of children

These techniques may help you through the problem-solving steps when you're working with common fractions:

- Look for keywords.
- Look for familiar patterns in the wording.
- Round fractions to whole numbers to estimate and to try to make sense of the problem.
- Draw a sketch or diagram of what the problem is describing.

Exercise 8

Solve the following word problems.

- a. Maria was angry with her children because they had eaten $\frac{2}{3}$ of the 24 cupcakes she had made to take to a meeting. How many cupcakes did the kids eat?
- b. Double this recipe for Awesome Chocolate Chip Cookies.

◦ $\frac{2}{3}$ cup butter

◦ $\frac{2}{3}$ peanut butter

◦ $\frac{5}{8}$ white sugar

◦ $\frac{5}{8}$ brown sugar

◦ 2 eggs

◦ $1\frac{3}{4}$ tsp vanilla

◦ $1\frac{1}{2}$ cup flour

◦ 1 cup rolled oats

◦ 1 tsp baking soda

◦ $\frac{1}{4}$ tsp salt

◦ $\frac{4}{7}$ chocolate chips

Cream butter and peanut butter well, and blend in sugar. Beat in eggs and vanilla. Add dry ingredients, blending very thoroughly. Add chocolate chips and mix. Drop by spoonful onto the cookie sheet; cookies will flatten during cooking. Bake in 350 F. oven for 10 to 12 minutes.

- c. Marni is trying to gradually cut down on the amount of coffee she drinks. Right now she allows herself $\frac{3}{4}$ of a cup of coffee at breakfast, $\frac{3}{4}$ cup at morning break time, and $\frac{3}{4}$ cup at lunch and another $\frac{3}{4}$ cup after dinner. How many cups of coffee is she drinking per day right now?
- d. Sam's truck uses $\frac{1}{3}$ of a tank of gas every time he drives to his girlfriend's house. His tank holds 75 litres. How many litres of gas does he use to drive to his girlfriend's place?
- e. If you do math for $1\frac{1}{2}$ hours every day you are at school, how many hours do you spend on math per month if you come to school twenty days in a month?
- f. David was complaining that his car insurance was the same price as $\frac{7}{8}$ of the cost of his car! His car cost \$1200. What did he pay for car insurance?
- g. Justine was building a dog shed. She spent $\frac{1}{3}$ of an hour on the project every evening for 8 days in a row. How much time did she spend on the project?

Exercise 8 Answers

- a. 16 cupcakes
b. Awesome Chocolate Chip Cookies (Doubled)

◦ $1\frac{1}{3}$ cup butter	◦ $3\frac{1}{2}$ teaspoons vanilla
◦ $1\frac{1}{3}$ cup peanut butter	◦ 3 cups flour
◦ $1\frac{1}{4}$ cup white sugar	◦ 2 cups rolled oats
◦ $1\frac{1}{4}$ cup brown sugar	◦ 2 teaspoons baking soda
◦ 4 eggs	◦ $\frac{1}{2}$ teaspoon salt
	◦ $1\frac{3}{4}$ cup chocolate chips

c. 3 cups	f. \$1050.00
d. 25 L	g. 2 hours
e. 30 hours	

3.7: Practice Questions

1. Find the products.

a. $\frac{1}{4} \times \frac{3}{8} =$	c. $\frac{7}{9} \times \frac{3}{14} =$	e. $5 \times \frac{4}{9} =$
b. $\frac{3}{8} \times \frac{5}{6} =$	d. $\frac{1}{4} \times 12 =$	f. $1\frac{1}{4} \times \frac{4}{5} =$

$$\begin{array}{ll} \text{g. } \frac{3}{5} \times 7\frac{2}{9} = & \text{i. } 1\frac{1}{2} \times 1\frac{3}{5} = \\ \text{h. } 6 \times 1\frac{2}{3} = & \text{j. } 2\frac{7}{8} \times 1\frac{9}{10} = \end{array}$$

2. Solve the following word problems.

- a. Frank is hosting a big party. He needs to multiply his favourite tomato sauce by five to make enough to serve his guests. Five times this recipe for him.

▪ $1\frac{1}{2}$ Tbsp Olive Oil;	▪ $1\frac{1}{3}$ tsp red chili flakes;
▪ 1 Onions;	▪ $1\frac{3}{4}$ diced tomatoes;
▪ $1\frac{1}{2}$ tsp salt;	▪ $1\frac{1}{4}$ brown sugar;
▪ 2 bay leaves;	
▪ 6 garlic cloves;	

- b. Joey practices $\frac{3}{4}$ of an hour each day. How much time does he spend practicing each week?
- c. A Haida longhouse measures $15\frac{1}{4}$ m by $18\frac{1}{3}$ m. What area of land does it cover?
- d. Find the area of the rectangle.



$$\frac{2}{5} \text{ cm}$$

$$1\frac{3}{4} \text{ cm}$$

3.7: Practice Answers

1. Find the products.

a. $\frac{3}{32}$

b. $\frac{5}{2} \cdot \frac{1}{6}$

d. 3

e. $2\frac{3}{32}$

f. 1

g. $4\frac{1}{3}$

h. 10

i. $2\frac{2}{5}$

j. $5\frac{37}{80}$

2. Solve the following word problems.

- a. Five times this recipe:

- $7\frac{1}{2}$ Tbsp Olive Oil;
- 5 Onions;
- $2\frac{1}{2}$ tsp salt;

- 10 bay leaves;
- 30 garlic cloves;
- $1\frac{2}{3}$ tsp red chili flakes;

■ $8\frac{3}{4}$ diced tomatoes;

■ $6\frac{1}{4}$ brown sugar;

b. $5\frac{1}{4}$

c. $279\frac{7}{12}$ m² in area

d. $\frac{7}{10}$ cm

Attribution

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3.8: Dividing Common Fractions

Think over what you know about dividing:

When we **divide**, we take the total amount and separate (divide it) into equal parts or groups.

Remember:

$$8 \div 4 = 2$$

- 8 is the dividend
- 4 is the divisor
- 2 is the quotient

Example A

$$8 \div 4 =$$

- The total amount is 8.
- The **divisor** is 4. How many groups of 4 are in 8?
Yes, 2.

$$8 \div 4 = 2$$

Example B

$$3 \div \frac{1}{2} =$$

- The total amount is 3.
- The total amount is $\frac{1}{2}$.
- How many $\frac{1}{2}$'s are in 3?

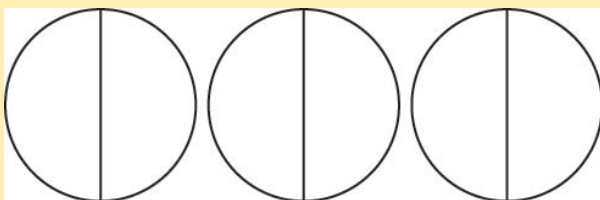


Figure 3.8.1

- There are 6 halves.

$$3 \div \frac{1}{2} = 6$$

Example C

$$2 \div \frac{2}{3}$$

- The total amount is 2.
- The total amount is $\frac{2}{3}$.
- How many $\frac{2}{3}$'s are in 2?
- Use different colours to shade in each group of two that you can find.



Figure 3.8.2

$$2 \div \frac{2}{3} = 3$$

Example D

$$1 \div \frac{1}{4}$$

- The total amount is 1. Divisor is $\frac{1}{4}$
- How many $\frac{1}{4}$'s in 1?
- Draw a shape. Divide it into quarters.
- How many $\frac{1}{4}$'s are there?
- There are 4 quarters.

$$1 \div \frac{1}{4} = 4$$

Example E

$$3 \div \frac{3}{8} =$$

- How many $\frac{3}{8}$ in 3.
- Use different colors to shade in each group of 3 that you can find.

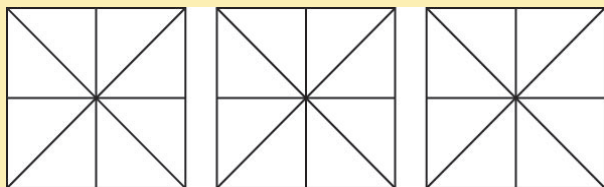


Figure 3.8.2

- Did you find 8 groups of $\frac{3}{8}$?

$$3 \div \frac{3}{8} = 8$$

The division of fractions by a fraction is difficult to picture, probably because it is not often used in everyday life. Here are some everyday examples for you to think about.

Example 1: You have half a dollar. Someone asks you to change it for quarters. How many quarters are there in half a dollar?

$$\frac{3}{8} \div \frac{1}{4} = 2$$

2 quarters in one half a dollar

Examples 2: It takes $\frac{1}{4}$ hour to solve a math problem. How many problems can you solve in $\frac{3}{4}$ of an hour?

$$\frac{3}{4} \div \frac{1}{4} = 3$$

3 problems in one $\frac{3}{4}$ of an hour

Reciprocals

Dividing by a number is the same as multiplying by its **reciprocal**. We use reciprocals when we divide fractions. Two numbers are reciprocals if they have a product of 1.

To find the reciprocal of a fraction, turn the fraction upside down (flip it over). This is called “inverting the fraction.”

Some people remember this by thinking of reciprocals as “refliprocals”!

Table 3.8.1 shows some fractions and their reciprocals.

Table 3.8.1: Fractions and Their Reciprocals

Fraction	Reciprocal
$\frac{1}{2}$	$\frac{2}{1}$
$\frac{3}{4}$	$\frac{4}{3}$
$\frac{7}{8}$	$\frac{8}{7}$
$\frac{2}{3}$	$\frac{3}{2}$
$\frac{1}{4}$	$\frac{4}{1}$

To find the reciprocal of a whole number:

1. Rename the whole number as a fraction with a denominator of 1.
2. Invert the fraction.
3. Check the reciprocal by multiplying the fraction by the reciprocal. The product will be one.

Table 3.8.2 shows some whole numbers with their reciprocals.

Table 3.8.2: Products of Whole Numbers With Fractions and Reciprocals

Whole Number	Fraction	Reciprocal	Check
3	$\frac{3}{1}$	$\frac{1}{3}$	$\frac{\cancel{3}1}{1} \times \frac{1}{\cancel{3}1} = 1$
6	$\frac{6}{1}$	$\frac{1}{6}$	$\frac{\cancel{6}1}{1} \times \frac{1}{\cancel{6}1} = 1$
10	$\frac{10}{1}$	$\frac{1}{10}$	$\frac{\cancel{10}1}{1} \times \frac{1}{\cancel{10}1} = 1$

To find the reciprocal of a mixed number:

1. Rename the mixed number as an improper fraction.
2. Invert the fraction.

Table 3.8.3 shows some mixed numbers with their reciprocals.

Table 3.8.3: Mixed Numbers With Their Fractions and Reciprocals

Mixed Number	Fraction	Reciprocal	Check
$1\frac{1}{2} =$	$\frac{3}{2}$	$\frac{2}{3}$	$\frac{\cancel{3}1}{21} \times \frac{\cancel{2}1}{\cancel{3}1} = \frac{6}{6} = 1$
$2\frac{1}{3}$	$\frac{7}{3}$	$\frac{3}{7}$	$\frac{\cancel{7}1}{31} \times \frac{\cancel{3}1}{\cancel{7}1} = 1$
$4\frac{3}{8}$	$\frac{35}{8}$	$\frac{8}{35}$	$\frac{\cancel{35}1}{81} \times \frac{\cancel{8}1}{\cancel{35}1} = 1$

Exercise 1

Write the reciprocal of these numbers.

Examples:

$$\frac{2}{5}$$

The reciprocal is

$$\frac{5}{2}$$

$$5 = \frac{5}{1}$$

The reciprocal is

$$\frac{1}{5}$$

$$2\frac{1}{2} = \frac{5}{2}$$

The reciprocal is

$$\frac{2}{5}$$

a. $\frac{5}{8}$

b. $\frac{1}{2}$

c. $\frac{9}{2}$

e. $2\frac{1}{2} = \frac{5}{2}$

f. $1\frac{1}{4}$

g. $8\frac{1}{3}$

Exercise 1 Answers

a. $\frac{8}{5}$

b. 2

c. $\frac{1}{9}$

d. $\frac{1}{2}$

e. $\frac{4}{5}$

f. $\frac{3}{25}$

Multiplying by the Reciprocal

To divide fractions, multiply by the reciprocal of the divisor:

1. Rewrite the division question.
 - Rename all mixed numbers as improper fractions.
 - Give any whole numbers a denominator of 1
2. Change the $\div$ sign to a $\times$ sign.
 - Invert (turn upside down) the divisor to make the reciprocal.
 - **Remember:** the divisor is always the number after the sign.
3. Simplify (cancel) and then multiply to find the answer.
4. Write the answer in the lowest terms.

Example F

$$\frac{3}{4} \div \frac{1}{2} =$$

Step 1: No whole numbers or mixed numbers.

Step 2:

$$\frac{3}{4} \div \frac{1}{2} = \frac{3}{4} \times \frac{2}{1} =$$

Step 3 & 4:

$$\frac{\cancel{3}}{\cancel{4}2} \times \frac{\cancel{2}1}{1} = \frac{3}{2} = 1\frac{1}{2}$$

Example G

$$\frac{7}{8} \div \frac{1}{4} =$$

Step 1: No whole numbers or mixed numbers.

Step 2:

$$\frac{7}{8} \div \frac{1}{4} = \frac{7}{8} \times \frac{4}{1} =$$

Step 3 & 4:

$$\frac{7}{\cancel{8}2} \times \frac{\cancel{4}1}{1} = \frac{7}{2} = 3\frac{1}{2}$$

Example H

$$5 \div \frac{2}{3} =$$

Step 1:

$$5 \div \frac{2}{3} = \frac{5}{1} \div \frac{2}{3} =$$

Step 2:

$$\frac{5}{1} \times \frac{3}{2} =$$

Step 3 & 4:

$$\frac{5}{1} \times \frac{3}{2} = \frac{15}{2} = 7\frac{1}{2}$$

Example I

$$3\frac{1}{2} \div 2\frac{3}{4} =$$

Step 1:

$$3\frac{1}{2} \div 2\frac{3}{4} = \frac{7}{2} \div \frac{11}{4}$$

Step 2:

$$\frac{7}{2} \times \frac{11}{4} =$$

Step 3 & 4:

$$\frac{7}{\cancel{2}1} \times \frac{\cancel{4}2}{11} = \frac{14}{11} = 1\frac{3}{11}$$

Exercise 2

Divide these fractions using the steps you have just learned.

Example:

$$\frac{4}{9} \div 4 = \frac{4}{9} \div \frac{4}{1} = \frac{4}{9} \times \frac{1}{4} = \frac{\cancel{4}^1}{9} \times \frac{1}{\cancel{4}_1} = \frac{1}{9}$$

$$\begin{array}{lll} \text{a. } \frac{7}{2} \div \frac{3}{5} = & \text{d. } \frac{1}{5} \div \frac{1}{2} = & \text{g. } \frac{6}{7} \div \frac{1}{6} = \\ \text{b. } \frac{5}{8} \div \frac{7}{16} = & \text{e. } \frac{5}{6} \div \frac{5}{3} = & \\ \text{c. } \frac{2}{3} \div \frac{8}{9} = & \text{f. } \frac{1}{3} \div \frac{3}{8} = & \end{array}$$

Exercise 2 Answers

$$\begin{array}{lll} \text{a. } 5\frac{5}{6} & \text{c. } \frac{3}{4} & \text{e. } \frac{1}{2} \\ \text{b. } 1\frac{3}{7} & \text{d. } \frac{2}{5} & \text{f. } \frac{8}{9} \end{array}$$

g. $5\frac{1}{7}$

Exercise 3

If you need more practice, try a few more of these division questions. If you are not having any trouble, go on to Exercise 4, which has mixed numbers in it.

Divide these fractions using the steps you have just learned.

Example:

$$\frac{1}{2} \div \frac{1}{8} = \frac{1}{2} \times \frac{8}{1} = \frac{1}{\cancel{2}1} \times \frac{\cancel{8}4}{1} = \frac{4}{1} = 4$$

a. $\frac{8}{9} \div \frac{3}{2} =$ c. $\frac{5}{6} \div \frac{3}{3} =$ e. $\frac{2}{3} \div \frac{1}{2} =$

b. $\frac{3}{4} \div \frac{3}{4} =$ d. $\frac{1}{3} \div \frac{3}{4} =$

Exercise 3 Answers

a. $\frac{16}{27}$

b. 1

c. $1\frac{5}{6}$

d. $\frac{4}{9}$

e. $1\frac{1}{3} = \frac{4}{3}$

Exercise 4

More practice: You might want to save some of this exercise for review before a test.

a. $8 \div \frac{1}{2} =$

e. $\frac{3}{5} \div \frac{1}{4} =$

i. $2\frac{3}{4} \div 1\frac{7}{8} =$

b. $2\frac{2}{5} \div \frac{1}{8} =$

f. $2\frac{4}{5} \div \frac{1}{5} =$

j. $5\frac{1}{10} \div 3\frac{3}{10} =$

c. $\frac{1}{6} \div \frac{1}{5} =$

g. $\frac{2}{5} \div \frac{1}{2} =$

k. $1\frac{5}{9} \div 3\frac{1}{3} =$

d. $\frac{1}{8} \div \frac{1}{5} =$

h. $\frac{1}{4} \div \frac{2}{3} =$

l. $\frac{1}{2} \div \frac{3}{8} =$

Exercise 4 Answers

a. 16

b. $19\frac{1}{5}$

c. $\frac{5}{6}$

d. $\frac{5}{8}$

e. $2\frac{2}{5}$

f. $\frac{14}{4}$

g. $\frac{5}{3}$

h. $\frac{3}{8}$

i. $1\frac{7}{15}$

j. $1\frac{6}{11}$

k. $\frac{7}{15}$

l. $1\frac{1}{3}$

Problems Which Use Division of Common Fractions

Look for word patterns and keywords in the division problems. Thinking about the problems using whole numbers instead of fractions may sometimes help you recognize the division pattern. Start your division equation with the **dividend**. The dividend is the total.

Keywords that point to division:

- separated split cut shared
- what is cost per...? **unit pricing**
- what is distance per...? average (speed, cost, weight, time)

Exercise 5

Solve the following word problems.

- a. Every fall 3 friends get together to make antipasto. Last year, they filled $4\frac{1}{2}$ ice cream buckets with antipasto and then shared it equally. How many buckets of antipasto did each person get?
- b. A pick-up truck load of split wood is $\frac{1}{2}$ cord of wood. If you shared a full truckload of wood with a neighbour, how much of a cord of firewood would you each get?
- c. The distance from Trail, BC to Vancouver, BC is 640 km via the Crowsnest Highway. The trip can be made in $7\frac{1}{2}$ hours in good weather. What average speed must be maintained?
- d. The sweater that Janet is knitting has a complicated pattern. It takes her $3\frac{3}{4}$ hours to finish 15 rows. How long does each row take?
- e. Marian had $\frac{12}{3}$ lemon pies left, which she wanted to share equally amongst 10 people. How much of a pie will each person be given?
- f. Jack wants to cut his piece of trim for his square windows into 4 equal parts. The trim is $2\frac{2}{5}$ metres

- long. What will the measurement be of each piece?
- g. Tony is sewing 3 identical pairs of pants for his son's dance performance. He bought metres of material. He uses up all of the material; how much material was used for each pair of pants?
- h. Joy has a $7\frac{1}{4}$ m long stick. She needs to split it into $\frac{1}{3}$ m pieces. How many pieces can she get?
- (Remember: your answer will be given with the **unit** of 'pieces' not metres!)

Exercise 5 Answers

a. $1\frac{1}{2}$ buckets	e. $\frac{1}{6}$ pie
b. $\frac{1}{4}$ cord	f. Each piece is $\frac{3}{5}$ metres long.
c. $85\frac{1}{3}$ km/h (85.3 km/h)	g. He uses $\frac{7}{9}$ metre for each pair.
d. $\frac{1}{4}$ hour or 15 minutes	h. She will get 21 pieces.

3.8: Practice Questions

1. Divide and be sure the answers are in the lowest terms.

a. $\frac{3}{4} \div \frac{1}{4} =$ b. $\frac{1}{4} \div 1\frac{1}{4} =$ c. $\frac{5}{8} \div \frac{15}{16} =$

$$\begin{array}{lll} \text{d. } 6 \div \frac{7}{9} = & \text{f. } 9\frac{3}{4} \div 2 = & \text{h. } 3\frac{3}{7} \div 2\frac{5}{14} = \\ \text{e. } \frac{5}{11} \div 11 = & \text{g. } 3 \div \frac{1}{3} = & \end{array}$$

2. Solve the following word problem.

- a. Joe is a school janitor. It takes him $\frac{3}{4}$ of an hour to clean one classroom. How many classrooms does he clean in his $7\frac{1}{2}$ hour shift?

3.8: Practice Answers

1. Divide and be sure the answers are in the lowest terms.

$$\begin{array}{l} \text{a. } 3 \\ \text{b. } \frac{5}{14} \\ \text{c. } \frac{2}{3} \end{array}$$

$$\begin{array}{l} \text{d. } 7\frac{5}{7} \\ \text{e. } \frac{5}{121} \\ \text{f. } 4\frac{7}{8} \end{array}$$

$$\begin{array}{l} \text{g. } \frac{9}{13} \\ \text{h. } 1\frac{5}{11} \end{array}$$

2. Solve the following word problem.

- a. 10 classrooms

Attribution

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3.9: Multiplying & Dividing Fractions Review

1. Write the multiplication equation you would use to find the fraction of the whole number. You do not have to calculate the answers.
 - a. More than $\frac{1}{3}$ of the children at the daycare are sick today. There are 15 children in the daycare. How many children are sick?
 - b. Rich answered $\frac{7}{8}$ of the questions on the test. The test had 48 questions. How many questions did Rich complete?
 - c. Canada Post delivered more than 11.6 billion pieces of mail across Canada in 2008. They have a target of getting $\frac{96}{100}$ letters delivered on time. How many letters were delivered on time in 2008?
 - d. We spend $\frac{1}{6}$ of our monthly income on food. Our take-home pay is \$1400. How much do we spend on groceries each month?
2. Write the multiplication equation you would use to multiply a fraction by a whole number. You do not have to calculate the answer.
 - a. We want to buy a good bottle of wine for dinner, but there are 4 bottles of wine from last night that are not used up yet. Let's take each $\frac{1}{6}$ bottle of wine that is left over and have that at dinner tonight. How much wine will we have?
 - b. There are four boxes of cereal open in the cupboard, and each one is $\frac{3}{4}$ full. How many full boxes would there be if they were all put together?

- c. Double a recipe that needs $\frac{1}{3}$ tsp of cinnamon.
- d. Mark is a frequent coffee drinker at the local coffee shop. He has a punch card and gets a hole punched out each time he buys a coffee. Once he has a card with 14 holes punched out, he gets a free coffee. He keeps losing his card and getting a new one. He has found them all, and he now has 3 cards, each with $\frac{2}{3}$ of the holes punched out. Does he have enough holes punched out to get a free coffee?

3. Multiply these fractions.

$$\begin{array}{lll} \text{a. } 7 \times \frac{2}{3} = & \text{d. } \frac{3}{8} \times 4 = & \text{g. } \frac{3}{2} \times 20 = \\ \text{b. } 3 \times \frac{2}{5} = & \text{e. } 5 \times \frac{1}{5} = & \text{h. } \frac{3}{8} \times 20 = \\ \text{c. } \frac{2}{3} \times 14 = & \text{f. } 3 \times \frac{3}{4} = & \end{array}$$

4. Multiply these fractions.

$$\begin{array}{ll} \text{a. } \frac{1}{6} \times \frac{1}{4} = & \text{c. } \frac{1}{4} \times \frac{1}{7} = \\ \text{b. } \frac{5}{10} \times \frac{1}{3} = & \text{d. } \frac{7}{10} \times \frac{2}{3} = \end{array}$$

5. Multiply these fractions. Simplify before multiplying, where possible.

$$\text{a. } \frac{2}{5} \times \frac{15}{7} \times \frac{3}{4} \times \frac{1}{9} \Rightarrow \text{b. } \frac{8}{9} \times \frac{1}{4} =$$

c. $\frac{3}{5} \times \frac{2}{9} \times \frac{1}{2} =$

6. Multiply these fractions. Simplify before multiplying, where possible.

a. $7\frac{1}{3} \times \frac{3}{7} =$

c. $2\frac{1}{5} \times \frac{5}{6} =$

b. $3\frac{1}{4} \times 3\frac{1}{2} =$

d. $9 \times \frac{2}{3} =$

7. Solve the following word problems.

- A recipe calls for $\frac{2}{3}$ of a cup of sugar. How much sugar should be used if only $\frac{1}{2}$ of the recipe is being made?
- The Wrights sold their house for \$240 000. The real estate company that helped them sell their house gets $\frac{3}{20}$ of this amount. How much money did the Wrights have to pay the real estate company?
- Find the **area** of the rectangle in Figure 3.9.1.

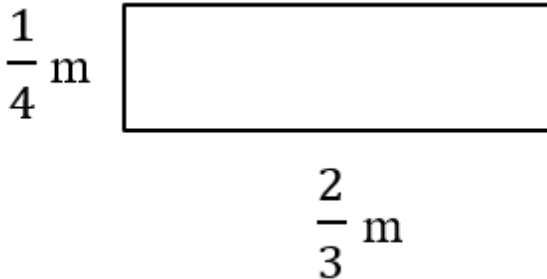


Figure 3.9.1

- d. Each turn of a screw sinks it $\frac{1}{2}$ of a centimetre deeper into the wood. Find out how deep the screw is after 7 turns.
- e. $\frac{3}{4}$ of 48 students in the gym are girls. How many girls are in the gym?
8. Divide these fractions. Simplify before multiplying, where possible.
- a. $\frac{7}{2} \div \frac{3}{4} =$
- b. $\frac{5}{8} \div \frac{7}{24} =$
- c. $\frac{2}{3} \div \frac{7}{9} =$
- d. $\frac{1}{5} \div \frac{1}{4} =$
9. Divide these fractions. Simplify before multiplying, where possible.
- a. $2\frac{4}{5} \div \frac{1}{5} =$
- b. $\frac{2}{5} \div \frac{1}{2} =$
- c. $\frac{1}{4} \div \frac{2}{3} =$
- d. $2\frac{3}{4} \div \frac{7}{8} =$
- e. $5\frac{1}{10} \div 3\frac{3}{10} =$
10. Solve the following word problems.
- a. If you want to share $3\frac{1}{3}$ cups of juice between 4 children, how much juice does each child get?
- b. Aaron ran $24\frac{3}{4}$ km in 3 days. How many kilometres did he run each day?
- c. Jordan is selling falafel balls at Smithers Mid-Summer

Music Festival this summer. She made $48\frac{3}{4}$ kg of falafel mix. How many $\frac{1}{16}$ kg falafel balls can she make?

- d. Sophie had $75\frac{1}{4}$ cm piece of wood trim. She wants to cut it into $2\frac{1}{2}$ cm pieces. How many pieces can she get?

3.9: Review Answers

1. Write the multiplication equation you would use to find the fraction of the whole number.

a. $\frac{1}{3} \times \frac{15}{1}$

b. $\frac{7}{8} \times \frac{48}{1}$

c. $\frac{96}{100} \times \frac{11.1 \text{ billion}}{1}$

d. $\frac{1}{6} \times \frac{1400}{1}$

2. Write the multiplication equation you would use to multiply a fraction by a whole number.

a. $4 \times \frac{1}{6}$

b. $4 \times \frac{3}{4}$

c. $2 \times \frac{1}{3}$

d. $3 \times \frac{2}{3}$

3. Multiply these fractions.

a. $4\frac{2}{3}$

b. $1\frac{1}{5}$

c. 7

d. $1\frac{1}{2}$

e. 1

f. $2\frac{1}{4}$

g. 30

h. $7\frac{1}{2}$

4. Multiply these fractions.

a. $\frac{1}{24}$

c. $\frac{1}{28}$

b. $\frac{1}{6}$

d. $\frac{7}{15}$

5. Multiply these fractions.

a. $\frac{6}{7}$

c. $\frac{2}{9}$

b. $\frac{1}{12}$

d. $\frac{1}{15}$

6. Multiply these fractions.

a. $3\frac{1}{7}$

c. $1\frac{5}{6}$

b. $11\frac{3}{8}$

d. 6

7. Solve the following word problems.

a. $\frac{1}{3}$ cup sugar

d. $3\frac{1}{2}$ cm

b. \$ 36 000

e. 36 students

c. $\frac{1}{6}$ m

8. Divide these fractions.

a. $4\frac{2}{3}$

b. $2\frac{1}{7}$

c. $\frac{6}{7}$

d. $\frac{4}{5}$

e. $\frac{1}{2}$

9. Divide these fractions.

a. $\frac{14}{5}$

b. $\frac{4}{3}$

c. $\frac{3}{8}$

d. $1\frac{7}{15}$

e. $1\frac{6}{11}$

10. Solve the following word problems.

a. $\frac{5}{6}$

b. $8\frac{1}{4}$ km in one day

c. 780 balls of falafel

d. $21\frac{1}{2}$

Attribution

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3.10: Adding Common Fractions

Vocabulary review:

$$4 + 5 = 9$$

- 4 and 5 are addends.
- 9 is the sum.

$$\frac{3}{7}$$

- 3 is the numerator.
- 7 is the denominator.

Like fractions are fractions that have the same denominator.

Example: $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, $\frac{4}{4}$, etc.

Adding and subtracting fractions has some different rules from multiplying and dividing.



Figure 3.10.1

There are two cakes that are left over (see Figure 3.10.1). There is 1 piece of each cake left. If you were to put all the pieces left onto one plate, how much cake would you have?

Shade in your answer in Figure 3.10.2.

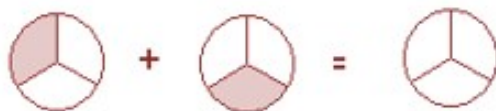


Figure 3.10.2

If you made your plate like Figure 3.10.3, then you are right!



Figure 3.10.3

Try the example in Figure 3.10.4.

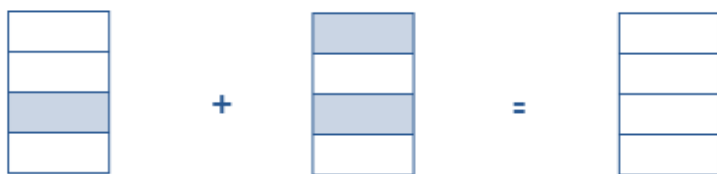


Figure 3.10.4

Figure 3.10.5 shows the answer.



Figure 3.10.5

What you are doing is adding two like fractions.

- You are moving pieces of fractions that are the same size into one whole shape. The pieces do not change size, so the denominator must stay the same size.
- When adding two fractions, your answer is a fraction.

Look back at the two examples (Figure 3.10.1 and Figure 3.10.4).

When you add fractions, does the denominator or the numerator stay the same?

Common fractions must have the same denominator when you add them together. Add the numerators and keep the denominators the same.

Look at the next two examples (Figure 3.10.6 and Figure 3.10.7):

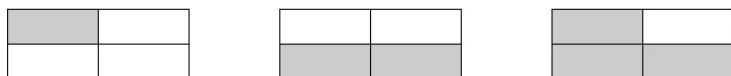


Figure 3.10.6

$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$



Figure 3.10.7

$$\frac{1}{5} + \frac{2}{5} + \frac{1}{5} = \frac{4}{5}$$

Exercise 1

Try a few for yourself.



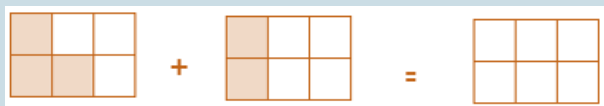
$$\frac{2}{9} + \frac{3}{9} = \frac{5}{9}$$



$$\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$$



$$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$



c.

$$\frac{3}{6} + \frac{3}{6} = \frac{6}{6}$$



d.

$$\frac{3}{8} + \frac{4}{8} = \frac{7}{8}$$

Exercise 1 Answers

a. $\frac{5}{9}$

b. $\frac{3}{4}$

c. $\frac{2}{3}$

d. $\frac{5}{6}$

e. $\frac{7}{8}$

Exercise 2

Now, find the answers to these additions without diagrams.

a. $\frac{2}{4} + \frac{1}{4} = \frac{\quad}{4}$

b. $\frac{1}{3} + \frac{1}{3} = \frac{\quad}{3}$

c. $\frac{1}{5} + \frac{1}{5} = \frac{\quad}{5}$

d. $\frac{2}{11} + \frac{7}{11} = \frac{\quad}{11}$

Exercise 2 Answers

a. $\frac{3}{4}$

b. $\frac{2}{3}$

c. $\frac{2}{5}$

d. $\frac{9}{11}$

Exercise 3

Add these common fractions.

$$\begin{array}{lll} \text{a. } \frac{1}{5} + \frac{2}{5} = & \text{c. } \frac{3}{7} + \frac{2}{7} = & \text{e. } \frac{14}{20} + \frac{3}{20} = \\ \text{b. } \frac{3}{6} + \frac{2}{6} = & \text{d. } \frac{3}{10} + \frac{6}{10} = & \text{f. } \frac{7}{37} + \frac{19}{37} = \end{array}$$

Exercise 3 Answers

$$\begin{array}{lll} \text{a. } \frac{3}{5} & \text{c. } \frac{5}{7} & \text{e. } \frac{17}{20} \\ \text{b. } \frac{5}{6} & \text{d. } \frac{9}{10} & \text{f. } \frac{26}{37} \end{array}$$

Sometimes, the **sum** of a fraction will need to be reduced (take a look at this example to remind yourself how to do this).

Example A

$$\frac{2}{8} + \frac{2}{8} = \frac{4}{8} \rightarrow \frac{\div 4}{\div 4} = \frac{1}{2}$$

Example B

$$\frac{3}{4} + \frac{3}{4} = \frac{6}{4} \rightarrow \frac{6 \div 2}{4 \div 2} = \frac{3}{2} = 1\frac{1}{2}$$

Exercise 4

Find the sums to the following additions. Make sure your answer is in the lowest terms.

$$\begin{array}{lll} \text{a. } \frac{1}{4} + \frac{1}{4} = & \text{c. } \frac{3}{10} + \frac{2}{10} = & \text{e. } \frac{3}{5} + \frac{1}{5} = \\ \text{b. } \frac{1}{3} + \frac{1}{3} = & \text{d. } \frac{7}{25} + \frac{8}{25} = & \text{f. } \frac{9}{27} + \frac{12}{27} = \end{array}$$

Exercise 4 Answers

$$\begin{array}{lll} \text{a. } \frac{1}{2} & \text{c. } \frac{1}{2} & \text{e. } \frac{4}{5} \\ \text{b. } \frac{2}{3} & \text{d. } \frac{3}{5} & \text{f. } \frac{7}{9} \end{array}$$

So far all your answers have been less than one (a **proper fraction**). Sometimes adding fractions can result in more than one whole.

Look at this example in Figure 3.10.8:



Figure 3.10.8

$$\frac{2}{4} + \frac{3}{4} = \frac{4}{4} \text{ and } \frac{1}{4} \text{ (or } \left(\frac{5}{4}\right))$$

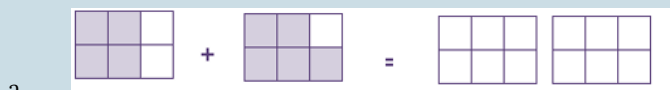
There are not enough parts in the first square to hold all your shaded parts, so you need to draw a second square to hold the extra shaded parts.

You would also have to convert this answer from an improper fraction to a mixed number:

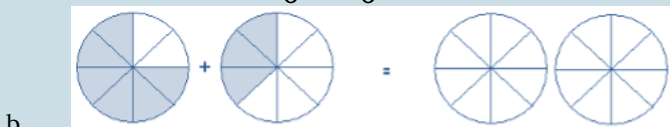
$$\frac{5}{4} = 1\frac{1}{4}$$

Exercise 5

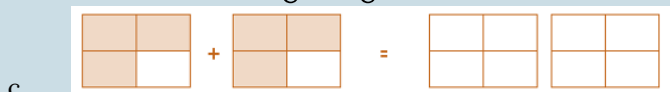
Try these additions. Remember to always reduce!



$$\frac{4}{6} + \frac{5}{6} =$$

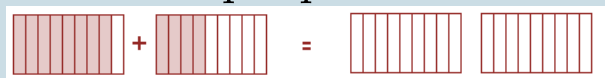


$$\frac{6}{8} + \frac{3}{8} =$$



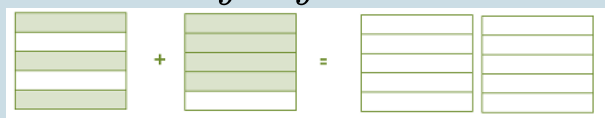
$$\frac{3}{4} + \frac{3}{4} =$$

d.



$$\frac{8}{9} + \frac{4}{9} =$$

e.



$$\frac{3}{5} + \frac{4}{5} =$$

Exercise 5 Answers

a. $1\frac{1}{2}$

c. $1\frac{1}{2}$

e. $1\frac{2}{5}$

b. $1\frac{1}{8}$

d. $1\frac{1}{3}$

Example C

Sometimes, you will have to add 3 or more fractions together (see Figure 3.10.9).



Figure 3.10.9

$$\frac{2}{3} + \frac{1}{3} + \frac{2}{3} = \frac{5}{3} = 1\frac{2}{3}$$

Example D

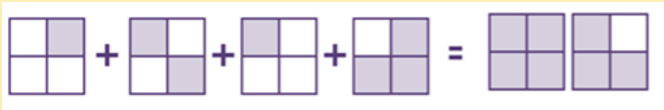


Figure 3.10.10

$$\frac{1}{4} + \frac{2}{4} + \frac{1}{4} + \frac{3}{4} = \frac{7}{4}$$

Exercise 6

Add these common fractions. Be sure your answers are in the lowest terms.

Example: $\frac{2}{3} + \frac{1}{3} = \frac{3}{3} = 1$

$$\text{a. } \frac{7}{10} + \frac{3}{10} =$$

$$\text{b. } \frac{3}{5} + \frac{2}{5} =$$

$$\text{c. } \begin{array}{r} \frac{3}{4} \\ + \frac{1}{4} \\ \hline \end{array}$$

$$\text{d. } \begin{array}{r} \frac{5}{6} \\ + \frac{5}{6} \\ \hline \end{array}$$

$$\text{e. } \begin{array}{r} \frac{4}{8} \\ + \frac{3}{8} \\ \hline \end{array}$$

$$\text{f. } \begin{array}{r} \frac{1}{8} \\ + \frac{3}{8} \\ \hline \end{array}$$

$$\text{g. } \begin{array}{r} \frac{2}{5} \\ + \frac{3}{5} \\ \hline \end{array}$$

$$\text{h. } \begin{array}{r} \frac{3}{6} \\ + \frac{1}{6} \\ \hline \end{array}$$

Exercise 6 Answers

$$\text{a. } 1$$

$$\text{b. } 1$$

$$\text{c. } 1$$

$$\text{d. } 1\frac{2}{3}$$

$$\text{e. } \frac{7}{8}$$

$$\text{f. } \frac{1}{2}$$

$$\text{g. } 1\frac{3}{5}$$

$$\text{h. } \frac{5}{6}$$

Adding Mixed Numbers

To add mixed numbers:

- Be sure the denominators are the same.
- Add the common fractions.
- Add the whole numbers.
- Simplify the common fraction.

Example E

$$\begin{array}{r} 3\frac{1}{8} \\ + 2\frac{3}{8} \\ \hline \end{array}$$

$$5\frac{4}{8} = 5\frac{1}{2}$$

$$\frac{4}{8} = \frac{4}{8} \left(\frac{\div 4}{\div 4} \right) = \frac{1}{2}$$

Example F

$$\begin{array}{r} 12\frac{1}{3} \\ + \quad 6\frac{1}{3} \\ \hline 18\frac{2}{3} \end{array}$$

Exercise 7

Add the following numbers. Reduce the answers to the lowest terms.

Example:

$$\begin{array}{r} 6\frac{1}{12} \\ + \quad 8\frac{5}{12} \\ \hline \end{array} \quad 14\frac{6}{12} = 14\frac{1}{2}$$

$$\begin{array}{r} \text{a.} \quad 22\frac{1}{6} \\ + \quad 14\frac{6}{12} \\ \hline 8\frac{1}{4} \end{array}$$

$$\begin{array}{r} \text{b.} \quad 3\frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} \text{c.} \quad 18\frac{1}{2} \\ + \quad 10 \\ \hline 4\frac{1}{10} \end{array}$$

$$\begin{array}{r} \text{d.} \quad 3 \\ \hline 10 \end{array}$$

Exercise 7 Answers

$$\text{a.} \quad 36\frac{1}{3}$$

$$\text{b.} \quad 11\frac{1}{2}$$

$$\text{c.} \quad 28\frac{1}{2}$$

$$\text{d.} \quad 4\frac{2}{5}$$

Exercise 8

Add these numbers. Give your answers in the lowest terms.

Example:

$$\begin{array}{r} 6\frac{4}{5} \\ + 3\frac{2}{5} \\ \hline \end{array} \quad 9\frac{6}{5} = 10\frac{1}{5}$$

a.

$$\begin{array}{r} 9\frac{1}{3} \\ + 2\frac{2}{3} \\ \hline 3\frac{3}{8} \end{array}$$

b.

$$\begin{array}{r} 12\frac{7}{8} \\ + 12\frac{7}{8} \\ \hline \end{array}$$

c.

$$\begin{array}{r} 100\frac{7}{10} \\ + 50\frac{5}{10} \\ \hline \end{array}$$

d.

$$\begin{array}{r} 3\frac{4}{7} \\ + 6\frac{5}{7} \\ \hline 8\frac{4}{5} \end{array}$$

e.

$$\begin{array}{r} 4\frac{4}{5} \\ + 4\frac{4}{5} \\ \hline \end{array}$$

Exercise 8 Answers

a. 12

b. $16\frac{1}{4}$

c. $151\frac{1}{5}$

d. $10\frac{2}{7}$

e. $12\frac{3}{5}$

The next question is:

What happens when two fractions in an addition (the **addends**) do not have the same **denominator**? If the addends do not have a common denominator, you will need to find equivalent fractions to make the addends have a common denominator.

Read on to find out how!

Multiples & Least Common Multiples (LCM)

When you learned the multiplication tables you learned the **multiples** of each number. Multiples are the answers when you multiply a whole number by 1, 2, 3, 4, 5, 6, 7, and so on.

Table 3.10.1 shows some multiples of 2 and 6.

Table 3.10.1: Multiples of 2 and 6

Multiplied By	Multiples of 2	Multiples of 6
1	2	6
2	4	12
3	6	18
4	8	24
5	10	30
6	12	36
7	14	42
8	16	48
9	18	54
10	20	60
11	22	66
12	24	72

You can keep going as high as you want.

The multiples of 2 are **2, 4, 6, 8, 10, 12, 14**, and so on.
The multiples of 6 are **6, 12, 18, 24, 30, 36**, and so on.

Exercise 9

Create a chart of the first ten multiples of each number.
This chart may be useful to you later.

Example: Multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

- a. 3
- b. 4
- c. 5
- d. 9

- e. 10
- f. 11
- g. 12

Exercise 9 Answers

Table 3.10.2: Exercise 9 Answers

Number	$\times 2$	$\times 3$	$\times 4$	$\times 5$	$\times 6$	$\times 7$	$\times 8$	$\times 9$	$\times 10$
2	4	6	8	10	12	14	16	18	20
3	6	9	12	15	18	21	24	27	30
4	8	12	16	20	24	28	32	36	40
5	10	15	20	25	30	35	40	45	50
9	18	27	36	45	54	63	72	81	90
10	20	30	40	50	60	70	80	90	100
11	22	33	44	55	66	77	88	99	110
12	24	36	48	60	72	84	96	108	120

This is a quick method to find the least common multiple (LCM):

- Least means smallest.
- Common means shared.

- Multiple means the answers when you multiply by 1, 2, 3, etc.

Example G

What is the least common multiple (LCM) of 3 and 5?

Multiples:

- Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30
- Multiples of 5: 5, 10, 15, 20, 25, 30...

The least common multiple of 3 and 5 is 15.

Example H

What is the LCM of 3 and 4?

Multiples:

- Multiples of 3: 3, 6, 9, 12, 15, 18, 21, 24, 27, 30...
- Multiples of 4: 4, 8, 12, 16, 20, 24, 28, 32

The least common multiple of 3 and 4 is 12.

Example I

What is the LCM of 4 and 8?

Multiples:

- Multiples of 4: 4, 8, 12, 16, 20...
- Multiples of 8: 8, 16, 24, 32, 40...

The least common multiple of 4 and 8 is 8.

Hint: Always check to see if the larger number is a multiple of the smaller number. If it is, then the larger number is the Least Common Multiple (LCM).

- LCM of 3 and 6 is 6
- LCM of 2 and 4 is 4
- LCM of 5 and 15 is 15

Exercise 10

Find the **least common multiple (LCM)** of these pairs of numbers.

Use your chart from Exercise 9 to help you. You may want to add the multiples of other numbers to that chart.

a. 3, 6	e. 5, 4	i. 25, 5
b. 2, 5	f. 4, 8	j. 2, 9
c. 12, 3	g. 8, 16	k. 6, 10
d. 6, 12	h. 4, 7	l. 8, 12

Exercise 10 Answers

a. 6	e. 20	i. 25
b. 10	f. 8	j. 18
c. 12	g. 16	k. 30
d. 12	h. 28	l. 24

Now that you know how to find an LCM, you can apply this knowledge to adding and subtracting fractions.

Least Common Denominator (LCD)

To find the Least Common Denominator (LCD) of common fractions, **find the least common multiple of the denominators.**

Example J

What is the least common denominator of $\frac{1}{2}$ and $\frac{3}{4}$?

- The denominators are 2 and 4.
- The **least common multiple** of 2 and 4 is **4**.

So, the **least common denominator (LCD)** for $\frac{1}{2}$ and $\frac{3}{4}$ is **4**.

Example K

What is the **LCD** of $\frac{3}{4}$ and $\frac{2}{3}$?

- The denominators are 4 and 3.
- The **least common multiple** of 4 and 3 is **12**.

So, the **least common denominator** for $\frac{3}{4}$ and $\frac{2}{3}$ is **12**.

Exercise 11

Find the Least Common Denominator (LCD) for these pairs of fractions.

Example: $\frac{5}{8}, \frac{2}{3}$

Denominators: 8, 3 LCD: 24

a. $\frac{1}{5}, \frac{1}{10}$

b. $\frac{1}{3}, \frac{3}{4}$

c. $\frac{2}{3}, \frac{1}{5}$

d. $\frac{5}{8}, \frac{1}{16}$

Exercise 11 Answers (only least common denominator is given)

a. 10
b. 12

c. 15
d. 16

You know how to find the least common denominator (**LCD**). The next step is to make **equivalent fractions** using the **LCD**.

Step 1: Find the least common denominator.

$$\begin{array}{r} \frac{3}{4} \\ + \frac{1}{3} \\ \hline \end{array}$$

LCD of 4 and 3 is 12.

Step 2: Write an = sign after each fraction, followed by the common denominator.

$$\begin{array}{r} \frac{3}{4} = \frac{\quad}{12} \\ + \frac{1}{3} = \frac{\quad}{12} \\ \hline \end{array}$$

Step 3: Rename the fractions as equivalent fractions with the LCD.

$$\begin{array}{r} \frac{3}{4} = \frac{\quad}{12} \\ 4 \text{ times what} = 12? \\ 4 \times 3 = 12 \end{array}$$

If the denominator was multiplied by 3, the numerator must be multiplied by 3.

$$\frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

Now rename the other fraction.

$$\frac{1}{3} = \frac{\quad}{12}$$

3 times what = 12?

$$3 \times 4 = 12$$

If this denominator was multiplied by 4, this numerator must be multiplied by 4.

$$\frac{1 \times 4}{3 \times 4} = \frac{4}{12}$$

Now, rename the other fraction.

Step 4: The question now looks like this and can be added.

$$\begin{array}{r} \frac{3}{4} = \frac{9}{12} \\ + \frac{1}{3} = \frac{4}{12} \\ \hline \end{array}$$

$$\frac{13}{12} = 1\frac{1}{12}$$

Example L

$$\frac{1}{4} + \frac{3}{8} =$$

Step 1 & 2: Find the least common denominator.

$$\begin{array}{r} \frac{1}{4} = \frac{}{8} \\ + \frac{3}{8} = \frac{3}{8} \\ \hline \end{array}$$

Step 3: Rename as equivalent fractions.

$$\begin{array}{r} \frac{1}{4} \left(\frac{\times 2}{\times 2} \right) = \frac{2}{8} \\ + \quad \frac{3}{8} \left(\frac{\times 1}{\times 1} \right) = \frac{3}{8} \\ \hline \end{array}$$

Step 4: Add and simplify the answer.

$$\begin{array}{r} \frac{1}{4} \left(\frac{\times 2}{\times 2} \right) = \frac{2}{8} \\ + \quad \frac{3}{8} \left(\frac{\times 1}{\times 1} \right) = \frac{3}{8} \\ \hline \frac{5}{8} \end{array}$$

Exercise 12

Add these common fractions. Express the answer in the lowest terms.

Examples:

$$\begin{array}{r}
 \frac{1}{2} \left(\frac{\times 4}{\times 4} \right) = \frac{4}{8} \\
 + \frac{3}{8} \left(\frac{\times 1}{\times 1} \right) = \frac{3}{8} \\
 \hline
 \frac{7}{8}
 \end{array}
 \qquad
 \begin{array}{r}
 \frac{1}{4} \left(\frac{\times 2}{\times 2} \right) = \frac{2}{8} \\
 + \frac{3}{8} \left(\frac{\times 1}{\times 1} \right) = \frac{3}{8} \\
 \hline
 \frac{5}{8}
 \end{array}$$

$$\begin{array}{lll}
 \text{a.} & \begin{array}{r} \frac{1}{5} \\ + \frac{1}{10} \\ \hline \end{array} & \text{c.} \begin{array}{r} \frac{1}{3} \\ + \frac{7}{12} \\ \hline \end{array} & \text{e.} \begin{array}{r} \frac{3}{10} \\ + \frac{2}{5} \\ \hline \end{array} \\
 \text{b.} & \begin{array}{r} \frac{5}{16} \\ + \frac{1}{4} \\ \hline \end{array} & \text{d.} \begin{array}{r} \frac{2}{3} \\ + \frac{1}{6} \\ \hline \end{array} & \text{f.} \begin{array}{r} \frac{1}{12} \\ + \frac{1}{4} \\ \hline \end{array}
 \end{array}$$

Exercise 12 Answers

$$\begin{array}{lll}
 \text{a.} & \frac{3}{10} & \text{c.} \frac{11}{12} & \text{e.} \frac{7}{10} \\
 \text{b.} & \frac{9}{16} & \text{d.} \frac{5}{6} & \text{f.} \frac{1}{3}
 \end{array}$$

Exercise 13

More practice. Do only as many as you think you need.

Examples:

$$\begin{array}{r} \frac{2}{3} \left(\frac{\times 4}{\times 4} \right) = \frac{8}{12} \\ \frac{1}{2} \left(\frac{\times 6}{\times 6} \right) = \frac{6}{12} \\ + \frac{3}{4} \left(\frac{\times 3}{\times 3} \right) = \frac{9}{12} \\ \hline \frac{23}{12} = 1 \frac{11}{12} \end{array}$$

$$\begin{array}{r} \frac{5}{24} \left(\frac{\times 1}{\times 1} \right) = \frac{5}{24} \\ \frac{1}{3} \left(\frac{\times 8}{\times 8} \right) = \frac{8}{24} \\ + \frac{3}{8} \left(\frac{\times 3}{\times 3} \right) = \frac{9}{24} \\ \hline \frac{22}{24} = 1 \frac{11}{12} \end{array}$$

a.
$$\begin{array}{r} \frac{5}{12} \\ + \frac{3}{4} \\ \hline \frac{3}{10} \end{array}$$

b.
$$\begin{array}{r} \frac{3}{4} \\ + \frac{4}{5} \\ \hline \end{array}$$

c.
$$\begin{array}{r} \frac{1}{2} \\ + \frac{7}{10} \\ \hline \frac{5}{6} \end{array}$$

d.
$$\begin{array}{r} \frac{3}{4} \\ + \frac{1}{3} \\ \hline \end{array}$$

e.
$$\begin{array}{r} \frac{7}{16} \\ + \frac{3}{4} \\ \hline \frac{4}{5} \end{array}$$

f.
$$\begin{array}{r} \frac{1}{3} \\ + \frac{1}{3} \\ \hline \end{array}$$

Exercise 13 Answers

$$\text{c. } 2 \\ \text{d. } 1\frac{17}{20}$$

$$\text{e. } 1\frac{3}{5} \\ \text{f. } 1\frac{1}{12}$$

$$\text{g. } 1\frac{3}{16} \\ \text{h. } 1\frac{2}{15}$$

Addition questions are often written with the fractions side by side instead of one fraction above the other. For example:

$$\frac{2}{3} + \frac{5}{8} =$$

You may solve as shown in this example or rewrite the question with the fractions one above the other.

$$\frac{2}{3} + \frac{5}{8} = \frac{2 \times 8}{3 \times 8} + \frac{5 \times 3}{8 \times 3} = \frac{16}{24} + \frac{15}{24} = \frac{31}{24} = 1\frac{7}{24}$$

or

$$\frac{2}{3} \left(\frac{\times 8}{\times 8} \right) = \frac{16}{24}$$

$$\frac{5}{8} \left(\frac{\times 3}{\times 3} \right) = \frac{15}{24}$$

$$\frac{31}{24} = 1\frac{7}{24}$$

Exercise 14

Find the sum. Do enough questions to be confident in your skill.

$$\begin{array}{r} \frac{1}{2} + \frac{1}{6} = \\ \text{Example: } \frac{1}{2} \left(\frac{\times 3}{\times 3} \right) + \frac{1}{6} = \\ \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3} \end{array}$$

$$\begin{array}{lll} \text{a. } \frac{1}{4} + \frac{7}{8} = & \text{d. } \frac{1}{3} + \frac{2}{3} = & \text{g. } \frac{1}{3} + \frac{5}{8} = \\ \text{b. } \frac{1}{5} + \frac{3}{5} = & \text{e. } \frac{1}{6} + \frac{3}{8} = & \\ \text{c. } \frac{1}{12} + \frac{2}{3} = & \text{f. } \frac{3}{4} + \frac{1}{2} = & \end{array}$$

Exercise 14 Answers

a. $1\frac{1}{8}$

b. $\frac{4}{5}$

c. $\frac{3}{4}$

d. $1\frac{13}{24}$

f. $1\frac{1}{4}$

g. $\frac{23}{14}$

You already know how to add mixed numbers which have the same (like) denominators.

To add mixed numbers with different denominators, you must:

1. Find the least common denominator (LCD) for the fractions.
2. Rename the fractions as equivalent fractions using the LCD
3. Be sure to bring the whole number across the equal sign when you rename.
4. Add the fractions.
5. Add the whole numbers.
6. Simplify the answer.
7. Remember that if the sum of the fractions is an improper fraction, you must rename it as a mixed number that is added to the whole number in your answer.

Example M

$$\begin{array}{r} 3\frac{3}{4} \left(\frac{\times 5}{\times 5} \right) = 3\frac{15}{20} \\ + \quad 6\frac{1}{5} \left(\frac{\times 4}{\times 4} \right) = 6\frac{4}{20} \\ \hline \\ = 9\frac{19}{20} \end{array}$$

Example N

$$\begin{array}{r} 3\frac{3}{4} \left(\frac{\times 3}{\times 3} \right) = 3\frac{3}{12} \\ 8\frac{2}{3} \left(\frac{\times 4}{\times 4} \right) = 8\frac{8}{12} \\ + \quad 2\frac{1}{2} \left(\frac{\times 6}{\times 6} \right) = 2\frac{6}{12} \\ \hline \\ = 13\frac{17}{12} \end{array}$$

$\frac{17}{12}$ is an improper fraction so we simplify it:

$$\frac{17}{12} = 1\frac{5}{12}$$

Therefore, the answer becomes:

$$13\frac{17}{12} = 13 + 1\frac{5}{12} = 14\frac{5}{12}$$

Exercise 15

Add. Express the sums in the lowest terms.

$$\begin{array}{rcl} & 1\frac{3}{8} \left(\begin{array}{c} \times 1 \\ \times 1 \end{array} \right) & = 1\frac{3}{8} \\ \text{Example: } & + 1\frac{1}{4} \left(\begin{array}{c} \times 2 \\ \times 2 \end{array} \right) & = 1\frac{2}{8} \\ \hline & & 2\frac{5}{8} \end{array}$$

$$\begin{array}{r} \text{a.} \quad \quad \quad 3\frac{1}{5} \\ + \quad 2\frac{3}{10} \\ \hline \end{array}$$

$$\begin{array}{r} \text{c.} \quad \quad \quad 8\frac{1}{4} \\ + \quad 4\frac{1}{3} \\ \hline \end{array}$$

$$\begin{array}{r} \text{b.} \quad \quad \quad 6\frac{2}{15} \\ + \quad 1\frac{3}{10} \\ \hline \end{array}$$

$$\begin{array}{r} \text{d.} \quad \quad \quad 5\frac{2}{3} \\ + \quad 6\frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 116\frac{5}{8} \\ + \quad 9\frac{1}{24} \\ \hline \end{array}$$

Exercise 15 Answers

a. $5\frac{1}{2}$

b. $7\frac{13}{30}$

c. $12\frac{7}{12}$

d. $11\frac{11}{12}$

e. $125\frac{2}{3}$

Exercise 16

Add. Express the sums in the lowest terms.

Example:

$$\begin{array}{r}
 4\frac{1}{2} \left(\frac{\times 6}{\times 6} \right) = 4\frac{6}{12} \\
 + \quad 2\frac{1}{3} \left(\frac{\times 4}{\times 4} \right) = 2\frac{4}{12} \\
 \hline
 6\frac{10}{12} = 6\frac{5}{6}
 \end{array}$$

a.	$ \begin{array}{r} 3\frac{2}{3} \\ + \quad 1\frac{1}{2} \\ \hline \end{array} $	c.	$ \begin{array}{r} 2\frac{1}{8} \\ + \quad 4\frac{3}{16} \\ \hline \end{array} $	e.	$ \begin{array}{r} 3\frac{3}{8} \\ 2\frac{3}{4} \\ + \quad 1\frac{1}{2} \\ \hline \end{array} $
b.	$ \begin{array}{r} 6\frac{1}{2} \\ + \quad 4\frac{1}{4} \\ \hline \end{array} $	d.	$ \begin{array}{r} 2\frac{1}{5} \\ + \quad 3\frac{2}{3} \\ \hline \end{array} $	f.	$ \begin{array}{r} 4\frac{3}{4} \\ 2\frac{1}{5} \\ + \quad 4\frac{1}{2} \\ \hline \end{array} $

Exercise 16 Answers

a. $5\frac{1}{6}$

c. $6\frac{5}{16}$

e. $7\frac{5}{8}$

b. $10\frac{3}{4}$

d. $12\frac{7}{15}$

f. $11\frac{9}{20}$

Problems Using Addition of Common Fractions

Exercise 17

Solve the word problems.

- a. The bathroom shelf is crowded with hand lotion bottles, each with a little lotion left inside. Everyone always likes to try the new bottle before the old one

is emptied! One bottle is $\frac{1}{3}$ full, another is $\frac{1}{4}$ full, one is only $\frac{1}{8}$ full and one is still $\frac{1}{2}$ full. How much lotion is in the bottles altogether?

- b. Sometimes, Joan thinks she will go crazy when she packs the lunches for her family. Little Sarah has decided she only wants $\frac{3}{4}$ of a sandwich, Megan wants $\frac{1}{4}$ of a sandwich, Joan's husband takes $1\frac{1}{2}$ sandwiches, and their son, who does heavy work, takes 3 sandwiches! How many sandwiches does Joan make?
- c. Dave paid the babysitter for the week. The sitter worked $3\frac{3}{4}$ hours on Monday, $4\frac{1}{4}$ hours on Tuesday and $6\frac{1}{2}$ hours on Friday. How many hours did the babysitter work looking after Dave's children that week?
- d. Quite a lot of watermelon was left after the watermelon-eating contest: $1\frac{1}{2}$ watermelons on one table, $2\frac{3}{4}$ of a watermelon on another table and $\frac{5}{8}$ of a watermelon on the third table. The organizers want to know exactly how much was left over so they will not buy so much next year. Calculate the amount of watermelon left over.

- e. Jeanette has a novel to read for English. She read $\frac{1}{2}$ of the book on the weekend and only had time to read $\frac{1}{8}$ of the book on Monday and another $\frac{1}{4}$ on Wednesday. How much of the book has she read?
- f. Dion walks around this route each day for exercise. How far does he walk each day? Is this a perimeter or area question?

$$\frac{1}{2} km$$



$$1\frac{2}{3} km$$

- g. How many metres of baseboard are needed for a rectangular room $4\frac{1}{2}$ m by $3\frac{1}{5}$ m? Deduct 1 m for the doorway. (TIP: Draw a picture)
- h. Sana is going to frame a large piece of art with a wooden frame. The art piece is $1\frac{1}{10}$ m by $\frac{3}{5}$ m. How much framing material should she buy?
- i. Find the perimeter of the following figure.
- j. Find the perimeter of a picture frame if one side is $12\frac{1}{10}$ cm and the other side measures $14\frac{1}{5}$ cm.
- k. Find the perimeter of this triangle.

Exercise 17 Answers

a. $1\frac{5}{24}$ bottles total	perimeter
b. $5\frac{1}{2}$ sandwiches	g. $14\frac{2}{5}$ m of material
c. $14\frac{1}{2}$ hours	h. $3\frac{2}{5}$ m of material
d. $4\frac{7}{8}$ watermelons	i. $15\frac{2}{3}$ cm
e. $\frac{7}{8}$ of the book	j. $52\frac{3}{5}$ cm k) $17\frac{11}{24}$ cm
f. He walks $4\frac{1}{3}$ km each day,	

3.10: Practice Questions

1. Add and express the answers in the lowest terms.

$$\begin{array}{r} \frac{1}{4} \\ \text{a. } + \frac{3}{4} \\ \hline \end{array}$$

$$\begin{array}{r} \frac{3}{8} \\ \text{c. } + \frac{3}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 1\frac{3}{5} \\ \text{b. } + 3\frac{4}{5} \\ \hline \end{array}$$

$$\begin{array}{r} 2\frac{1}{6} \\ \text{d. } + 3\frac{5}{12} \\ \hline \end{array}$$

$$\begin{array}{r} 6\frac{3}{4} \\ + 2\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} 6\frac{7}{8} \\ + 9\frac{1}{3} \\ \hline \end{array}$$

2. Solve the following word problems.

- a. The flight from Vancouver to Sandspit took $1\frac{1}{4}$ hours.

The wait in Sandspit was $1\frac{1}{2}$ hours, and the flight from there to Ketchikan, Alaska was $\frac{3}{4}$ of an hour. How long did it take to make the trip from Vancouver, BC to Ketchikan, Alaska?

- b. Dave built $\frac{1}{8}$ of the fence around his house on Monday,

$\frac{1}{4}$ of it on Tuesday and another $\frac{1}{4}$ on Wednesday. How much of the fence has he built?

- c. John bought snacks in bulk for the class party. His items weighed $\frac{2}{5}$ kg of chips, $\frac{3}{5}$ kg of peanuts, $\frac{1}{2}$ kg of cheese and $1\frac{1}{4}$ kg of fresh veggies. How much did all his snacks weigh?

- d. Clarence is making a frame for his favourite photo. The frame needs to be $\frac{1}{8}$ m by $\frac{5}{6}$ m. How much material should he buy?

3.10: Practice Answers

1. Add and express the answers in the lowest terms.

a. $1\frac{2}{5}$

c. $1\frac{1}{8}$

e. $9\frac{1}{4}$

d. $5\frac{7}{12}$

f. $16\frac{5}{24}$

2. Solve the following word problems.

a. $3\frac{1}{2}$ hr

c. $2\frac{3}{4}$ kg of food

b. $\frac{5}{8}$ of the fence

d. $1\frac{11}{12}$ m of material

Attribution

All figures in this chapter are from Topic A: Adding Common Fractions in Adult Literacy Fundamental Mathematics: Book 5 – 2nd Edition by Liz Girard, Wendy Tagami, and Leanne Caillier-Smith, via BCcampus.

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3.II: Subtracting Common Fractions

Example A

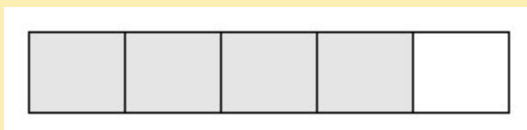


Figure 3.11.1

The shaded part $\left(\frac{4}{5}\right)$ in Figure 3.11.1 is the amount that you are starting with.

Now, cross out (pretend you are *taking away*) 3 shaded parts $\left(\frac{3}{5}\right)$ (see Figure 3.11.2).

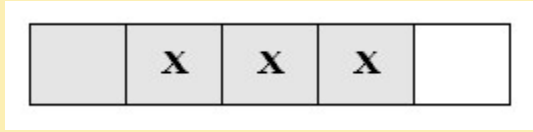


Figure 3.11.2

Example B

Step 1: Draw a pizza.

Step 2: Slice it into 8 equal pieces.

Step 3: Draw pieces of pineapple on 5 pieces.

What fraction of the pizza has pineapple? $\frac{5}{8}$

Step 4: Cross out 2 pineapple pieces to show they have been eaten.

How much of the pineapple pizza is left?

$$\begin{array}{r}
 \frac{5}{8} \quad (\text{amount you started with}) \\
 - \frac{2}{8} \quad (\text{amount eaten, "taken away"}) \\
 \hline
 \frac{3}{8} \quad \text{of the pizza is left with pineapple on it}
 \end{array}$$

Common fractions must have the same denominator when you subtract one from the other. **Subtract the numerators** and keep the same denominators.

Exercise 1

Subtract to find the difference. Express the difference in the lowest terms.

Examples:

$$\begin{array}{r}
 \frac{3}{5} \\
 - \frac{1}{5} \\
 \hline
 \frac{2}{5}
 \end{array}$$

$$\begin{array}{r}
 \frac{7}{8} \\
 - \frac{3}{8} \\
 \hline
 \frac{4}{8} = \frac{1}{2}
 \end{array}$$

$$\begin{array}{r} \\ \\ \\ \text{a. } - \\ \\ \hline \end{array}$$

$$\begin{array}{r} \\ \\ \\ \text{d. } - \\ \\ \hline \end{array}$$

$$\begin{array}{r} \\ \\ \\ \text{b. } - \\ \\ \hline \end{array}$$

$$\begin{array}{r} \\ \\ \\ \text{e. } - \\ \\ \hline \end{array}$$

$$\begin{array}{r} \\ \\ \\ \text{c. } - \\ \\ \hline \end{array}$$

Exercise 1 Answers

$$\begin{array}{r} \\ \text{a. } \\ \\ \\ \hline \end{array}$$

$$\begin{array}{r} \\ \text{c. } \\ \\ \\ \hline \end{array}$$

$$\text{e. } \frac{2}{2} = 1$$

$$\begin{array}{r} \\ \text{b. } \\ \\ \\ \hline \end{array}$$

$$\begin{array}{r} \\ \text{d. } \\ \\ \\ \hline \end{array}$$

You know how to find the least common denominator (LCD) and rewrite fractions in an equivalent form using the LCD.

You must use those skills when you wish to subtract fractions with different denominators.

Example C

$$\frac{4}{5} - \frac{3}{10} =$$

Denominators are 5 and 10. The least common multiple is 10, so the least common denominator is 10.

Step 1: Write equivalent fractions using the LCD.

$$\frac{4}{5} \left(\frac{\times 2}{\times 2} \right) = \frac{8}{10}$$

Step 2: Subtract the numerators.

$$\frac{8}{10} - \frac{3}{10} = \frac{5}{10}$$

Step 3: Simplify the answer.

$$\frac{5}{10} \left(\frac{\div 5}{\div 5} \right) = \frac{1}{2}$$

Exercise 2

Subtract and simplify the answers.

Examples:

$$\begin{array}{r} \frac{5}{6} \\ - \frac{2}{3} \\ \hline \frac{1}{6} \end{array}$$

$$\begin{array}{r} \frac{5}{6} \\ - \frac{2}{3} \\ \hline \frac{1}{6} \end{array}$$

a.
$$\begin{array}{r} \frac{1}{4} \\ - \frac{1}{12} \\ \hline \end{array}$$

c.
$$\begin{array}{r} \frac{15}{16} \\ - \frac{5}{8} \\ \hline \end{array}$$

e.
$$\begin{array}{r} \frac{3}{8} \\ - \frac{1}{4} \\ \hline \end{array}$$

b.
$$\begin{array}{r} \frac{7}{10} \\ - \frac{3}{5} \\ \hline \end{array}$$

d.
$$\begin{array}{r} \frac{7}{16} \\ - \frac{1}{4} \\ \hline \end{array}$$

f.
$$\begin{array}{r} \frac{5}{6} \\ - \frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} \frac{7}{8} \\ \text{g. } \frac{1}{2} \\ \hline \end{array} \quad \begin{array}{r} \frac{5}{12} \\ \text{j. } \frac{1}{4} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{8} \\ \text{m. } \frac{1}{16} \\ \hline \end{array}$$

$$\begin{array}{r} \frac{5}{8} \\ \text{h. } \frac{1}{3} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{2} \\ \text{k. } \frac{1}{6} \\ \hline \end{array} \quad \begin{array}{r} \frac{4}{5} \\ \text{n. } \frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} \frac{2}{3} \\ \text{i. } \frac{1}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{4} \\ \text{l. } \frac{1}{10} \\ \hline \end{array}$$

Exercise 2 Answers

$$\begin{array}{r} \frac{1}{6} \\ \text{c. } \frac{1}{10} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{16} \\ \text{f. } \frac{1}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{7}{24} \\ \text{i. } \frac{13}{24} \\ \hline \end{array}$$

$$\begin{array}{r} \frac{5}{16} \\ \text{e. } \frac{1}{16} \\ \hline \end{array} \quad \begin{array}{r} \frac{3}{8} \\ \text{h. } \frac{1}{8} \\ \hline \end{array} \quad \begin{array}{r} \frac{1}{6} \\ \text{k. } \frac{1}{6} \\ \hline \end{array}$$

$$\begin{array}{l} \text{l. } \frac{1}{3} \\ \text{m. } \frac{13}{20} \end{array}$$

$$\begin{array}{l} \text{n. } \frac{1}{16} \\ \text{o. } \frac{3}{10} \end{array}$$

Subtracting mixed numbers is very similar to adding mixed numbers:

1. Find the least common denominator if the fractions do not have the same denominator already.
2. Rename the fractions as equivalent fractions using the LCD. Don't forget to keep the whole number with the problem.
3. Subtract the second denominator from the first. Keep the same denominator. Subtract the whole numbers.
4. Simplify the answer.

Example D

$$\begin{array}{r} 4\frac{1}{2} \left(\frac{\times 3}{\times 3} \right) = 4\frac{3}{6} \\ - \quad 3\frac{1}{6} = 3\frac{1}{6} \\ \hline 1\frac{2}{6} = 1\frac{1}{3} \end{array}$$

Example E

$$\begin{array}{r} 12\frac{3}{4} \left(\frac{\times 3}{\times 3} \right) = 12\frac{9}{12} \\ - \quad 2\frac{2}{3} \left(\frac{\times 4}{\times 4} \right) = \frac{8}{12} \\ \hline 12\frac{1}{12} \end{array}$$

Exercise 3

Work through all these questions carefully.

$$\begin{array}{r} 16\frac{2}{3} = 16\frac{16}{24} \\ - 4\frac{3}{8} = 4\frac{9}{24} \\ \hline \end{array}$$

Example:

$$12\frac{7}{24}$$

$$\begin{array}{r} 9\frac{7}{12} \\ - 9\frac{5}{12} \\ \hline \end{array}$$

a.

$$\begin{array}{r} 22\frac{5}{6} \\ - 18\frac{2}{5} \\ \hline \end{array}$$

c.

$$\begin{array}{r} 1\frac{7}{100} \\ - \frac{1}{2} \\ \hline \end{array}$$

e.

$$\begin{array}{r} 6\frac{3}{4} \\ - 2\frac{1}{3} \\ \hline \end{array}$$

b.

$$\begin{array}{r} 3\frac{7}{8} \\ - 2\frac{3}{4} \\ \hline \end{array}$$

d.

$$\begin{array}{r} 9\frac{1}{4} \\ - 7 \\ \hline \end{array}$$

f.

$$\begin{array}{r} 1\frac{5}{8} \\ - 1\frac{1}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 19\frac{5}{6} \\ - 11\frac{5}{12} \\ \hline \end{array}$$

Exercise 3 Answers

$$\text{a. } \frac{1}{6}$$

$$\text{d. } 1\frac{1}{8}$$

$$\text{g. } 1\frac{7}{24}$$

$$\text{b. } 4\frac{5}{12}$$

$$\text{e. } 1\frac{1}{5}$$

$$\text{h. } 8\frac{5}{12}$$

$$\text{c. } 4\frac{13}{30}$$

$$\text{f. } 2\frac{1}{4}$$

Subtracting Mixed Numbers From Whole Numbers

This is the start of a new process! You already have all the skills to do this, but the process is new.

Example F

Let's look at some apples.

You have 3 whole apples and you want to give your son 1 apple and your daughter half an apple (See Figure 3.11.3). How will you do this?



Figure 3.11.3

Yes! You will cut one apple in half (see Figure 3.11.4).



Figure 3.11.4

Now, you have $2\frac{2}{2}$ apples. And you can easily give away $1\frac{1}{2}$ of them. Cross out $1\frac{1}{2}$ apples in the drawing.

How much is left?

Here is the arithmetic for what you just did.

$$3 = 2\frac{2}{2} - 1\frac{1}{2} = 1\frac{1}{2}$$

$1\frac{1}{2}$ apples are left

Example G

Figure 3.11.5 shows 6 cans of pop to share among your friends.



Figure 3.11.5

4 people want a whole can, but one person is on a diet and only wants $\frac{1}{4}$ of a can. How much pop will be left?

What will you do? You will open a can and think of that can as $\frac{4}{4}$ (see Figure 3.11.6).



Figure 3.11.6

You have $5\frac{4}{4}$ cans of pop and can give out 4 whole cans

and $\frac{1}{4}$ can of pop. Cross out the 4 whole cans and $\frac{1}{4}$ of a can in the drawing.

How many cans are left?

Here is the arithmetic:

$$\begin{array}{r} 6 = 5\frac{4}{4} \\ - 1\frac{1}{4} \\ \hline 1\frac{3}{4} \end{array}$$

$1\frac{3}{4}$ cans are left.

Example H

Draw five apple pies. Plan to give away $3\frac{2}{3}$ of the pies.

How many pies are left?

To do that, cut one pie into thirds. Then cross out 2 whole pies and $\frac{2}{3}$.

Here is the arithmetic:

$$\begin{array}{r} 5 = 4\frac{3}{3} \\ - 3\frac{2}{3} \\ \hline 1\frac{1}{3} \text{ pies left} \end{array}$$

Remember:

$$1 = \frac{1}{1} = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} = \frac{5}{5} = \frac{6}{6} = \frac{7}{7} = \frac{8}{8} = \frac{9}{9} = \frac{10}{10}$$

and so on.

To subtract a mixed number from a whole number:

1. “Borrow” one from the whole number.
2. Rename the one as an improper fraction with the same denominator as the fraction you are taking away. (Remember to change the whole number to one less.)
3. Subtract the mixed numbers.

Example I

$$18 - 12\frac{3}{4} =$$

“Borrow” 1 from 18 and change to $\frac{4}{4}$.

$$\begin{array}{r} \cancel{18} 17 = 17\frac{4}{4} \\ - 12\frac{3}{4} \\ \hline 5\frac{1}{4} \end{array}$$

Try it yourself:

$$1 - \frac{4}{5} =$$

Exercise 4

Subtract and express in the lowest terms. Remember to change your whole number to a mixed number.

Example:

$$\begin{array}{r} 5 = 4\frac{2}{2} \\ - 1\frac{1}{2} \\ \hline 2\frac{1}{2} \end{array}$$

a. $\begin{array}{r} 9 \\ - 4\frac{3}{10} \\ \hline \end{array}$ c. $\begin{array}{r} 25 \\ - 20\frac{1}{4} \\ \hline \end{array}$ e. $\begin{array}{r} 8 \\ - \frac{3}{4} \\ \hline \end{array}$

b. $\begin{array}{r} 12 \\ - 11\frac{5}{8} \\ \hline \end{array}$ d. $\begin{array}{r} 3 \\ - 2\frac{2}{3} \\ \hline \end{array}$ f. $\begin{array}{r} 32 \\ - 28\frac{1}{2} \\ \hline \end{array}$

$$\begin{array}{r} \text{g.} \quad \quad \quad 5 \\ \quad \quad \quad 3\frac{1}{3} \\ \hline \end{array} \quad \begin{array}{r} \text{h.} \quad \quad \quad 8 \\ \quad \quad \quad 4\frac{4}{9} \\ \hline \end{array}$$

Exercise 4 Answers

$$\text{a.} \quad 4\frac{7}{10}$$

$$\text{b.} \quad \frac{3}{8}$$

$$\text{c.} \quad 4\frac{3}{4}$$

$$\text{d.} \quad \frac{1}{3}$$

$$\text{e.} \quad 7\frac{1}{4}$$

$$\text{f.} \quad 3\frac{1}{2}$$

$$\text{g.} \quad 1\frac{2}{3}$$

$$\text{h.} \quad 3\frac{5}{9}$$

Renaming to Subtract Mixed Numbers

Example J

Look at the $3\frac{1}{4}$ chocolate bars in Figure 3.11.7.



Figure 3.11.7

You need to give $2\frac{3}{4}$ chocolate bars to the kids on the soccer team. How will you do this?

You will have to cut up one of the whole chocolate bars into 4 pieces, or $\frac{4}{4}$ (see Figure 3.11.8).



Figure 3.11.8

Now you have 2 whole bars, $\frac{4}{4}$ of a bar and $\frac{1}{4}$ of a bar which equals $2\frac{5}{4}$ of a bar. It will be easy to give away

(subtract) $2\frac{3}{4}$ bars. Cross out $2\frac{3}{4}$ bars.

How much is left?

Here is the arithmetic:

$$3\frac{1}{4} - 2\frac{3}{4} =$$

$$3\frac{1}{4} = 2\frac{4}{4} + \frac{1}{4} = 2\frac{5}{4}$$

$$\begin{array}{r} 2\frac{5}{4} \\ - 2\frac{3}{4} \\ \hline \frac{2}{4} = \frac{1}{2} \text{ chocolate bar left} \end{array}$$

Example K

Figure 3.11.9 has 5 cherry pies.



Figure 3.11.9

You promised to send $3\frac{5}{8}$ pies to the spring party at the school. What will you do? Cut one of the pies into eighths (see Figure 3.11.10).



Figure 3.11.10

Do that, and then cross out $3\frac{5}{8}$ pies.

How much pie is left?

$$\begin{array}{r}
 5\frac{1}{8} = 4\frac{8}{8} + \frac{1}{8} = 4\frac{9}{8} \\
 \quad \quad \quad - \quad 3\frac{5}{8} \\
 \hline
 \quad \quad \quad 1\frac{4}{8} = 1\frac{1}{2} \text{ pies left}
 \end{array}$$

Renaming a Mixed Number So You Can Subtract

Step 1: Check to see if renaming is needed. That is, check that the fraction in the mixed number you are starting with is less than the fraction you want to take away.

$$4\frac{1}{3} - 2\frac{2}{3} = \left(\frac{1}{3} \text{ is less than } \frac{2}{3} \right)$$

Step 2: “Borrow” one from the whole number. Remember to change the whole number to 1 less.

Step 3: Rename the “borrowed” one as an improper fraction with the same denominator as the other fractions.

$$4\frac{1}{3} = 3\frac{3}{3} + \frac{1}{3}$$

$\frac{3}{3}$ is the 1 that you borrowed from the four.

Step 4: Add the renamed one to the fraction that is part of the same mixed number.

$$3\frac{3}{3} + \frac{1}{3} = 3\frac{4}{3}$$

Step 5: Subtract, as usual, expressing your answer in the lowest terms.

$$\begin{array}{r} 4\frac{1}{3} = 3\frac{4}{3} \\ - 2\frac{2}{3} = 2\frac{2}{3} \\ \hline 1\frac{2}{3} \end{array}$$

Example L

$$5\frac{2}{3} - 2\frac{4}{5}$$

Step 1:

$$\frac{2}{5} \text{ is less than } \frac{4}{5}$$

Step 2 and 3: Borrow one.

$$5\frac{2}{5} = 4\frac{5}{5} + \frac{2}{5} = 4\frac{7}{5}$$

Step 4 and 5:

$$\begin{array}{r} 5\frac{2}{5} = 4\frac{5}{5} + \frac{2}{5} = 4\frac{7}{5} \\ - \quad 2\frac{4}{5} = 2\frac{4}{5} \\ \hline 2\frac{3}{5} \end{array}$$

Example M

$$1\frac{2}{4} - \frac{3}{4}$$

Step 1:

$\frac{2}{4}$ is less than $\frac{3}{4}$ so we need to rename to subtract.

Step 2, 3, 4 & 5:

$$\begin{array}{r} 1\frac{2}{4} = \frac{4}{4} + \frac{2}{4} = \frac{6}{4} \\ - \qquad \frac{3}{4} = \frac{3}{4} \\ \hline \frac{3}{4} \end{array}$$

Exercise 5

Subtract. Be sure the answers are in the lowest terms.

Example:

$$\begin{array}{r} 20\frac{1}{4} = 19\frac{5}{4} \\ - 10\frac{3}{4} \\ \hline 9\frac{2}{4} = 9\frac{1}{2} \end{array}$$

a.
$$\begin{array}{r} 3\frac{1}{3} \\ - 1\frac{2}{3} \\ \hline \end{array}$$

c.
$$\begin{array}{r} 8\frac{1}{3} \\ - 1\frac{2}{3} \\ \hline \end{array}$$

e.
$$\begin{array}{r} 5\frac{2}{7} \\ - 1\frac{3}{7} \\ \hline \end{array}$$

b.
$$\begin{array}{r} 56\frac{2}{5} \\ - 20\frac{4}{5} \\ \hline \end{array}$$

d.
$$\begin{array}{r} 4\frac{1}{5} \\ - 2\frac{3}{5} \\ \hline \end{array}$$

f.
$$\begin{array}{r} 12\frac{5}{9} \\ - 10\frac{7}{9} \\ \hline \end{array}$$

$$\begin{array}{r} \frac{3}{5} \\ \text{g. } \underline{ \frac{4}{5}} \end{array} \qquad \begin{array}{r} \frac{3}{8} \\ \text{h. } \underline{ \frac{7}{8}} \end{array}$$

Exercise 5 Answers

$$\begin{array}{lll} \text{a. } 1 \frac{2}{3} & \text{d. } 1 \frac{3}{5} & \text{g. } 8 \frac{4}{5} \\ \text{b. } 35 \frac{3}{5} & \text{e. } 3 \frac{6}{7} & \text{h. } \frac{1}{2} \\ \text{c. } 6 \frac{2}{3} & \text{f. } 1 \frac{7}{9} & \end{array}$$

Here is the last step for the subtraction of fractions. Often, mixed numbers to be subtracted do not have the same denominators—they are **unlike fractions**.

To subtract unlike fractions:

1. Write equivalent fractions using the LCD.
2. Decide if you need to “borrow” or rename before you subtract.
3. Subtract and simplify the answer.

Example N

$$4\frac{1}{3} - 2\frac{5}{6} =$$

$$\begin{array}{r} 4\frac{1}{3} = 4\frac{2}{6} = 3\frac{6}{6} + \frac{2}{6} = 3\frac{8}{6} \\ - 2\frac{5}{6} \\ \hline 1\frac{3}{6} = 1\frac{1}{2} \end{array}$$

Example O

$$9\frac{1}{10} - 4\frac{1}{4} =$$

$$\begin{array}{r}
 9\frac{1}{10} = 9\frac{2}{20} = 8\frac{20}{20} + \frac{2}{20} = 8\frac{22}{20} \\
 - 4\frac{1}{4} = 4\frac{5}{20} = 4\frac{5}{20} \\
 \hline
 4\frac{17}{20}
 \end{array}$$

Exercise 6

Subtract. Be sure the answers are in the lowest terms.

Example:

$$\begin{array}{r}
 9\frac{3}{8} = 9\frac{3}{8} = 8\frac{11}{8} \\
 - 7\frac{1}{2} = 7\frac{4}{8} = 7\frac{4}{8} \\
 \hline
 1\frac{7}{8}
 \end{array}$$

$$\begin{array}{r} 7\frac{1}{16} \\ - 4\frac{1}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 6\frac{3}{5} \\ - 4\frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 18\frac{1}{6} \\ - 12\frac{2}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 15\frac{1}{6} \\ - 12\frac{7}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 9\frac{1}{4} \\ - 7\frac{3}{8} \\ \hline \end{array}$$

$$\begin{array}{r} 20\frac{2}{6} \\ - 16\frac{2}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 5\frac{1}{3} \\ - 2\frac{1}{2} \\ \hline \end{array}$$

Exercise 6 Answers

$$\text{a. } 2\frac{15}{16}$$

$$\text{d. } 2\frac{7}{20}$$

$$\text{g. } 5\frac{1}{2}$$

$$\text{b. } 2\frac{7}{24}$$

$$\text{e. } 1\frac{7}{8}$$

$$\text{c. } 3\frac{2}{3}$$

$$\text{f. } 2\frac{5}{6}$$

Exercise 7

A subtraction review.

a.
$$\begin{array}{r} \frac{5}{8} \\ - \frac{1}{4} \\ \hline \end{array}$$

d.
$$\begin{array}{r} 7\frac{1}{3} \\ - 4\frac{5}{6} \\ \hline \end{array}$$

b.
$$\begin{array}{r} 8\frac{3}{4} \\ - 4\frac{1}{3} \\ \hline \end{array}$$

e.
$$\begin{array}{r} 9\frac{1}{5} \\ - 6\frac{1}{3} \\ \hline \end{array}$$

c.
$$\begin{array}{r} 13\frac{1}{3} \\ - 12\frac{5}{6} \\ \hline \end{array}$$

f.
$$\begin{array}{r} 7 \\ - 1\frac{7}{8} \\ \hline \end{array}$$

g.
$$\begin{array}{r} 19\frac{2}{3} \\ - 18 \\ \hline \end{array}$$

h. $-\frac{5}{8} - \frac{1}{2}$

Exercise 7 Answers

a. $\frac{3}{8}$

d. $2\frac{1}{2}$

g. $1\frac{1}{3}$

b. $4\frac{5}{12}$

e. $2\frac{13}{15}$

h. $\frac{1}{8}$

c. $\frac{1}{2}$

f. $5\frac{1}{8}$

Problems Using Subtraction of Common Fractions

Subtraction problems may ask you to:

- Find the difference between the two amounts.
 - “how much more is...”
 - “how much less is...”
- Take away, give away, or lose.
- Decide how much is left or how much remains.

Drawing a sketch and estimating the answer using whole numbers may also be helpful.

Exercise 8

Solve the following word problems.

- a. The New Earth Diaper Company stocks went from $5\frac{7}{8}$ to $7\frac{3}{8}$ this week. How much did the stocks increase in value?
- b. Jean is knitting an afghan which will be made from 5 long pieces. She has finished $3\frac{2}{3}$ of the pieces. How many pieces does she still have to knit?
- c. Dave said he worked in the garden for $6\frac{1}{4}$ hours, but his wife saw him snoozing under a tree for $1\frac{1}{2}$ hours! How long did Dave really work?
- d. Maureen left $\frac{2}{3}$ of a big lasagne casserole in the fridge, hoping it would be enough for a quick dinner that night. But alas, when she got home, only $\frac{1}{4}$ of the big lasagne casserole remained. How much of the lasagne was eaten while she was out?

- e. In the first half of 1992, the Bank of Canada's Prime Rate dropped steadily. It started the year at $8\frac{1}{2}\%$ and was at a low $6\frac{3}{4}\%$ in July. How many percentage points did the prime rate drop? (Note: treat the % just like a unit in this problem.)
- f. Mark is $1\frac{3}{4}$ metres tall. His partner is $1\frac{1}{3}$ metres tall. How much taller is Mark than his partner?
- g. A teenager can drink $3\frac{3}{4}$ litres of water each day. If this teenager drinks $2\frac{1}{2}$ litres of water by lunch, how much more water will he drink in the day?
- h. Joan bought $13\frac{1}{2}$ metres to do her sewing project. She has used $8\frac{1}{5}$ metres so far. How much material does she have left?

Exercise 8 Answers

- a. $1\frac{1}{2}$ or \$1.50 c. $4\frac{3}{4}$ hours e. $1\frac{3}{4}$ percentage points
- b. $1\frac{1}{3}$ pieces d. $\frac{5}{12}$ of the lasagna f. $\frac{5}{12}$ metres taller

g. $1\frac{1}{4}$ litres left to drink

h. $5\frac{3}{10}$ metres left

3.II: Practice Questions

1. Subtract these fractions. Simplify the answers when necessary.

a.
$$\begin{array}{r} \frac{7}{8} \\ - \frac{1}{8} \\ \hline \end{array}$$

b.
$$\begin{array}{r} \frac{11}{15} \\ - \frac{4}{15} \\ \hline \end{array}$$

c.
$$\begin{array}{r} \frac{7}{8} \\ - \frac{3}{16} \\ \hline \end{array}$$

d.
$$\begin{array}{r} \frac{2}{3} \\ - \frac{1}{6} \\ \hline \frac{4}{7} \end{array}$$

e.
$$\begin{array}{r} \frac{1}{14} \\ - \frac{4}{5} \\ \hline \end{array}$$

f.
$$\begin{array}{r} \frac{3}{8} \\ - \frac{4}{5} \\ \hline \end{array}$$

g.
$$\begin{array}{r} 2\frac{1}{2} \\ - \frac{5}{5} \\ \hline \end{array}$$

h.
$$\begin{array}{r} 4\frac{7}{8} \\ - \frac{5}{8} \\ \hline \end{array}$$

i.
$$\begin{array}{r} 10\frac{3}{5} \\ - 3\frac{3}{10} \\ \hline 7\frac{1}{5} \end{array}$$

j.
$$\begin{array}{r} \frac{4}{5} \\ - 3\frac{1}{8} \\ \hline 12\frac{1}{8} \end{array}$$

k.
$$\begin{array}{r} 11 \\ - 9\frac{1}{5} \\ \hline \end{array}$$

l.
$$\begin{array}{r} \frac{1}{4} \\ - \frac{1}{4} \\ \hline \end{array}$$

$$\begin{array}{r} 5\frac{1}{4} \\ - 1\frac{2}{3} \\ \hline \end{array}$$

$$\begin{array}{r} 10\frac{1}{2} \\ - 2\frac{2}{5} \\ \hline \end{array}$$

$$\begin{array}{r} 6\frac{2}{3} \\ - 4\frac{3}{8} \\ \hline \end{array}$$

3.11: Practice Answers

1. Subtract these fractions.

$$\text{a. } \frac{3}{4}$$

$$\text{b. } \frac{7}{15}$$

$$\text{c. } \frac{11}{16}$$

$$\text{d. } \frac{1}{2}$$

$$\text{e. } \frac{1}{2}$$

$$\text{f. } \frac{17}{40}$$

$$\text{g. } 1\frac{1}{2}$$

$$\text{h. } \frac{1}{8}$$

$$\text{i. } 7\frac{3}{10}$$

$$\text{j. } 3\frac{2}{5}$$

$$\text{k. } 1\frac{1}{8}$$

$$\text{l. } 8\frac{19}{20}$$

$$\text{m. } 3\frac{7}{12}$$

$$\text{n. } 8\frac{1}{10}$$

$$\text{o. } 2\frac{7}{24}$$

Attribution

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3.12: Problems Using Common Fractions

Review again the five steps for problem solving:

1. Read, find the question.
2. Get the necessary information from the problem.
3. Decide on the arithmetic operation.
4. Estimate the answer, using rounded numbers.
Does the answer to the problem seem sensible?
5. Solve the problem using the actual numbers.
Check. Is the answer close to the estimate?

Fraction problems with mixed numbers are easily estimated by rounding the fraction to the nearest whole number.

If the problem uses proper fractions, it is harder to estimate the answer using a rounded number because proper fractions will round off to either 0 or 1, which isn't too useful.

However, to figure out the operation to use (Step 3), it sometimes helps to substitute whole numbers for the proper fraction; this may help you to make sense of the problem. Drawing a sketch might also be helpful.

Exercise 1

Decide on the arithmetic operation ($+$, $-$, $\times$, $\div$), then solve.

- a. The following recipe for Macaroni and Cheese is very tasty. It feeds six people. Look at the recipe and then complete the chart according to these questions.
 - i. You want to make enough macaroni and cheese for 9 people. That is $\frac{9}{6}$ ($1\frac{1}{2}$) of the recipe. Figure out all the quantities.
 - ii. Figure out the quantities for $\frac{3}{4}$ the recipe, so it is the right amount for 3 people.

Table 3.12.1: Question A Complete the Chart

Ingredients for 6 People	Quantities for 9 People ($1\frac{1}{2}$ x)	Quantities for 3 People ($\frac{1}{2}$ x)
$1\frac{3}{4}$ cup elbow macaroni		
$\frac{3}{4}$ cup chopped onion		
$\frac{1}{2}$ cup chopped green pepper		
10 sliced mushrooms		
3 tbsp. butter or margarine		
$1\frac{1}{2}$ tbsp. flour		
1 tsp. dry mustard		
$\frac{3}{4}$ tsp. salt		

Ingredients for 6 People	Quantities for 9 People ($1\frac{1}{2}$ x)	Quantities for 3 People ($\frac{1}{2}$ x)
$\frac{1}{4}$ tsp. oregano		
$1\frac{1}{2}$ cups milk		
$1\frac{1}{2}$ cups shredded cheddar cheese		
$\frac{1}{2}$ cup fine dry breadcrumbs		

- b. Jack spent his school day this way: $1\frac{1}{4}$ hours on English, $1\frac{3}{4}$ hours on math, $\frac{1}{2}$ hour on science and 2 hours on lunch and coffee breaks. How long was his school day?
- c. The canoe trip usually takes $4\frac{1}{4}$ hours for the 34-kilometre trip. What is the average time per kilometre?

- d. The “Walkyerbunsoff” Club members walk $4\frac{3}{4}$ kilometres around the shopping mall six mornings a week.
- How far do they walk each week?
 - How far do they walk in one year if they keep the same schedule? (52 weeks = 1 year)
- e. The test has 30 multiple choice questions to be completed in $\frac{3}{4}$ of an hour. How much time can be spent on each question? (It might be easier to work with minutes. 1 hour = 60 minutes, so $\frac{3}{4}$ of an hour = ? minutes.)
- f. The gas tank was filled before the family left Hazelton. When they pulled into Prince Rupert that afternoon, the gas gauge showed they had used $\frac{5}{8}$ of the gas.
- What fraction of the gas was left?
 - The full gas tank holds 54 litres. How many litres of gas did they use during the trip from Hazelton to Prince Rupert?
- g. About $\frac{2}{5}$ of the population of our city has an Italian background. The population is 6 500. How many people in our city have an Italian background?
- h. The four children carefully planted 3 rows of corn

together and promised to share the work and the corn. Only $2\frac{1}{4}$ rows of corn came up. How much of a row does each child need to look after?

- i. Gail was supposed to babysit for $2\frac{3}{4}$ hours, but she didn't feel well, so her sister Debbie said she would come for $\frac{1}{2}$ of the time. How long did each sister babysit?
- j. The stocks closed at $6\frac{1}{8}$ on Monday. On Tuesday, they fell $\frac{1}{4}$. What was the value of the stocks at closing on Tuesday?
- k. The same stocks had a good day on Wednesday. They rose $\frac{3}{4}$ from the Tuesday closing price (see question j). What was the value of the stocks at closing on Wednesday?
- l. Victor and his family heat their house with wood. Last year, they cut and hauled $12\frac{1}{2}$ cords of wood from the bush. At $\frac{1}{2}$ cord per pick-up truck load, how many trips did they have to make with their one truck?
- m. Canadian Mortgage Housing Corporation (CMHC) suggests that when you buy a home, the cost of the

house should be no more than $2\frac{1}{2}$ times your annual income. Between them, John and Pam Miller have a gross annual income of \$68 000. About how much would CMHC say they should spend on a house?

- n. The loaded logging trucks cover $38\frac{1}{2}$ km from the logging site down the steep logging roads to the highway in $1\frac{1}{4}$ hours. What is their average speed in kilometres per hour?

Exercise 1 Answers

- a. Tasty Macaroni and Cheese

Table 3.12.2: Question A Answers

Ingredients for 6 People	Quantities for 9 People ($1\frac{1}{2} \times$)	Quantities for 3 People ($\frac{1}{2} \times$)
$1\frac{3}{4}$ cup elbow macaroni	$2\frac{5}{8}$ cups	$\frac{7}{8}$ cup
$\frac{3}{4}$ cup chopped onion	$1\frac{1}{8}$ cup	$\frac{3}{8}$ cup
$\frac{1}{2}$ cup chopped green pepper	$\frac{3}{4}$ cup	$\frac{1}{4}$ cup
10 sliced mushrooms	15	5
3 tbsp. butter or margarine	$4\frac{1}{2}$ tbsp.	$1\frac{1}{2}$ tbsp.
$1\frac{1}{2}$ tbsp. flour	$2\frac{1}{4}$ tbsp.	$\frac{3}{4}$ tbsp.
1 tsp. dry mustard	$1\frac{1}{2}$ tsp.	$\frac{1}{2}$ tsp.
$\frac{3}{4}$ tsp. salt	$1\frac{1}{8}$ tsp.	$\frac{3}{8}$ tsp.

Ingredients for 6 People	Quantities for 9 People ($1\frac{1}{2} \times$)	Quantities for 3 People ($\frac{1}{2} \times$)
$\frac{1}{4}$ tsp. oregano	$\frac{3}{8}$ tsp.	$\frac{1}{8}$ tsp.
$2\frac{1}{2}$ cups milk	$3\frac{3}{4}$ cups	$1\frac{1}{4}$ cups
$2\frac{1}{2}$ cups shredded cheddar cheese	$3\frac{3}{4}$ cups	$1\frac{1}{4}$ cups
$\frac{1}{2}$ cup fine dry breadcrumbs	$\frac{3}{4}$ cup	$\frac{1}{4}$ cup

- b. $5\frac{1}{2}$ hours

c. $\frac{1}{8}$ hours/km or $7\frac{1}{2}$ min/km

d. Answers

e. $1\frac{1}{2}$ minutes/question

f. Answers
- i. $\frac{3}{8}$ tank

ii. $33\frac{3}{4}$

g. 2 600 people

h. $\frac{3}{4}$ of a row

i. $1\frac{3}{8}$ hours

j. $5\frac{7}{8}$

k. $6\frac{5}{8}$

- l. 25 trips
- m. \$170 000
- n. 31 km/h

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3.13: Adding & Subtracting Common Fractions Review

1. Add these common fractions, make sure to reduce your answer to the lowest terms.

$$\begin{array}{lll} \text{a. } \frac{1}{3} + \frac{1}{3} = & \text{d. } \frac{3}{8} + \frac{1}{8} = & \text{g. } \frac{3}{7} + \frac{5}{7} = \\ \text{b. } \frac{3}{7} + \frac{2}{7} = & \text{e. } \frac{4}{8} + \frac{2}{8} = & \text{h. } \frac{7}{21} + \frac{7}{21} = \\ \text{c. } \frac{4}{5} + \frac{1}{5} = & \text{f. } \frac{1}{12} + \frac{3}{12} = & \end{array}$$

2. Add these common fractions, make sure to reduce your answer to the lowest terms.

$$\begin{array}{lll} \text{a. } \frac{1}{3} + \frac{3}{6} = & \text{d. } \frac{3}{5} + \frac{3}{4} = & \text{g. } \frac{1}{8} + \frac{1}{3} = \\ \text{b. } \frac{1}{2} + \frac{2}{5} = & \text{e. } \frac{3}{7} + \frac{6}{8} = & \text{h. } \frac{3}{4} + \frac{8}{9} = \\ \text{c. } \frac{1}{4} + \frac{1}{3} = & \text{f. } \frac{1}{2} + \frac{13}{16} = & \end{array}$$

3. Add. Express the sum in the lowest terms.

$$\begin{array}{lll} \text{a. } 6\frac{3}{5} + 2\frac{3}{7} = & \text{d. } 8\frac{1}{5} + 1\frac{2}{7} = & \text{g. } 12\frac{1}{2} + 1\frac{1}{3} = \\ \text{b. } 3\frac{1}{4} + 4\frac{1}{2} = & \text{e. } 1\frac{14}{15} + 3\frac{1}{3} = & \text{h. } 9\frac{2}{5} + 1\frac{7}{10} = \\ \text{c. } 3\frac{3}{8} + 4\frac{1}{4} = & \text{f. } 4\frac{2}{3} + 1\frac{1}{4} = & \text{i. } 9\frac{4}{5} + 3\frac{1}{7} = \end{array}$$

j. $2\frac{1}{5} + 7\frac{4}{15} =$

4. Subtract. Express your answers in the lowest terms.

a.
$$\begin{array}{r} \frac{2}{5} \\ - \frac{1}{5} \\ \hline \end{array}$$

c.
$$\begin{array}{r} \frac{2}{3} \\ - \frac{1}{3} \\ \hline \end{array}$$

b.
$$\begin{array}{r} \frac{5}{8} \\ - \frac{1}{8} \\ \hline \end{array}$$

d.
$$\begin{array}{r} \frac{5}{12} \\ - \frac{3}{12} \\ \hline \end{array}$$

5. Subtract and express your answer in the lowest terms

a.
$$\begin{array}{r} \frac{17}{22} \\ - \frac{6}{22} \\ \hline \end{array}$$

c.
$$\begin{array}{r} \frac{3}{4} \\ - \frac{1}{12} \\ \hline \end{array}$$

e.
$$\begin{array}{r} \frac{1}{2} \\ - \frac{3}{8} \\ \hline \end{array}$$

b.
$$\begin{array}{r} \frac{3}{4} \\ - \frac{1}{2} \\ \hline \end{array}$$

d.
$$\begin{array}{r} \frac{9}{10} \\ - \frac{4}{5} \\ \hline \end{array}$$

f.
$$\begin{array}{r} \frac{15}{16} \\ - \frac{3}{8} \\ \hline \end{array}$$

g.

$$\frac{\frac{9}{16}}{\frac{1}{8}}$$

h.

$$\frac{\frac{1}{6}}{\frac{1}{12}}$$

6. Subtract. Be sure to reduce your answers to the lowest terms.

a. $\begin{array}{r} 20\frac{2}{4} \\ - 10\frac{3}{4} \\ \hline \end{array}$	c. $\begin{array}{r} 4 \\ - 1\frac{1}{5} \\ \hline \end{array}$	e. $\begin{array}{r} 5\frac{1}{3} \\ - 2\frac{2}{3} \\ \hline \end{array}$
b. $\begin{array}{r} 1\frac{2}{3} \\ - 5\frac{5}{6} \\ \hline \end{array}$	d. $\begin{array}{r} 4\frac{1}{5} \\ - 1\frac{2}{5} \\ \hline \end{array}$	f. $\begin{array}{r} 9\frac{3}{5} \\ - 4\frac{4}{5} \\ \hline \end{array}$

7. Subtract the following fractions. Express your answer in the lowest terms.

a. $\frac{15\frac{1}{6}}{3\frac{2}{5}}$ b. $\frac{7}{4\frac{1}{3}}$ c. $\frac{9\frac{1}{4}}{7\frac{5}{8}}$

d. $\begin{array}{r} 13\frac{1}{4} \\ - 3\frac{4}{9} \\ \hline \end{array}$	f. $\begin{array}{r} 7\frac{1}{3} \\ - 4\frac{5}{6} \\ \hline \end{array}$	h. $\begin{array}{r} 4\frac{3}{5} \\ - 3\frac{3}{4} \\ \hline \end{array}$
e. $\begin{array}{r} 13 \\ - 1\frac{1}{5} \\ \hline \end{array}$	g. $\begin{array}{r} 18\frac{1}{6} \\ - 14\frac{2}{9} \\ \hline \end{array}$	i. $\begin{array}{r} 22 \\ - 1\frac{7}{12} \\ \hline 19\frac{1}{8} \\ - 14\frac{1}{3} \\ \hline \end{array}$

8. Solve the following word problems.

- a. Cheryl walks for $\frac{1}{2}$ of an hour on Tuesdays and Thursdays, and $\frac{2}{3}$ of an hour on Mondays and Wednesdays. On Fridays, Saturdays and Sundays, she walks for $1\frac{1}{4}$ hours each day. How much time does she walk each week?
- b. The car trip took $2\frac{2}{3}$ hours for 300km. What was the average speed (in km per hour)?
- c. The kids spent each day of their summer vacation at the lake! They would play in the water for $\frac{3}{4}$ of an

- hour in the morning, $\frac{2}{3}$ of an hour after lunch, and then $1\frac{1}{2}$ hours before dinner. How many hours did they spend in the water during their 14-day vacation?
- d. A freight truck has $26\frac{1}{4}$ kg of paper, $4\frac{3}{6}$ kg of pencils, $37\frac{1}{3}$ kg of file folders. How much weight was it carrying?
- e. A flight from Fort Nelson to Vancouver takes $2\frac{5}{6}$ of an hour. If the plane has been flying for $1\frac{1}{4}$ of an hour, how much longer will the flight be?
- f. According to Statistics Canada, Terrace's annual rainfall is $970\frac{6}{25}$ mm, and Smithers' annual rainfall is $513\frac{1}{4}$ mm, how much more rain does Terrace get than Smithers each year?
- g. A park is $\frac{121}{2}$ km wide and $\frac{251}{3}$ km long. What is the area of the park?

3.13: Review Answers

- Add these common fractions, make sure to reduce your answer to the lowest terms.

a. $\frac{2}{3}$

b. $\frac{5}{7}$

c. 1

d. $\frac{1}{2}$
 $\frac{3}{4}$

f. $\frac{1}{3}$
g. $1\frac{1}{7}$

h. $\frac{2}{3}$

2. Add these common fractions, make sure to reduce your answer to the lowest terms.

a. $\frac{5}{6}$

e. $1\frac{5}{28}$

i. $1\frac{29}{63}$

b. $\frac{9}{10}$
 $\frac{7}{12}$

f. $1\frac{5}{16}$

j. $\frac{47}{60}$

c. $\frac{7}{12}$

g. $\frac{11}{24}$

d. $1\frac{7}{20}$

h. $\frac{59}{72}$

3. Add.

a. $9\frac{1}{35}$

e. $5\frac{4}{15}$

i. $12\frac{33}{35}$

b. $7\frac{3}{4}$

f. $5\frac{11}{12}$

j. $9\frac{7}{15}$

c. $7\frac{5}{8}$

g. $13\frac{5}{6}$

d. $9\frac{17}{35}$

h. $11\frac{1}{10}$

4. Subtract.

a. $\frac{1}{5}$

b. $\frac{1}{2}$

c. $\frac{1}{3}$

d. $\frac{1}{6}$

5. Subtract and express your answer in the lowest terms

a. $\frac{1}{2}$

d. $\frac{1}{10}$

g. $5\frac{7}{16}$

b. $\frac{1}{4}$

e. $\frac{1}{8}$

h. $\frac{1}{12}$

c. $\frac{2}{3}$

f. $5\frac{9}{16}$

6. Subtract.

a. $9\frac{3}{4}$

c. $3\frac{4}{5}$

e. $4\frac{2}{3}$

b. $\frac{5}{6}$

d. $2\frac{4}{5}$

f. $4\frac{4}{5}$

7. Subtract the following fractions.

a. $11\frac{23}{30}$

e. $11\frac{4}{5}$

i. $20\frac{5}{12}$

b. $2\frac{2}{3}$

f. $2\frac{1}{2}$

j. $4\frac{19}{24}$

c. $1\frac{5}{8}$

g. $3\frac{17}{18}$

d. $9\frac{29}{36}$

h. $\frac{17}{20}$

8. Solve the following word problems.

- a. Cheryl walked $6\frac{1}{12}$ hours each week.
- b. The average was $112\frac{1}{2}$ km/hour
- c. $40\frac{5}{6}$ hours in the lake
- d. $68\frac{1}{12}$ kg
- e. $1\frac{7}{12}$ hours left on the flight
- f. Terrace gets $456\frac{99}{100}$ cm more rain per year than Smithers.
- g. $316\frac{2}{3}$ km²

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CHAPTER 4: DECIMAL NOTATION

4.1: Part of the Whole Thing

Chapter 4: Learning Outcomes

By the end of this chapter, you should be able to:

- Write fractions as decimals and decimals as fractions.
- Add, subtract, multiply and divide rational numbers.
- Use order of operations.
- Graph rational numbers on the number line.

This is the beginning of an adventure with numbers that represent part of the whole thing. These numbers can be shown in a few different ways.

For example, 50 can be shown as a:

- Decimal fraction — 0.50
- Common fraction — $\frac{1}{2}$ or $\frac{50}{100}$
- Percent fraction — 50%

When we talk about fractions in any of the three ways listed above, we are talking about numbers in relation to the whole thing. What do we mean by “the whole thing”? We mean one complete thing.

An example would be one full jug of juice (see Figure 4.1.1). That is 1 whole thing.



Figure 4.1.1

Once someone starts taking some juice, less than the whole thing remains. Now, half of the juice is gone (see Figure 4.1.2).



Figure 4.1.2

The remaining amount can be written as:

- half of one whole thing
- 0.5
- $\frac{1}{2}$

- 50%

Now, almost all the juice has been taken (see Figure 4.1.3).



Figure 4.1.3

The remaining amount can be written as:

- 0.25
- $\frac{1}{4}$
- 25%

Now, there are two full jugs of juice (see Figure 4.1.4). This shows **two** whole things.



Figure 4.1.4

A fraction does not tell us much unless we know what the fraction is part of—we need to know exactly what the whole thing is! If someone says to you, “Sure, let’s go! I still have half!” you really need to know, “Half of what?”

The answer could be:

- $\frac{1}{2}$ a tank of gas.
- $\frac{1}{2}$ a paycheck.
- $\frac{1}{2}$ a vacation.
- $\frac{1}{2}$ an hour.

Fractions have meaning only when we understand the whole thing.

Decimal Fractions

Decimal fractions are one way to consider parts of the whole thing. The whole thing = 1.

You use decimal fractions every time you think about money. The dollars are written as whole numbers and the cents are written as a decimal fraction of a dollar (see Figure 4.1.5).

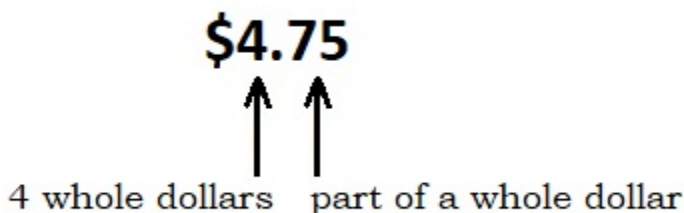


Figure 4.1.5

A decimal fraction has a decimal point that separates the whole number from the fraction. The decimal point looks like this: •

A whole pizza might be divided into eight pieces, ten pieces, or twelve pieces. However, for decimal fractions, the whole is always divided into ten pieces, called tenths. This is because we use a decimal system based on the number ten (“deci” is the Latin word for ten). The tenths are also divided into ten pieces to make hundredths. And then the hundredths are divided by ten to make thousandths, and so on.

Decimal fractions are often used in our daily lives, especially for money and measurement. For example:

- The total was \$12.24.
- It is 3.5 kilometres from my house to the store.

- It costs \$1.99 per kilogram for apples.
- She caught a 4.8-kilogram salmon.

You will be working with decimal fractions in the first two units of this book.

For example:

- A weight and price tag on meat packages (see Figure 4.1.6).

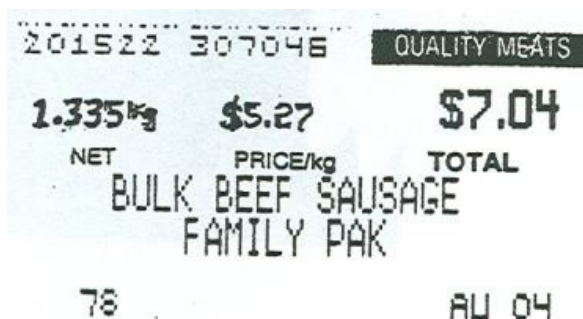


Figure 4.1.6

- The price of gas (see Figure 4.1.7).



Figure 4.1.7

- The price of groceries in a flyer (see Figure 4.1.8).

SAFeway

14⁹⁹ BUTTERBALL BONELESS TURKEY BREAST

8⁹⁹ /lb CURED HAM

1²⁹ Cucumbers

2⁴⁹ /lb Grapes

2⁴⁹ /lb Little Potato Company

2⁴⁹ /lb Carrots

2⁴⁹ /lb Stuffed BUTTERBALL Farci

1⁷⁷ /lb BUTTERBALL

2⁹⁹ /lb Carrots

2/\$5 Dempsters

4⁹⁷ Natrol

2/\$9 Coca-Cola

99¢ Stove Top

6⁹⁹ Armstrong

PROUDLY CANADIAN safeway.ca

Figure 4.1.8

Common Fractions

Common fractions are a second way we will work with parts of the whole thing.

They are written with two numbers, one above the other, with a line in between. The line may be straight, or it may be on an angle.

$$\frac{1}{2} \text{ or } \frac{1}{2}$$

The **numerator** is the top number in a common fraction. The numerator tells how many parts. The **denominator** is the bottom number (see Figure 4.1.9). The denominator tells how many equal parts there are in the whole thing.

$$\begin{array}{ccc} \text{Numerator} & \rightarrow & 3 \\ & & \hline & & 4 \leftarrow \text{Denominator} \end{array}$$

Figure 4.1.9

Example A

The whole thing is 1 pizza. This pizza has been cut into 8 equal pieces (see Figure 4.1.10). The denominator is 8.

As a fraction, the whole thing is $\frac{8}{8}$ (eight-eighths).



Figure 4.1.10

If I ate 3 pieces, that would be $\frac{3}{8}$ of the pizza.

Here are some things to keep in mind while you complete the following exercise:

- one quarter = $0.25 = \frac{1}{4}$
- one third = $0.333.. = \frac{1}{3}$
- one half = $0.5 = \frac{1}{2}$
- two thirds = $0.666... = \frac{2}{3}$
- three quarters = $0.75 = \frac{3}{4}$

Exercise 1

Look at the pictures and use a fraction to answer the questions.

- a. How much gas is left?



**Figure
4.1.11**

a.



**Figure
4.1.12**

b.

- b. Do you want more coffee?

- a. No thanks, I still have _____ of a cup.



**Figure
4.1.13**

- b. Sure, I only have _____ a cup.



**Figure
4.1.14**

- c. Yes please, I'm down to _____ of a cup.



**Figure
4.1.15**

- c. Do we need more juice? Yes, there is just _____ of the juice.



**Figure
4.1.16**

Exercise 1 Answers

Answers may differ because the fraction is approximate.
Ask your instructor to check any different answers.

a. Gas

a. $\frac{1}{2}$ or $\frac{2}{4}$ or 0.5

b. $\frac{1}{4}$ or 0.25

b. Coffee

a. $\frac{3}{4}$ or 0.75

b. $\frac{1}{2}$ or 0.5

c. $\frac{1}{4}$

c. Juice — $\frac{1}{3}$

Fractions as a Percent

A third and useful way to think about parts of the whole thing is as a **percent**.

Percent fractions are written with a number and a percent sign.

1% 12% 50% 99%

In percent fractions, there is always a denominator of 100. That makes the arithmetic much easier and helps us to understand the fraction.

For example, if you got $\frac{14}{20}$ on a test last week and $\frac{13}{17}$ on a test this week, it is hard to get a sense of how you are doing. But if you know you got 70% last week and 76% this week, it is easier to see your improvement.

In percent fractions, the whole thing is 100%, so 100% equals 1.

Statistics and general information are often reported in percent fractions.

For example:

- Pie charts created from collected data (see Figure 4.1.7).

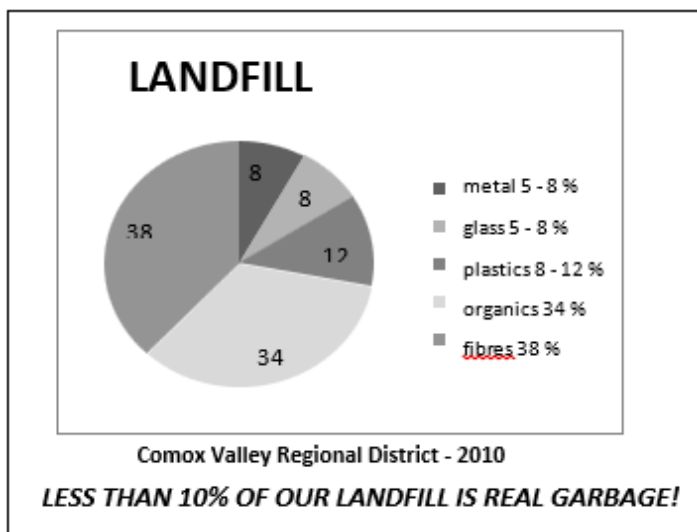


Figure 4.1.17

- Investments with your bank (see Figure 4.1.8).

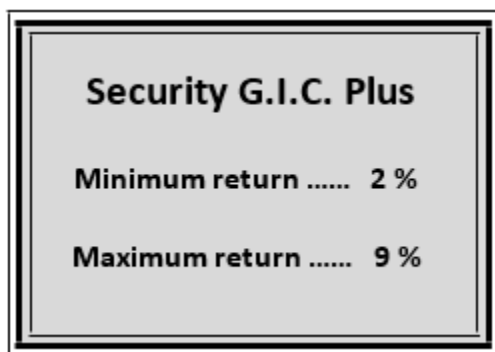


Figure 4.1.18

- The chance of certain weather conditions occurring (see Figure 4.1.9).



Figure 4.1.19

What Is a Decimal Fraction?

As you know, fractions describe part of the whole thing—a fraction is smaller than 1. And as you also know, 1 (the whole thing) can be many things. For example, it can be:

- 1 dollar
- 1 city
- 1 school
- 1 pay cheque
- 1 year
- 1 second
- 1 loaf of bread
- 1 ferry ride
- etc.

A decimal might represent part of a year, part of the population of Canada, part of an hour, or part of anything.

Decimal fractions are different from common fractions in several ways:

A decimal point separates whole numbers from the fraction:

- $\frac{1}{10} = 0.1$
- $\frac{34}{100} = 0.34$
- $\frac{5}{10} = 0.5$

$$\frac{1}{8} \Leftarrow \text{denominator}$$

$$\frac{3}{4} \Leftarrow \text{denominator}$$

Figure 4.1.20

In a decimal fraction, there is no denominator (like in Figure 4.1.20).

We tell the size of the denominator by looking at how many numerals are placed after the decimal point.

Decimal fraction denominators are always ten or ten multiplied by tens. Decimal means “based on the number ten.”

- 0.4 has a denominator of 10
- 0.44 has a denominator of 100
- 0.444 has a denominator of 1 000
- 0.4444 has a denominator of 10 000
- 0.44444 has a denominator of 100 000
- 0.444444 has a denominator of 1 000 000

A whole number and a decimal can be written together. This is called a **mixed decimal**.

4.35 100.47 \$12.39

Every whole number has a decimal point after it, even though we usually do not bother to write the decimal point unless a part of the whole (fraction) follows the whole number.

We can also put zeros to the right of the decimal point of any whole number without changing its value. Get used to thinking of a decimal point after every whole number!

- $3 = 3. = 3.0000000$
- $275 = 275. = 275.0$
- $100 = 100. = 100.0000000000$
- $\$8 = \$8. = \$8.00$

Tip: In math, we use the word decimal to mean decimal fraction.

Attribution

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4.2: Reading & Writing Decimals

Remember the **place value** chart of whole numbers? Complete the following exercise for a refresher.

Exercise 1

352 is the number on the first line of the chart below. The 3 is in the hundreds spot, the 5 is in the tens spot, and the 2 is in the ones spot.

Place the following numbers on the place value chart (see Table 4.2.1):

- 4 984
- 836 466
- 70 834

Table 4.2.1: Exercise 1 Blank Place Value Chart

Hundred Thousands	Ten Thousands	One Thousand	Hundreds	Tens	Ones	(decimal point)
—	—	—	3	5	2	.
—	—	—	—	—	—	—
—	—	—	—	—	—	—
—	—	—	—	—	—	—

Exercise 1 Answers

Check with your instructor to see if you have placed the numbers in the chart correctly.

Turning a Decimal Into Words

Have you ever wondered what goes to the right of the decimal in a place value chart? That is where the decimal numbers go! (The parts of the whole.)

Table 4.2.2 shows a place value chart for decimals:

Table 4.2.2: Place Value Chart For Decimals

Hundreds	Tens	Ones	Decimal	Tenths	Hundredths	Thousandths	Ten thousandths
—	—	3	.	4	5	3	—
—	—	0	.	9	6	—	—

See the words to the right of the decimal point? They look different than the usual whole number words you are used to. These are all the names for the decimal places. You will see them in the next lesson.

The first number on the chart above is 3.453. We say, “Three point four five three,” or, “Three and four hundred fifty-three thousandths.”

- 3 is in the **ones** spot.

- 4 is in the **tenths** spot.
- 5 is in the **hundredths** spot.
- 3 is in the **thousandths** spot.

The second number is 0.96. We say, “Zero point nine six,” or, “Zero and ninety-six hundredths.”

- 0 is in the **ones** spot.
- 9 is in the **tenths** spot.
- 6 is in the **hundredths** spot.

Tenths Place

Common fractions with a **denominator** of 10 are written as a decimal with one place to the right of the decimal point. This is the tenths place.

We often shorten “places to the right of the decimal point” to “decimal places.” We can say that tenths have one decimal place.

- $\frac{6}{10} = 0.6 = \text{six tenths}$
- $\frac{3}{10} = 0.3 \text{ three tenths}$

An easy way to remember this is that there’s one zero in the denominator, so there is one decimal place taken up.

Exercise 2

Write each fraction as a decimal.

Examples:

$$\frac{4}{10} = 0.4 = \text{four tenths}$$

$$\frac{1}{10} = 0.1 = \text{one tenth}$$

a. $\frac{2}{10} = \underline{\quad} = \underline{\hspace{1cm}}$ c. $\frac{7}{10} = \underline{\quad} = \underline{\hspace{1cm}}$

b. $\frac{9}{10} = \underline{\quad} = \underline{\hspace{1cm}}$

Now, enter each common fraction in the place value chart below (see Table 4.2.3). The first one is done for you.

Table 4.2.3: Exercise 2 Blank Place Value Chart

Hundr eds	Te ns	O nes	Deci mal	Ten ths	Hundred ths	Thousand ths	Thousan
—	—	0	.	4	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—

Exercise 2 Answers

- a. 0.2, two tenths
- b. 0.9, nine tenths

- c. 0.7, seven tenths

Decimals with one **digit** to the right of the decimal point have an unwritten denominator of ten. This means that the whole thing is broken into 10 equal parts. Each part is called a tenth.

When we write decimals, we put a zero to the left of the decimal point to show there is no whole number. This zero keeps the decimal point from being “lost” or unnoticed.

For example, .2 should be written as 0.2.

Exercise 3

Write each decimal as a common fraction and in words.

Examples:

$$0.3 = \frac{3}{10} = \text{three tenths}$$

$$0.4 = \frac{4}{10} = \text{four tenths}$$

a. $0.8 = \underline{\quad} = \underline{\quad}$
b. $0.7 = \underline{\quad} = \underline{\quad}$

c. $0.1 = \underline{\quad} = \underline{\quad}$

Exercise 3 Answers

a. $\frac{8}{10}$ eight tenths

b. $\frac{7}{10}$ seven tenths

c. $\frac{1}{10}$ one tenth

Hundredths Place

Decimals with two digits to the right of the decimal point have an unwritten denominator of one hundred. This means that the whole thing is broken into 100 equal parts. Each part is called a hundredth.

Exercise 4

Write each decimal as a common fraction and in words.

Examples:

$0.34 = \frac{34}{100}$	$0.71 = \frac{71}{100}$
thirty-four hundredths	seventy-one hundredths

a. $0.06 = \frac{\quad}{\quad} = \frac{\quad}{\quad}$ c. $0.33 = \frac{\quad}{\quad} = \frac{\quad}{\quad}$
b. $0.56 = \frac{\quad}{\quad} = \frac{\quad}{\quad}$ d. $0.4 = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

Now, place the above decimal numbers in a place value chart (see Table 4.2.4). The first two are done for you. Then, ask your instructor to mark it.

Examples:

$0.34 = \frac{34}{100}$	$0.71 = \frac{71}{100}$
hundredths	hundredths

- a. $0.06 = \frac{\quad}{\quad} = \frac{\quad}{\quad}$ c. $0.33 = \frac{\quad}{\quad} = \frac{\quad}{\quad}$
b. $0.56 = \frac{\quad}{\quad} = \frac{\quad}{\quad}$ d. $0.4 = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

Now, place the above decimal numbers in a place value chart (see Table 4.2.4). The first two are done for you. Then, ask your instructor to mark it.

Table 4.2.4: Exercise 4 Example Place Val

Hundr eds	Te ns	O nes	Deci mal	Ten ths	Hundred ths	Thou
—	—	0	.	3	4	
—	—	0	.	7	1	
—	—	—	.	—	—	
—	—	—	.	—	—	
—	—	—	.	—	—	
—	—	—	.	—	—	

Exercise 4 Answers

a. $\frac{6}{100}$ six hundredths

b. $\frac{56}{100}$ fifty-six hundredths

c. $\frac{33}{100}$ hundredths thirty-three

d. $\frac{40}{100}$ forty hundredths

Common fractions with a denominator of one hundred are written as decimals with two decimal places.

- $\frac{23}{100} = 0.23$
- $\frac{99}{100} = 0.99$
- $\frac{4}{100} = 0.04$

Look at that last example. The 0 must be used after the decimal point in 0.04 to hold the tenths place. This makes it clear that the denominator is one hundred. There are two zeros in the denominator, so there must be two decimal places taken up.

This is called prefixing zeros.

Exercise 5

Write these common fractions as decimals.

Examples:

$$\frac{34}{100} = 0.34$$

$$\frac{70}{100} = 0.70$$

a. $\frac{85}{100}$
b. $\frac{11}{100}$
c. $\frac{21}{100}$

d. $\frac{5}{100}$
e. $\frac{6}{100}$
f. $\frac{45}{100}$

g. $\frac{50}{100}$
h. $\frac{1}{100}$

Exercise 5 Answers

- a. 0.85
b. 0.11
c. 0.21

- d. 0.05
e. 0.06
f. 0.45

- g. 0.50
h. 0.01

Thousandths Place

Decimals with three digits to the right of the decimal point (three decimal places) have an unwritten denominator of one thousand. This means that the whole thing is broken into 1 000 equal parts. Each part is one thousandth.

Look carefully at how thousandths are written. Watch for the prefixing zeros that may be needed to hold the tenth decimal place or the hundredth decimal place.

- $0.472 = \text{four hundred seventy-two thousandths} = \frac{472}{1\,000}$
- $0.085 = \text{eighty-five thousandths} = \frac{85}{1\,000}$
- $0.003 = \text{three thousandths} = \frac{3}{1\,000}$

There are three zeros in the denominator, so there must be three decimal places taken up.

Exercise 6

Write each decimal as a common fraction and in words.
Practice saying them out loud.

Examples:

$$0.006 = \frac{6}{1\,000} = \text{six thousandths}$$

$$0.142 = \frac{142}{1\,000} = \text{one hundred forty-two thousandths}$$

- a. $0.238 = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$ c. $0.600 = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$
 b. $0.562 = \underline{\hspace{1cm}} = \underline{\hspace{1cm}}$

Exercise 6 Answers

- a. $\frac{238}{1\,000}$, two hundred thirty-eight thousandths
 b. $\frac{562}{1\,000}$, five hundred sixty-two thousandths
 c. $\frac{600}{1\,000}$, six hundred thousandths

Exercise 7

Write each common fraction as a decimal.

Examples:

$$\frac{736}{1\,000} = 0.736$$

$$\frac{84}{1\,000} = 0.084$$

a.
$$\frac{210}{1\,000}$$

c.
$$\frac{106}{1\,000}$$

e.
$$\frac{592}{1\,000}$$

b.
$$\frac{6}{1\,000}$$

d.
$$\frac{116}{1\,000}$$

f.
$$\frac{962}{1\,000}$$

Now, place the above decimal numbers in a place value chart (see Table 4.2.5). The first two are done for you. Then, ask your instructor to mark it.

Table 4.2.5: Exercise 7 Example Place Value Chart

Hundr eds	Te ns	O nes		Ten ths	Hundred ths	Thousand ths	Ten thousandths
—	—	0	.	7	3	6	—
—	—	0	.	1	4	2	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—

Exercise 7 Answers

a. 0.736

b. 0.084

c. 0.210

d. 0.006
e. 0.106

f. 0.116
g. 0.592

h. 0.962

Ten-Thousandths Place

Decimals with four decimal places have an unwritten denominator of ten-thousand. The whole thing is being thought of as having 10 000 equal parts. Each part is one ten-thousandth.

- $0.1458 = \frac{1458}{10\,000}$ = one thousand four hundred fifty-eight ten-thousandths
- $0.0581 = \frac{581}{10\,000}$ = five hundred eighty-one ten-thousandths

Notice that there are four zeros in the denominator. That means there must be four decimal places taken up.

Exercise 8

Write each decimal as a common fraction and in words. Practice saying these aloud to someone else; they can be real tongue twisters!

Examples:

$$0.2489 = \frac{2489}{10000} = \text{two thousand four hundred eighty-nine ten-thousandths}$$

$$0.1111 = \frac{1111}{10000} = \text{one thousand one hundred eleven ten-thousandths}$$

a. 0.0236

b. 0.4015

c. 0.2306

d. 0.0003

Exercise 8 Answers

a. $\frac{236}{10\,000}$, two hundred thirty-six ten-thousandths

b. $\frac{4015}{10\,000}$, four thousand fifteen ten-thousandths

c. $\frac{2306}{10\,000}$, two thousand three hundred six ten-thousandths

d. $\frac{3}{10\,000}$, three ten-thousandths

Exercise 9

Write these common fractions as decimals.

Examples:

$$\frac{1489}{10\,000} = 0.1489$$

$$\frac{2}{10\,000} = 0.0002$$

a.
$$\frac{386}{10\,000}$$

b.
$$\frac{9137}{10\,000}$$

c.
$$\frac{4}{10\,000}$$

d.
$$\frac{916}{10\,000}$$

Now, place the above decimal numbers in the place value chart below (see Table 4.2.6). The first two are done for you. Then, ask your instructor to mark it.

Table 4.2.6: Exercise 9 Example Place Value Chart

Hundr eds	Te ns	O nes		Ten ths	Hundred ths	Thousand ths	Ten Thousandths
—	—	0	.	1	4	8	9
—	—	0	.	0	0	0	2
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—

Exercise 9 Answers

- a. 0.0386
b. 0.9137

- c. 0.0004
d. 0.0916

Mixed Decimals

Mixed decimals are a whole number and a decimal written together.

$4.3 = 4\frac{3}{10}$ = four and three tenths

$27.27 = 27\frac{27}{100}$ = twenty-seven and twenty-seven hundredths

$8.104 = 8\frac{104}{1\,000}$ = eight and one hundred four thousandths

The digits to the left of the decimal point are whole numbers. The digits to the right of the decimal point are fractions. We say “and” for the decimal point.

Look at the **mixed decimals** from the examples above in the place value chart below (see Table 4.2.7):

Table 4.2.7: Mixed Decimal Place Value Chart

Hundreds	Tens	Ones	Decimal	Tenths	Hundredths	Thousandths	Ten Thousandths
—	—	4	.	3	—	—	—
—	2	7	.	2	7	—	—
—	—	8	.	1	0	4	—

Turning Words Into a Decimal

Step 1: Read the number. Does the word “and” show that this is a mixed decimal?

- If it does, the whole number is before the word “and.” Write the whole number with the decimal point after it.
- If there is no whole number, write a 0 with the decimal point after it.

Step 2: Decide how many decimal places you need. Look and listen for the “ths” ending. The word with “ths” is the understood denominator. It may help if you draw a little line for each decimal place that you need.

- Tenth**ths** need one decimal place.
- Hundred**ths** need two decimal places.
- Thousand**ths** need three decimal places.
- Ten-thousand**ths** need four decimal places.

Step 3: Write the decimal so the last digit is on the last little line, and fill any remaining lines with zeros.

Example A

a. Seven hundredths

- It is not a mixed decimal, so write: 0.
- “Hundredths” means two decimal places: 0. _
—
- Fill in the numbers: 0.07

b. Eight thousandths

- It is not a mixed decimal, so write: 0.
- “Thousandths” means three decimal places: 0. _
— —
- Fill in the numbers: 0.008

c. Twenty-six thousandths

- It is not a mixed decimal, so write: 0.
- “Thousandths” means three decimal places: 0.

— — —

- Fill in the numbers: 0.026

d. Four hundred six thousandths

- It is not a mixed decimal, so write: 0.
- “Thousandths” means three decimal places: 0.

— — —

- Fill in the numbers: 0.406

Reading & Writing Money

Dollars

We write money with a dollar sign, a whole number, and a decimal with two decimal places.

$$\$1.00 = 1 \text{ dollar}$$

What do we call $\frac{1}{100}$ of a dollar? Right! One cent.

- \$2.33 = two dollars and thirty-three cents
- \$427.05 = four hundred twenty-seven dollars and five cents
- \$0.62 = sixty-two cents
- \$0.03 = three cents

Exercise 10

Write the amount of money in words.

Example: \$212.63 – two hundred twelve dollars and sixty-three cents

a. \$47.01	c. \$82.50
b. \$9.28	d. \$100.05

Write with numerals, using \$.

Example: twenty-seven dollars and six cents – \$27.06

a. one hundred sixty-c.	one thousande.	seven cents
two dollars	dollars and seventy-f.	five hundred dollars
b. thirteen dollars and	seven cents	and ninety cents
sixty cents	d. sixty-nine cents	

Exercise 10 Answers

a. forty-seven dollars	and fifty cents	g. \$1 000.77
and one cent	d. one hundred dollarsh.	\$0.69
b. nine dollars and	and five cents	i. \$0.07
twenty-eight cents	e. \$162.00	j. \$500.90
c. eighty-two dollarsf.	\$13.60	

“Centum” is a Latin word that means hundred! Here are English words that start with “cent”:

- centurion — commander of a hundred soldiers
- century — a hundred years
- centennial — a hundredth anniversary
- centigrade — having a hundred degrees
- cent — one hundredth of a dollar
- centimetre — one-hundredth of a metre
- centipede — wormlike creatures with a hundred legs

When we read \$12.25 as “twelve dollars and twenty-five **cents**,” we are using the Latin word for “one hundredths.”

We could also write our money like this, as we do on cheques (although it looks funny!):

$$\begin{aligned} \$14.75 &= \$14 \frac{75}{100} \\ \$12.25 &= \$12 \frac{25}{100} \end{aligned}$$

Cents

We have another way of writing money. We often write money that is less than one dollar using a cent sign which is a c for cent with a line through it ¢.

• \$0.05 = 5¢	• \$0.25 = 25¢
• \$0.33 = 33¢	• \$0.99 = 99¢
• \$0.10 = 10¢	• \$1.08 = 108¢

It is incorrect to use both a dollar sign and a cent sign. Instead of \$4.53¢, do \$4.53 or 453¢.

It is incorrect to use a cent sign with a decimal point. Instead of 4.53¢, do \$4.53 or 453¢.

Important information:

- We do not need to use a decimal point with the cent sign. A decimal point would indicate a fraction or part of one cent.
- For example, If a sign said “Ice cream cones 0.50¢,” those ice cream cones would only cost half a cent each!
- Pay careful attention to the way amounts of money are written.

Exercise 11

Rewrite these using the other common way of writing money. Remember to use the ¢ or \$ as needed.

Examples:

$$\$0.75 = 75\text{¢}$$

$$83\text{¢} = \$0.83$$

a. $\$0.01 =$

b. $47\text{¢} =$

c. $\$0.04 =$

d. $3\text{¢} =$

e. $\$0.40 =$

f. $101\text{¢} =$

g. $\$0.29 =$

h. $50\text{¢} =$

i. $\$0.80 =$

j. $99\text{¢} =$

k. $\$1.00 =$

l. $175\text{¢} =$

m. $\$1.10 =$

Exercise 11 Answers

a. 1¢	f. \$1.01	k. 100¢
b. \$0.47	g. 29¢	l. \$1.75
c. 4¢	h. \$0.50	m. 110¢
d. \$0.03	i. 80¢	
e. 40¢	j. \$0.99	

Exercise 12

Correct the following ways of writing money.

Example: .50¢ = 50¢

a. .99¢ =	c. ¢0.40 =
b. .20¢ =	

Exercise 12 Answers

a. 99¢	c. 40¢
b. 20¢	

Exercise 13

Complete the chart so that each question has the amount written as a decimal, a common fraction, and in words (see Table 4.2.8). The first two are done.

Table 4.2.8: Exercise 13 Fill In The Place Value Chart

Num ber	Deci mal	Fraction	In Words
a	.048	$\frac{48}{1000}$	forty eight thousandths
b	0.7	$\frac{7}{10}$	seven tenths
c	—	—	four hundredths
d	0.006	—	—
e	—	$16\frac{2}{1000}$	—
f	—	—	twelve and fifteen hundredths
g	$463.\overset{0}{\underset{3}{}}$	—	—
h	—	$213\frac{25}{1000}$	—
i	—	—	seventy-five and twenty-eight thousandths
j	$1833.\overset{0}{\underset{018}{}}$	—	—
k	—	$12\frac{418}{10000}$	—

1	—	—	nine tenths

Exercise 13 Answers

Table 4.2.9: Exercise 13 Answers

Num ber	Deci mal	Fraction	In Words
c	0.04	$\frac{4}{100}$	four hundredths
d	0.006	$\frac{6}{1\,000}$	six thousandths
e	16.002	$16\frac{2}{1\,000}$	sixteen and two thousandths
f	12.15	$12\frac{15}{100}$	twelve and fifteen hundredths
g	$463.\overset{0}{3}$	$463\frac{3}{100}$	four hundred sixty-three and three hundredths
h	$213.\overset{02}{5}$	$213\frac{25}{1\,000}$	two hundred thirteen and twenty-five thousandths
i	75.028	$75\frac{28}{1\,000}$	seventy-five and twenty-eight thousandths
j	$1\,833.\overset{018}{0}$	$1833\frac{18}{1\,000}$	one thousand eight hundred thirty-three and eighteen thousandths
k	$12.\overset{041}{8}$	$12\frac{418}{10\,000}$	twelve and four hundred eighteen ten-thousandths
l	0.9	$\frac{9}{10}$	nine tenths

4.2: Practice Questions

1. Write as decimals.

a. $\frac{3}{10}$

b. $\frac{24}{1\,000}$

c. $\frac{36}{1\,000}$

d. $\frac{206}{10\,000}$

e. $3\frac{123}{1\,000}$

f. $\frac{2}{100}$

g. $6\frac{3}{10}$

h. $4\frac{11}{1\,000}$

i. $6\frac{250}{1\,000}$

j. $9\frac{47}{10\,000}$

2. Change these decimals to common fractions.

a. 0.5

b. 0.04

c. 0.37

d. 0.010

e. 3.0918

f. 3.025

g. 0.164

h. 2.1498

3. Write as common fractions and as decimals.

a. one hundredth

b. forty-seven

c. two hundred seventy-

one thousandths

d. forty-one thousandths

e. one hundred twenty

ten-thousandths

f. four and four tenths

two hundred sixty and

fourteen

thousandths

i. seven and two hundred

eleven thousandths

j. forty and six

hundredths

five dollars and sixty-

three cents

4. Write the amount of money with numerals, using a \$ sign.

a. five dollars and sixty cents

b. seventy-two cents

c. fifty-six cents

d. six cents

e. one hundred twenty-four cents

4.2: Practice Answers

1. Write as decimals.

a. 0.3	e. 3.123	i. 6.250
b. 0.24	f. 0.02	j. 9.0047
c. 0.036	g. 6.3	
d. 0.0206	h. 4.011	

2. Change these decimals to common fractions.

a. $\frac{5}{10}$	d. $\frac{10}{1\ 000}$	g. $\frac{164}{1\ 000}$
b. $\frac{4}{100}$	e. $3\frac{918}{10\ 000}$	h. $2\frac{1498}{10\ 000}$
c. $\frac{37}{100}$	f. $3\frac{25}{1\ 000}$	

3. Write as common fractions and as decimals.

a. $\frac{1}{100}$ and 0.01	d. $\frac{41}{1\ 000}$ and 0.041	i. $7\frac{211}{1\ 000}$ and 7.211
b. $\frac{47}{100}$ and 0.47	e. $\frac{120}{10\ 000}$ and 0.0120	j. $40\frac{6}{100}$ and 40.06
c. $\frac{271}{1\ 000}$ and 0.271	f. $4\frac{4}{10}$ and 4.4	j. $5\frac{63}{100}$ and \$5.63
	g. $260\frac{14}{10\ 000}$ and 260.0014	

4. Write the amount of money with numerals, using a \$ sign.

a. \$5.60	d. \$0.06
b. \$0.72	e. \$1.24
c. \$0.56	

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4.3: Comparing Decimals

More About Zeros...

A whole number can have a decimal point and many zeros after it without changing its value.

$$47 = 47.0 = 47.000 = 47.0000000000000000$$

Zeros are used to hold a place when we write whole numbers.

- In 100, the 0s hold the tens place and the ones place.
- In 1 206, the 0 holds the tens place.

In decimals, any zero to the right of the decimal point and to the left of another **digit** is important because the zero is holding a place and giving the decimal the correct value.

- In 4.306, the 0 holds the hundredths place.
- In 17.0002, the 0's hold the tenth, hundredth and thousandth places.

A zero is usually placed to the left of the decimal point if there is no whole number.

$$0.5 \quad 0.937$$

Which Zeros Are Not Needed?

- Zeros at the beginning, or the far left, of whole numbers are not needed.
 - 00004 = 4
 - 00100 = 100

- Zeros at the end, or the far right, of decimal are not needed.
 - $3.210 = 3.21$
 - $34.062000 = 34.062$

Zeros on the outside edges of **mixed decimals** do not change the value of the number and are unnecessary.

- **00**28.971**0** = 28.971
- **00**100.003**000** = 100.003
- 890.407 = **00**890.407**0000000**

Exercise 1

Cross out the zeros that are not needed.

a. 70.0390	e. 60.0205	i. 03.5830
b. 00.2906	f. 4 020.0020	j. 9.0
c. 06.30	g. 400.080	k. 0.002030
d. 087.50	h. 340.600	l. 50.300

Exercise 1 Answers

a. 709.039	e. 60.0205	i. 3.583
b. 0.2906	f. 4 020.002	j. 9.0
c. 6.3	g. 400.08	k. 0.00203
d. 87.5	h. 340.6	l. 50.3

Zeros at the end of a decimal do not change the value.

$$6. = 6.0 = 6.00$$

Zeros at the beginning of a whole number do not change the value.

$$8 = 08 = 00008$$

Zeros between a decimal point and a digit do change the value.

Example A

405 is very different than 45. And 0.05 (five hundredths) is very different than 0.5 (five tenths).

Comparing Decimals

You have probably heard the old saying: “You cannot compare apples to oranges!” and it’s true. It is tough to compare things that do not have much in common. So before we compare decimals, we give the decimals something in common — the same number of decimal places, which gives them a commonly understood **denominator**.

Before comparing decimals, put zeros at the end or cross out any unnecessary zeros so the decimals have a common (same) number of decimal places. If you write the decimals that you are comparing right underneath each other, your eye will often tell you which is the larger amount or if the amounts are equal.

Example B

Compare 0.43 and 0.4. Which is larger?

Step 1:

- 0.43 has two decimal places; it is forty-three hundredths = $\frac{43}{100}$.
- 0.4 has one decimal place; it is four tenths = $\frac{4}{10}$.

Step 2: Give them a common number of decimal places.

Step 3: Add a zero to 0.4 to make it 0.40; now we read it as forty hundredths = $\frac{40}{100}$.

Now, which is larger?

$$\frac{40}{100} \text{ or } \frac{43}{100}.$$

You can easily see that 0.43 is the larger amount.

Review of Greater Than > and Less Than <

An easy way to remember these signs is to think that the big (wide)

end of the sign is closer to the bigger (greater) number, and the small end of the sign is closer to the smaller number.

- 0.43 is larger than 0.40, $0.43 > 0.40$
- 0.52 is smaller than 0.60, $0.52 < 0.60$

Exercise 2

Which is greater? Draw a box around the bigger decimal fraction in each pair and write a greater than $>$ or a less than $<$ sign to make a true statement.

Examples:

$$0.6 < 0.65$$

$$14 > 1.4$$

a. $0.17 \underline{\hspace{1cm}} 0.02$

d. $0.51 \underline{\hspace{1cm}} 0.159$

g. $630.3 \underline{\hspace{1cm}} 630.03$

b. $0.009 \underline{\hspace{1cm}} 0.09$

e. $0.03 \underline{\hspace{1cm}} 0.0352$

h. $0.80 \underline{\hspace{1cm}} 0.6989$

c. $0.23 \underline{\hspace{1cm}} 0.215$

f. $0.07 \underline{\hspace{1cm}} 0.063$

Exercise 2 Answers

a. $>$

d. $>$

g. $>$

b. $<$

e. $<$

h. $>$

c. $>$

f. $>$

Comparing Decimals Using a Number Line

You can compare decimals using a number line. A number line organizes what you are thinking about on paper — or on a ruler. You can plot your decimals on the number line and then be able to see which number is larger. Take a look:

Example C

Step 1: Try to put the following numbers in order without looking at the number line below:

2.347 2.3 2.37 2.33 2.39 2.341 2.41

Step 2: Look at the number line and see if you ordered your numbers correctly (see Figure 4.3.1).

The number line has a jagged edge, which means it does not start at zero. It starts in the middle of a ruler. Using a number line can help you see your work and think about it at the same time.

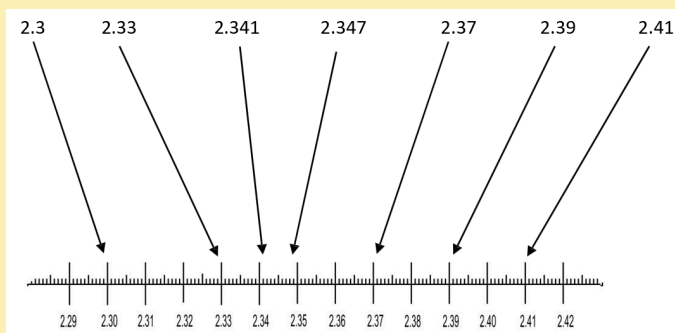


Figure 4.3.1

Exercise 3

Try plotting the following decimals on the number line in Figure 4.3.2 below:

4.59 4.32 4.7 5.23 4.47 4.3 4.17



Figure 4.3.2

Exercise 4

And now, plot these numbers on an empty number line (see Figure 4.3.3). (You need to fill in the numbers yourself. Plot 7.3 on the first large vertical line on the left)

7.35 7.3 8.2 7.53 7.98 8.34 7.9 7.5



Figure 4.3.3

This is one way to organize and order decimals, please use it if the system is helpful for you.

Exercise 5

Draw a circle around the smallest decimal fraction in each group.

a. 0.3, 0.03, 0.23	d. 0.62, 0.6103, 0.5
b. 0.04, 0.14, 0.41	e. 0.041, 0.04, 0.4
c. 0.016, 0.2, 0.216	f. 100.1, 100.13, 100.01

Exercise 5 Answers

a. 0.03	c. 0.016	e. 0.04
b. 0.04	d. 0.5	f. 100.01

Exercise 6

Identify whether the pair of numbers is equal (=) or not equal ($\neq$).

Examples:

$$0.8 = 0.80$$

$$0.25 \neq 0.205$$

a. $5.503 \underline{\hspace{0.5cm}} 5.330$

c. $0.75 \underline{\hspace{0.5cm}} 0.750$

e. $0.040 \underline{\hspace{0.5cm}} 0.40$

b. $9.3200 \underline{\hspace{0.5cm}} 9.32$

d. $23.3 \underline{\hspace{0.5cm}} 2.33$

f. $0.8010 \underline{\hspace{0.5cm}} 0.801$

Exercise 6 Answers

a. $\neq$

c. $=$

e. $\neq$

b. $=$

d. $\neq$

f. $=$

4.3: Practice Questions

1. Cross out the unnecessary zeros.

a. 0401.02

c. 68.3020

b. 1 000.001

d. 050.200

2. Draw a box around the larger amount in each pair.

a. 0.8 or 0.08

c. 0.006 or 0.6

b. 0.004 or 0.04

d. 33.006 or 33.06

3. Put the correct sign between the decimals. Choose = or > or <.

a. $2.3 \underline{\hspace{0.5cm}} 23$

c. $4.7 \underline{\hspace{0.5cm}} 4.700$

e. $042.9002 \underline{\hspace{0.5cm}} 042.9$

b. $0.2 \underline{\hspace{0.5cm}} 0.003$

d. $0.25 \underline{\hspace{0.5cm}} 0.250$

f. $0.4092 \underline{\hspace{0.5cm}} 0.411$

4.3: Practice Answers

1. Cross out the unnecessary zeros.

a. 401.02	c. 68.302
b. 1 000.001	d. 50.2

2. Draw a box around the larger amount in each pair.

a. 0.8	c. 0.6
b. 0.04	d. 33.06

3. Put the correct sign between the decimals.

a. <	c. =	e. >
b. >	d. =	f. <

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4.4: Rounding Numbers

If a pair of jeans cost \$49.98, what amount would you say if someone asks what you paid for them? You would probably say, “They cost around \$50.”

We often round cents to dollars as we go about our lives. You may already have an idea of how to do this. For example, answer these questions.

- About how much do your groceries cost each month?
- About how much does it cost to fill a small car’s gas tank?

Look at your answers. The amount for groceries may be quite large. When you estimated your answer, how did you round the amount?

For example, if your real monthly grocery bill was \$481.73, you might have said \$482 or perhaps \$480. Maybe you even have estimated to the nearest hundred dollars and said, “About \$500 a month for groceries.” All those estimates would be correct.

The amount for a tank of gas is less than a month’s groceries. How did you estimate?

For example, a small car may take \$54.72 of gas.

- If you estimated to the nearest dollar, you would say, “About \$55.”
- If you estimated to the nearest ten dollars, you would say, “About \$50.”
- If you rounded to the nearest dollar, you would say, “54 dollars.”

We round a number in different ways depending on several things:

- The size of the number we are rounding.
- What we are going to do with the number after we have rounded it off.
- Our own convenience.

Carefully review the place value for whole numbers:

- hundred thousands
- ten thousands
- one thousand
- hundreds
- tens
- ones
- decimal

Rounding Whole Numbers

We round down if the digit to the right is less than 5. We round up if the digit to the right is 5 or more.

- Rounding numbers gives an approximate amount; it is not an accurate figure.
- Use a different form of the equal sign ($\approx$), which means “approximate equality.”

Review: Rounding to the Nearest Ten

- Underline the tens digit
- Look at the digit in the ones place (to the right). You can put an arrow above it to help you find it later.
 - If the ones digit is **5 or more, round up**. Write the ones digit as zero and increase the tens digit by one.
 - If the ones digit is **less than 5, round down**. The tens digit does not change, and the ones digit is written as a zero.

Example A

↓
23

23 is rounded down to 20. The tens digit stays the same.

$$23 \approx 20$$

Here's another example:

Example B

↓
287

287 is rounded up to 290. The tens digit increases by 1.

$$287 \approx 290$$

Exercise 1

Round each of the following to the nearest ten. Use the “approximate equality” sign $\approx$.

Examples:

$$46 \approx 50$$

$$111 \approx 110$$

- a. $7 \approx$
- b. $116 \approx$
- c. $71 \approx$
- d. $89 \approx$
- e. $96 \approx$

- f. $4 \approx$
- g. $385 \approx$
- h. $108 \approx$
- i. $73 \approx$
- j. $17 \approx$

- k. $361 \approx$
- l. $8 \approx$
- m. $49 \approx$
- n. $148 \approx$

Exercise 1 Answers

a. 10	f. 0	k. 360
b. 120	g. 390	l. 10
c. 70	h. 110	m. 50
d. 90	i. 70	n. 150
e. 100	j. 20	

Review: Rounding to the Nearest Hundred

- Underline the hundreds digit.
- Look at the digit in the tens place (to the right). You can put an arrow above it to help you find it later.
 - If the tens digit is **5 or more, round up**. Write the tens and ones digit as zero and increase the hundreds digit by one.
 - If the tens digit is **less than 5, round down**. The hundreds digit does not change and the tens and ones digit is written as a zero.

Example C

↓
473

473 is rounded up to 500.

$$473 \approx 500$$

Round down if the tens digit is less than 5 and up if it is 5 or more:

- 728 rounded to the nearest hundred is 700. (The tens digit is 2, which is less than 5, so the hundreds digit stays the same.)
- 758 rounded to the nearest hundred is 800. (The tens digit is 5, which is five or more, so the hundreds digit increases by 1.)

Exercise 2

Round each of the following to the nearest HUNDRED.
Use the “approximate equality” sign $\approx$.

Example: $330 \approx 300$

a. $330 \approx 300$

b. 908

c. 2 795

d. 1 260

e. 742

f. 127

g. 302

h. 945

i. 865

j. 275

k. 590

l. 1 240

m. 214

n. 4 450

o. 98

p. 996

Exercise 2 Answers

a. 900

b. 2 800

c. 1 300

d. 700

e. 100

f. 300

g. 900

h. 900

i. 300

j. 600

k. 1 200

l. 200

m. 4 500

n. 100

o. 1 000

Review: Rounding to the Nearest Thousand

- Underline the thousands digit
- Look at the digit in the hundreds place (to the right). You can put an arrow above it to help you find it later.

- If the hundreds digit is **5 or more, round up**. Write the hundreds, tens and ones digit as zero and increase the thousands digit by one.
- If the hundreds digit is **less than 5, round down**. The thousands digit does not change and the hundreds, tens and ones digit is written as a zero.

Example D



3485

3 485 is rounded down to 3 000.

$$3\,485 \approx 3\,000$$

Round down if the hundreds digit is less than 5 and round up if it is 5 or more:

- 2 109 rounded to the nearest thousand is 2 000. (The hundreds digit is 1, which is less than 5.)
- 2 643 rounded to the nearest thousand is 3 000. (The hundreds digit is 6, which is more than 5.)
- 0 940 rounded to the nearest thousand is 1 000. (The hundreds digit is 9, which is more than 5, so the thousands digit increases from 0 to 1.)
- 0 465 rounded to the nearest thousand is 0. (The hundreds digit is 4, which is less than 5, so the thousands digit stays at

0.)

Exercise 3

Round each of the following to the nearest thousand. Use the “approximate equality” sign $\approx$.

Example: $1\,760 \approx 2\,000$

- a. 6 250
- b. 850
- c. 320
- d. 5 544
- e. 1 234
- f. 492
- g. 6 199

- h. 9 883
- i. 1 045
- j. 7 856
- k. 500
- l. 1 780
- m. 495
- n. 9 300

- o. 700
- p. 2 449
- q. 5 555
- r. 8 914
- s. 85 455
- t. 6 475

Exercise 3 Answers

- a. 2 000
- b. 6 000
- c. 1 000
- d. 0
- e. 6 000
- f. 1 000
- g. 5 000

- h. 6 000
- i. 10 000
- j. 1 000
- k. 8 000
- l. 1 000
- m. 2 000
- n. 0

- o. 9 000
- p. 1 000
- q. 2 000
- r. 6 000
- s. 9 000
- t. 85 000
- u. 6 000

Rounding Decimals to Whole Numbers

Remember, decimals are part of the whole thing. We can round the decimal to the nearest whole number. Rounding to whole numbers means rounding off to the ones place.

When rounding to the whole number:

- Underline the ones digit
- Look at the digit in the tenths place (to the right). You can put an arrow above it to help you find it later.
 - If the tenths digit is **5 or more, round up**. Increase the ones digit by one. Do not write a decimal or any decimal digits.
 - If the tenths digit is **less than 5, round down**. The ones digit does not change. Do not write a decimal or any decimal digits,

Example E



37.482

37.482 rounded to the nearest whole number is 37. (The tenths digit is 4, which is less than 5.)

$$37.482 \approx 37$$

$$\downarrow$$

$$37.\underline{9}06$$

37.906 rounded to the nearest whole number is 38. (The tenths digit is 9, which is more than 5.)

$$37.906 \approx 38$$

Example F

- a. Round to a whole number.

$$\downarrow$$

$$42.123 \rightarrow 42.\underline{1}23 \approx 42$$

- b. Round 960.802 to the nearest whole number.

$$\downarrow$$

$$960.802 \rightarrow 960.\underline{8}02 \approx 961$$

- c. Round 39.5 to the nearest ones.

$$\downarrow$$

$$39.5 \rightarrow 39.\underline{5} \approx 40$$

Zeros Again

You know that zeros at the end of a decimal do not change the value of the amount. You can add as many as you like.

But when a decimal has been rounded, drop any zeros after the place where you have rounded.

Instead of $39.52 \approx 40.0$, do $39.52 \approx 40$

Instead of $960.802 \approx 961.000$, do $960.802 \approx 961$

Exercise 4

Round each of the following to the nearest whole number. Use the “approximate equality” sign $\approx$.

Example: $11.3 \approx 11$

- a. 2.679
- b. 403.8
- c. 7.6

- d. 65.91
- e. 22.2
- f. 3.76

- g. 9.2
- h. 1.7
- i. 0.6

j. 2.63	p. 3.61	v. 2.01
k. 5.09	q. 12.3	w. 5.55
l. 19.8	r. 34.5	x. 10.3
m. 2.1	s. 17.82	y. 9.9
n. 0.7	t. 2.45	z. 8.15
o. 74.2	u. 1.792	

Exercise 4 Answers

a. 3	j. 3	s. 18
b. 404	k. 5	t. 2
c. 8	l. 20	u. 2
d. 66	m. 2	v. 2
e. 22	n. 1	w. 6
f. 4	o. 74	x. 10
g. 9	p. 4	y. 10
h. 2	q. 12	z. 8
i. 1	r. 35	

Important Information:

If these exercises on rounding are becoming tiresome, please do not despair—there is a purpose.

When you do operations ($+$ $-$ $\times$ $\div$) with decimals, you will often end up with answers in the ten-thousandths place even though you only need the accuracy of a tenth or hundredth place decimal.

If you do decimal operations on a calculator, you may end up with 6 decimal places (millionths). This result is not too practical if you are working with money and only want two decimal places!

You will know how to round the answer to the decimal place you need for that question or situation.

Rounding Decimals to the Nearest Tenth

To round decimals to the nearest tenth:

- Underline the tenths place digit.
- Look at the digit (to the right) in the hundredths place. You can put an arrow above it to help you find it later.
 - If the hundredths digit is **less than 5**, the tenths digit does not change, and the hundredths digit (and all other decimal numbers after the hundredths) is not written at all.
 - If the hundredths digit is **5 or more**, increase the tenths digit by one and write no more decimals in the hundredths spot or after.

Example G

Round to the nearest tenth.

$$\begin{array}{c} \downarrow \\ 13.432 \rightarrow 13.\underline{4}32 \approx 13.4 \end{array}$$

$$\begin{array}{c} \downarrow \\ 13.476 \rightarrow 13.\underline{4}76 \approx 13.5 \end{array}$$

$$\begin{array}{c} \downarrow \\ 0.263 \rightarrow 0.\underline{2}63 \approx 0.3 \end{array}$$

$$\begin{array}{c} \downarrow \\ 234.0399 \rightarrow 234.\underline{0}399 \approx 234.0 \end{array}$$

Keep the 0 because you have accurately rounded off to that zero. It is called a significant figure.

Exercise 5

Round each of the following to the nearest tenth.

Example: $4.23 \approx 4.2$

a. 5.18	j. 2.15	s. 9.45
b. 8.54	k. 1.44	t. 12.36
c. 16.09	l. 3.172	u. 202.305
d. 3.52	m. 9.99	v. 2.66
e. 4.14	n. 5.09	w. 9.492
f. 6.24	o. 4.111	x. 7.388
g. 1.76	p. 6.046	y. 5.249
h. 1.74	q. 0.71	z. 2.45
i. 7.19	r. 3.63	

Exercise 5 Answers

a. 5.2	j. 2.2	s. 9.5
b. 8.5	k. 1.4	t. 12.4
c. 16.1	l. 3.2	u. 202.3
d. 3.5	m. 10.0	v. 2.7
e. 4.1	n. 5.1	w. 9.5
f. 6.2	o. 4.1	x. 7.4
g. 1.8	p. 6.0	y. 5.2
h. 1.7	q. 0.7	z. 2.5
i. 7.2	r. 3.6	

Rounding Decimals to the Nearest Hundredth

Rounding decimals to the nearest hundredth is similar to rounding to the nearest tenth:

- Underline the hundredths place digit.
- Look at the digit (to the right) in the thousandths place. You can put an arrow above it to help you find it later.
 - If the thousandths digit is **less than 5**, the hundredths digit does not change, and the thousandths digit (and all other decimal numbers after the hundredths) is not written at

all.

- If the thousandths digit is **5 or more**, increase the hundredths digit by one and write no more decimals in the thousandths spot or after.

Example H

Round to the nearest hundredth.



$$35.4524 \rightarrow 35.4\underline{5}24 \approx 35.45$$



$$35.4567 \rightarrow 35.4\underline{5}67 \approx 35.46$$



$$47.9873 \rightarrow 47.9\underline{8}73 \approx 47.99$$



$$23.99609 \rightarrow 23.99\underline{6}09 \approx 24.00$$

Keep these zeros because you have accurately rounded off to them. These zeros are significant.

Exercise 6

Round to the nearest hundredth. Keep significant zeros!

Example: $128.409 \approx 128.41$

- a. 0.909
- b. 98.024
- c. 3.001
- d. 76.3333

- e. 0.229
- f. 100.999
- g. 0.756

Exercise 6 Answers

- a. 0.91
- b. 98.02
- c. 3.00
- d. 76.33

- e. 0.23
- f. 101.00
- g. 0.76

More Dollars and Cents

A cent is what fraction of a dollar?

Yes, a cent is $\frac{1}{100}$ of a dollar (one hundredth).

You may be asked to round amounts of money to the nearest cent. What you are actually doing is rounding to the nearest hundredth of a dollar.

- $\downarrow$
\$3.286 $\approx$ \$3.29
- $\downarrow$
\$14.923 $\approx$ \$14.92

one cent = one hundredth of a dollar

Exercise 7

Round to the nearest cent.

Examples:

$$\$42.008 \approx \$42.01$$

$$\$0.233 \approx \$0.23$$

a. \$25.255	d. \$100.999
b. \$10.141	e. \$0.9834
c. \$0.706	f. \$2.8977

Exercise 7 Answers

a. \$42.01	e. \$0.71
b. \$0.23	f. \$101.00
c. \$25.26	g. \$0.98
d. \$10.14	h. \$2.90

Rounding Decimals to the Nearest Thousandth

Example 1

a. Round to the nearest thousandth (1 000th).

$$2.0486 = 2.0486 \approx 2.049$$

b. Round to the nearest thousandth (1 000th).

$$29.4324 = 29.4324 \approx 29.432$$

Use rounded numbers to estimate answers in daily situations, in math problem-solving, and to get an idea of the answer before you figure something out on a calculator. Numbers that are rounded off make calculations simpler.

Exercise 8

Round the following numbers as called for at the left of the chart.

Example: Round to the nearest tenth — $2.34 \approx 2.3$

1. Round to the nearest tenth.	b. 490
a. 3.75	c. 1260
b. 1.028	5. Round to the nearest hundredth.
2. Round to the nearest thousandth.	a. 1.732
a. 0.1234	b. 2.466
b. 1.8032	c. 3.074
c. 7.0052	6. Round to the nearest ten.
3. Round to the nearest whole number.	a. 68
a. 21.1	b. 32
b. 2.7	c. 824
c. 12.05	7. Round to the nearest thousandth.
4. Round to the nearest hundred.	a. 0.7286
a. 275	b. 0.5027
	c. 1.2345

Exercise 8 Answers

1. Round to the nearest tenth.	a. 3.8
	b. 1.0

2. Round to the nearest thousandth. a. 0.123 b. 1.803 c. 7.005	5. Round to the nearest hundredth. a. 1.73 b. 2.47 c. 3.07
3. Round to the nearest whole number. a. 21 b. 3 c. 12	6. Round to the nearest ten. a. 70 b. 30 c. 820
4. Round to the nearest hundred. a. 300 b. 500 c. 1300	7. Round to the nearest thousandth. a. 0.729 b. 0.503 c. 1.235

Exercise 9

Round the numbers to estimate the answer. Circle the estimate that is the best answer.

Example: $47 \times 52 \approx 240, 2\,500, 250, 2\,600$ Estimation:
 $50 \times 50 = 2\,500$

a. $3.2 \times 4.875 \approx 6, 8, 15, 17$ b. $4\,149 \div 20 \approx 2\,000, 200, 20, d.$ c. $2\,895 + 2\,895 \approx 600, 6\,000, e.$	4 000, 5 000 $91 \times 79 \approx 720, 800, 8\,000,$ 80 000 $347 \div 50 \approx 7, 70, 700, 8$

f.	4 892 - 3 012 $\approx$ 1500, 1 000, 2 000, 2 500	averages 286 km per week. Estimate how many
g.	Nathan drives to Terrace and back once a week. He	kilometres he drives in one year (52 weeks).

Exercise 9 Answers

- 15
- 200
- 6 000
- 8 000
- 7
- 2 000
- 15 000 km

4.4: Practice Questions

1. Round to the nearest hundred.
 - a. 749
 - b. 691
2. Round to the nearest whole number.
 - a. 0.831
 - b. 6.24
3. Round to the nearest tenth.
 - a. 8.29
 - b. 6.533
4. Round to the nearest hundredth.
 - a. 34.792
 - b. 6.459

- a. Estimation: $7 \text{ hours} \times \$8 \approx \$56$

Attribution

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4.5: Working With Decimals Review

This section is for extra practice and review. If you are unsure about how to do something, look back at the lesson on that skill.

Reading & Writing Decimals

1. Write as decimals.

a. $\frac{4}{\frac{10}{25}}$

b. $\frac{\frac{100}{37}}{100}$

c. $\frac{\frac{100}{3}}{100}$

d. $\frac{\frac{100}{207}}{10000}$

e. $\frac{10000}{10000}$

f. $3\frac{787}{1000}$

g. $\frac{11}{100}$

h. $5\frac{12}{10000}$

i. $38\frac{12}{100}$

j. $78\frac{43}{10000}$

2. Write these decimals as common fractions.

a. 0.7

b. 0.06

c. 0.49

d. 3.078

e. 47.397

f. 2.173

3. Write as common fractions and as decimals.

a. three tenths

b. fifty-one hundredths

c. three hundred and sixty-one thousandths

d. fifty one thousandths

- e. four hundred thirty-one ten thousandths
 - f. seven and seven tenths
 - g. nine hundred seventy and eighty-nine hundredths
 - h. nine and four hundred twelve thousandths
 - i. six hundredths
4. Write the amount of money with numerals, using a \$ sign.
- a. seven dollars and seventy-eight cents
 - b. eighty-eight cents
 - c. five hundred dollars and five cents
 - d. ninety-nine dollars and ninety cents
 - e. five thousand three hundred twenty-two dollars eighty-nine cents
5. Complete the chart in Table 4.5.1. The first two are done for you as examples.

Table 4.5.1: Question 5 Fill In The Chart

#	Decimal	Fraction	In Words
a	0.0005	$\frac{5}{10000}$	five ten thousandths
b	0.07	$\frac{7}{100}$	seven hundredths
c	—	$\frac{86}{1000}$	—
d	—	$7\frac{11}{100}$	—
e	—	—	fourteen and seventeen thousandths
f	647.8	—	—
g	—	$103\frac{62}{1000}$	—
h	75.13	—	—
i	—	—	forty-two and three tenths
j	0.789	—	—
k	—	—	ten and five hundred sixty-seven thousandths

Comparing Decimals

6. Cross out the zeros that are not needed.

- a. 0.5060
- b. 07.0307
- c. 900.380
- d. 05.200

7. Compare the decimals and then put the correct sign between

the decimals (<, >, =).

- a. 4.3 _____ 43 d. 042.9003 _____ f. 4.01 _____ 5.01
b. 78.9 _____ 7.89 042.9
c. 3.5 _____ 3.05 e. 17.34 _____ 13.34

Rounding Decimals

8. Round the following decimals.

- a. To the nearest tenth.
i. 3.84
ii. 0.09
iii. 3.97
- b. To the nearest hundredth.
i. 2.754
ii. 4.3856
iii. 5.9754
iv. 1.8032
v. 37.439
- c. To the nearest thousandth.
i. 0.1376
ii. 2.7499
iii. 0.00057
- d. To the nearest whole number.
i. 0.39
ii. 78.78
iii. 4.44
iv. 80.745
v. 901.399

4.5: Review Answers

1. Write as decimals.

- a. 0.4 e. 0.0207 i. 38.12
b. 0.25 f. 3.787 j. 78.0043
c. 0.37 g. 0.11
d. 0.03 h. 5.012

2. Write these decimals as common fractions.

- a. $\frac{7}{10}$ c. $\frac{49}{100}$ e. $47\frac{379}{1000}$
b. $\frac{6}{100}$ d. $3\frac{78}{1000}$ f. $2\frac{173}{1000}$

3. Write as common fractions and as decimals.

- a. $\frac{3}{10}$, 0.3 d. $\frac{51}{1000}$, 0.051 g. $970\frac{89}{100}$, 970.89
 b. $\frac{51}{100}$, 0.51 e. $\frac{431}{10000}$, 0.0431 h. $9\frac{412}{1000}$, 9.412
 c. $300\frac{61}{1000}$, 300.061 f. $7\frac{7}{10}$, 7.7 i. $\frac{6}{100}$, 0.06

4. Write the amount of money with numerals, using a \$ sign.

- a. \$7.78 c. \$500.05 e. \$5 322.89
 b. \$0.88 d. \$99.90

5. Complete the chart (see Table 4.5.2).

Table 4.5.2: Question 5 Answers

#	Decimal	Fraction	In Words
a	0.0005	$\frac{5}{10000}$	five ten thousandths
b	0.07	$\frac{7}{100}$	seven hundredths
c	0.086	$\frac{86}{1000}$	eighty six thousandths
d	7.11	$7\frac{11}{100}$	seven and eleven hundredths
e	14.017	$14\frac{17}{1000}$	fourteen and seventeen thousandths
f	647.8	$647\frac{8}{10}$	six hundred forty seven and eight tenths
g	103.062	$103\frac{62}{1000}$	one hundred forty seven and eight tenths
h	75.13	$75\frac{13}{100}$	seventy five and thirteen hundredths
i	42.3	$42\frac{3}{10}$	forty-two and three tenths
j	0.789	$\frac{789}{1000}$	seven hundred eighty nine thousandths
k	10.567	$10\frac{567}{1000}$	ten and five hundred sixty-seven thousandths

6. Cross out the zeros that are not needed.

- a. 0.506
- b. 7.0307

- c. 900.38
- d. 5.2

7. Compare the decimals and then put the correct sign between the decimals ($<$, $>$, $=$).

- a. $<$
- b. $>$

- c. $>$
- d. $>$

- e. $>$
- f. $<$

8. Round the following decimals.

a. To the nearest tenth.

- i. 3.8
- ii. 0.1
- iii. 4.0

c. To the nearest thousandth.

- i. 0.138
- ii. 2.750
- iii. 0.001

b. To the nearest thousandth.

- i. 2.75
- ii. 4.39
- iii. 5.98
- iv. 1.80
- v. 37.44

d. To the nearest whole number.

- i. 0
- ii. 79
- iii. 4
- iv. 81
- v. 901

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4.6: Adding Decimals

Line Up the Decimals

Review place value in whole numbers and in decimal fractions.

Table 4.6.1 shows a place value chart for decimals:

Table 4.6.1: Place Value Chart For Decimals

Hundr eds	Te ns	O nes	· Ten ths	Hundred ths	Thousand ths	Ten Thousandths
—	—	3	. 4	5	3	—
—	—	0	. 9	6	—	—

Vocabulary review:

Use the example in Figure 4.6.1 below and the glossary to help you write the definitions.

addend		sum
↓ ↓		↓
3 + 5	=	8

Figure 4.6.1

Definition of Addends:

Definition of Sum:

When adding decimals, you must be very careful to add together the **digits** with the same place value:

- Add thousandths to thousandths.
- Add hundredths to hundredths.
- Add tenths to tenths.

The best way to do this is to **line up your decimals**.

How to Add Decimals

Step 1: Write the **addends** underneath each other so the decimal points are in a straight column. Notice how lining up the decimals also lines up the place values.

$$0.43 + 0.2 + 0.804 + 0.1 =$$

Rewrite as:

$$\begin{array}{r}
 0.43 \\
 0.2 \\
 0.804 \\
 + 0.1 \\
 \hline
 \end{array}$$

Step 2: Put zeros at the end of the decimal fractions so that all fractions have the same number of decimal places.

$$\begin{array}{r}
 0.430 \\
 0.200 \\
 0.804 \\
 + 0.100 \\
 \hline
 \end{array}$$

Step 3: It may help you to put the decimal point in the answer line before you do the addition.

$$\begin{array}{r}
 0.430 \\
 0.200 \\
 0.804 \\
 + 0.100 \\
 \hline
 .
 \end{array}$$

Step 4: When adding **mixed decimals**, be sure to keep the whole number places lined up as well. As you know, ones add to ones, tens to tens, hundreds to hundreds, and so on. Put a decimal point after any whole numbers.

$$\begin{array}{r}
 62.100 \\
 14.580 \\
 62.1 + 14.58 + 6. + 7.311 + 6.23 = 6.000 \\
 7.311 \\
 + 6.230 \\
 \hline
 \end{array}$$

Step 5: Add each column, starting with the decimal place farthest to the right. If the total of a column is ten or more, carry the ten

number to the next column as you are used to doing with whole numbers.

$$\begin{array}{r}
 11 \\
 0.470 \\
 0.298 \\
 0.100 \\
 + 5.600 \\
 \hline
 6.468
 \end{array}$$

Exercise 1

Rewrite each question in columns and add.

Examples:

$$0.24 + 0.73 + 0.51 = \quad 0.821 + 0.14 + 0.019 =$$

$$\begin{array}{r}
 1 \\
 0.24 \\
 0.73 \\
 + 0.51 \\
 \hline
 1.48
 \end{array}$$

$$\begin{array}{r}
 1 \\
 0.821 \\
 0.140 \\
 + 0.019 \\
 \hline
 0.980
 \end{array}$$

- a. $0.084 + 0.291 + 0.652 =$ c. $0.625 + 0.845 + 0.33 =$
 b. $0.195 + 0.982 + 0.605 =$ d. $0.442 + 0.782 + 0.254 =$

Exercise 1 Answers

a. 1.027

b. 1.782

c. 1.8

d. 1.478

Exercise 2

Rewrite in columns and add.

Examples:

$$\begin{array}{r} 1.003 + 141.2 + 0.47 = \\ \begin{array}{r} 1.003 \\ 141.200 \\ + \quad 0.470 \\ \hline 142.673 \end{array} \end{array} \qquad \begin{array}{r} 5.6 + 3.59 + 1\,829 = \\ \begin{array}{r} 5.60 \\ 3.59 \\ + \quad 1829.00 \\ \hline 1839.19 \end{array} \end{array}$$

$$\begin{array}{ll} \text{c. } 4.6 + 2.59 + 1\,629 = & \text{f. } 321.6 + 485.791 + 0.001 = \\ \text{d. } 0.391 + 80 + 4.63 = & \text{g. } 0.22 + 46.84 + 95.6 = \\ \text{e. } 16.34 + 211.684 + 75.0697 = & \text{h. } 0.042 + 0.018 + 19.1 = \end{array}$$

Exercise 2 Answers

a. 142.673	e. 303.0937
b. 1838.19	f. 807.392
c. 1636.19	g. 142.66
d. 85.021	h. 19.16

Remember that the **perimeter** is the measurement around the outside edge of an object. To find the perimeter, you add the lengths of all the sides together.

Exercise 3

Find the perimeter of the squares described in each question. The measure of one side has been given. Draw a picture of each square to help visualize the question.

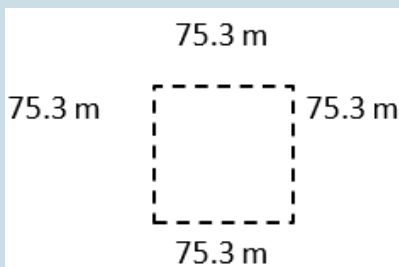
Example:

$$S = 75.3 \text{ m}$$

$$P = S + S + S + S$$

$$P = 75.3 + 75.3 + 75.3 + 75.3 \text{ m}$$

$$P = 301.2 \text{ m}$$



a. $S = 12.4 \text{ mm}$

$P =$

b. $S = 100.73 \text{ km}$

$P =$

c. $S = 50.2 \text{ cm}$

d. $S = 16.5 \text{ m}$

e. $S = 3.25 \text{ m}$

Exercise 3 Answers

a. 49.6 mm

b. 402.92 km

c. 200.8 cm

d. 66 m

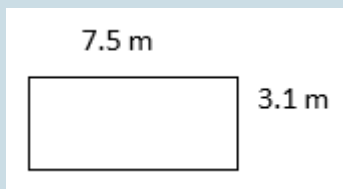
e. 13 m

Exercise 4

Find the perimeter of the rectangles described below.
Draw your own rectangle if there is no picture.

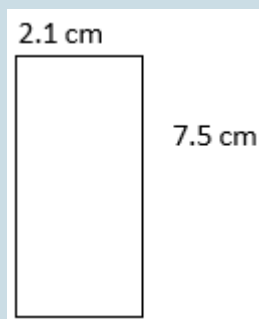
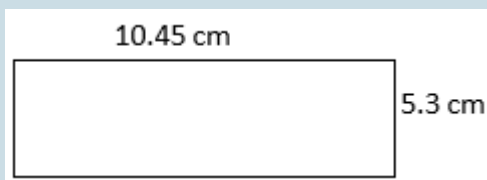
a. $P = \text{side} + \text{side} + \text{side} + \text{side}$

$P =$

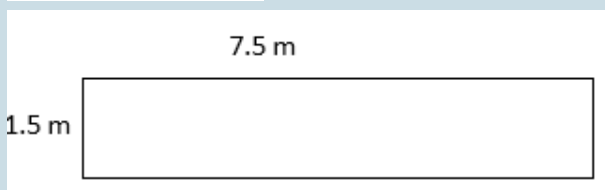


b. $P = \text{side} + \text{side} + \text{side} + \text{side}$

$P =$



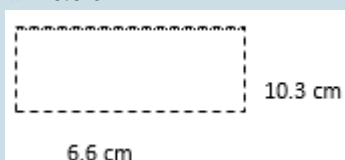
c.



d.

e. $l = 10.3 \text{ cm}$

$w = 6.6 \text{ cm}$



f. $l = 100.03 \text{ km}$

$w = 70.96 \text{ km}$

g. $l = 9.75 \text{ cm}$

$w = 3.5 \text{ cm}$

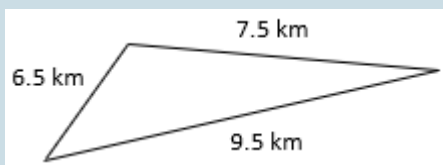
Exercise 4 Answers

a. 21.2 m	d. 18 m	g. 26.5 cm
b. 31.5 cm	e. 33.8 cm	
c. 19.2 cm	f. 341.98 km	

Exercise 5

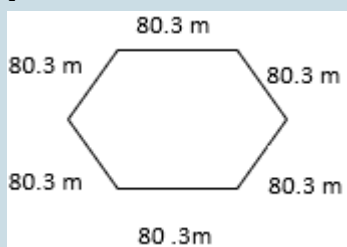
Find the perimeter of the polygons described below. Be sure the measurements are in the same unit value. Use a formula for each calculation. The formula work is started in the first two for you.

- a. $P = \text{Side} + \text{Side} + \text{Side}$
 $P = 6.5 \text{ km} + 7.5 \text{ km} + 9.5 \text{ km}$
 $P =$

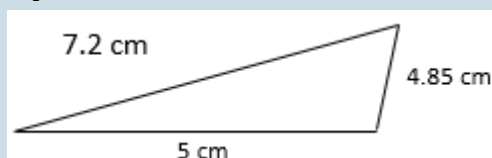


- b. $P = \text{Side} + \text{Side} + \text{Side} + \text{Side} + \text{Side} + \text{Side}$
 $P = \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$
 $P =$

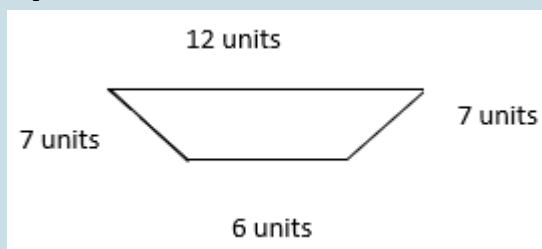
P =



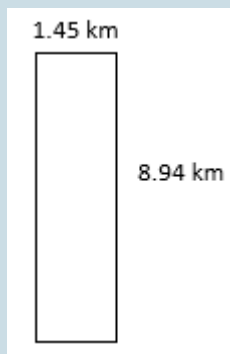
c. P =



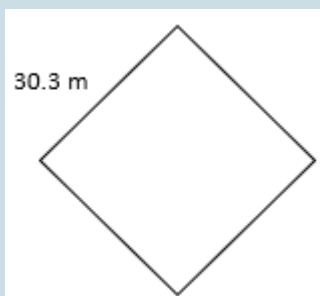
d. P =



e. P =



f. $P =$



Exercise 5 Answers

a. 23.5 km

b. 481.8 m

c. 17.05 cm

d. 32 units

e. 20.78 km

f. 121.2 m

How to Solve Word Problems

In math, word problems describe real-life situations that involve numbers.

Often, the most difficult part of a word problem is knowing what we should do. Once we know what to do, it is much easier to figure out how to do it.



Figure 4.6.2

It is sort of like driving. You may be all ready to go, but before you get into the car, turn on the engine, or put your foot on the gas pedal, you need to know **where** you are going and figure out how to get there (see Figure 4.6.2 and 4.6.3).

The first thing to do is decide on your destination.

How do you “decide on a destination” for a math problem?

Use these steps to solve word problems:

Step 1: Question.

- Read the problem carefully.
- What is the question being asked?
- Does the problem have two parts?
- Draw a picture to help you see the problem described.



Figure 4.6.3

Step 2: Information.

- What information is necessary to solve the problem?
- Often, you are given extra numbers and information that are not needed.
- Keep the question (destination) in mind.
- Circle the information you need.
- Write that information on the picture you drew.

Step 3: Operation.

- Decide what arithmetic operation to use.
- Do you need to add, subtract, multiply, or divide?
- If the problem has two parts, decide what operation you need to do for each part.
 - Keywords often point to the operation needed.
 - Drawing a diagram or sketch is always helpful.
- Write an equation (a number sentence).

Step 4: Estimate.

- Estimate the answer to the problem.
 - Round the numbers so you can work with them quickly.
 - Use the operation you chose in Step 3 and come to a quick answer.
- Does this estimated answer make sense?
- Does it answer the question of the problem?
- Think carefully before you do Step 5.

Step 5: Solve.

- Solve the problem using the actual numbers.
- Check your arithmetic.
- Compare your answer in Step 4 to your estimate from Step 3.
- Think again about the question. Does your solution make sense?
- When you write your answer, include the units.
 - For example: 3 dozen eggs, 7.5 kilometres, 68 people

See Table 4.6.2 for common metric units and abbreviations:

Table 4.6.2: Common Metric Units and Abbreviations

Unit	Abbreviation
kilometre	km
metre	m
centimetre	cm
kilogram	kg
gram	g
litre	L

Example A

The nutrition information on a box of cereal says that a regular serving contains 2.8 g of protein, 0.2 g of fat, 25 g of carbohydrate, and 1.9 g of “other nutrients.” Give the total number of grams in a regular serving.

Step 1: Question.

How many grams are in a regular serving?

Draw a picture (see Figure 4.6.4).

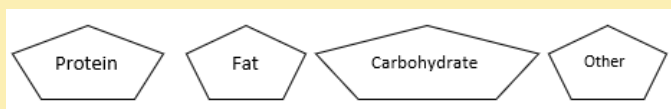


Figure 4.6.4

Step 2: Information.

What information is necessary to solve the problem?

- A regular serving contains 2.8 g protein, 0.2 g fat, 25 carbohydrate, and 1.9 g “other nutrients.”
- Add the information to your picture.

Step 3: Operation.

The problem lists four different amounts and asks you to find one total. “Total” is a keyword that points to addition.

Add the operation you need to your picture (see Figure 4.6.5).

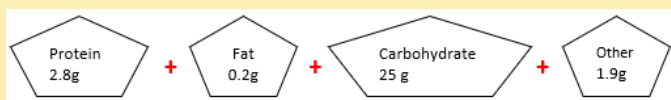


Figure 4.6.5

Next, write an equation:

$$2.8 + 0.2 + 25 + 1.9 = \text{total number of grams}$$

Step 4: Estimate.

Round off each amount and add them together to get an estimated answer.

$$2.8 \text{ g} \approx 3 \text{ g}, 0.2 \text{ g} \approx 0 \text{ g}, 25 \text{ g} \approx 25 \text{ g}, 1.9 \text{ g} \approx 2 \text{ g}$$

$$3 \text{ g} + 0 \text{ g} + 25 \text{ g} + 2 \text{ g} = 30 \text{ g}$$

Does this answer make sense?

A slice of bread is about 35 grams. A serving of cereal would probably weigh a similar amount. Yes, this makes sense.

Step 5: Solve.

$$2.8 \text{ g} + 0.2 \text{ g} + 25.0 \text{ g} + 1.9 \text{ g} = 29.9 \text{ g}$$

A regular serving of cereal is 29.9 grams ← (include the units)

Check:

- Check your arithmetic.
- Compare your answer to your estimate (estimate = 30, answer = 29.9).
- Does the answer make sense?
- Did you include the units in your answer?

Review keywords pointing to addition:

• sum	• complete
• plus	• entire
• total	• in all
• total amount	• added to
• altogether	• increased by
• combined	

Word Problems Using Addition of Decimals

Exercise 6

Use your skills in adding decimal fractions to do the following problems. Underline keywords in the problems that will help you to recognize addition problems. Remember to draw a picture first!

Example:

A road construction crew finished surfacing 5.7 km of highway one week and 4.4 km the next week. How many kilometres did they complete in those two weeks?

- $5.7 \text{ km} + 4.4 \text{ km}$
- Estimation: $6 \text{ km} + 4 \text{ km} = 10 \text{ km}$

• Actual Solution:
$$\begin{array}{r} 5.7 \text{ km} \\ + 4.4 \text{ km} \\ \hline 10.1 \text{ km} \end{array}$$

- Answer: The road crew completed 10.1 km of road surfacing in two weeks.
- a. Lisa rode her bike 4 km to her auntie's house for a little visit. Then she rode 1.5 km to the grocery store

to get apples and 0.7 km to Walmart to get bread and chips. From Walmart, she rode 3.6 km home. How far did she go on her bike altogether? (the picture has been drawn in Figure 4.6.6, but you will need to fill in the information)

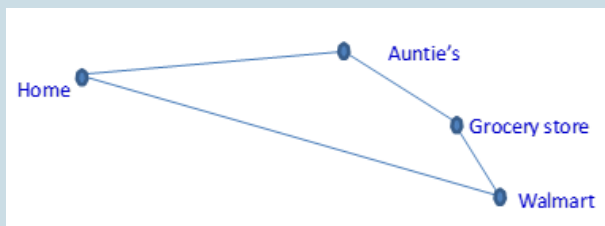


Figure 4.6.6

- Estimation:
 - Actual Solution:
 - Answer:
- b. When Crystalle bought her used car, the odometer showed 12 686.4 km. In one year, she drove 13 294.8 km. What did the odometer read then? (draw your own picture)
- Estimation:
 - Actual Solution:
 - Answer:
- c. A recipe for spaghetti sauce uses 1.25 kg of ground beef, 2.4 kg of fresh tomatoes, 1 kg of onions, 0.75 kg of celery, 0.5 kg of green pepper and lots of garlic and herbs. What is the total weight of the ingredients, not counting the garlic and herbs?

- Estimate:
 - Actual Solution:
 - Answer:
- d. This month, your power bill has increased by \$24.67, your phone bill has an extra \$13.43, and your cable bill is up \$2.24. It has been an expensive month! What is the combined extra cost that you are paying this month?
- Estimate:
 - Actual Solution:
 - Answer:
- e. Wanda texted her boyfriend Curtis and asked him to pick up some groceries on the way home. He answered back, “Only have \$20. Is that enough?” Wanda wanted him to buy milk (\$4.86), mac and cheese (\$1.99), a bag of apples (\$3.99) and laundry detergent (\$8.57). Does Curtis have enough money?
- Estimate:
 - Actual Solution:
 - Answer:

Exercise 6 Answers

- b. Estimation: $2 + 2 + 1 + 4 = 9$
 Actual Solution: $2.4 + 1.5 + 0.7 + 3.6 = 8.2$
 Answer: Lisa rode her bike 8.2 km altogether.
- c. Estimation: $13\ 000\text{ km} + 13\ 000\text{ km} = 26\ 000\text{ km}$
 Actual Solution: $12\ 686.4\text{ km} + 13\ 294.8\text{ km} = 25\ 981.2$

km

Answer: The odometer read 25 981.2 km after one year.

- d. Estimation: $1 \text{ kg} + 2 \text{ kg} + 1 \text{ kg} + 1 \text{ kg} + 1 \text{ kg} = 6 \text{ kg}$

Actual Solution: $1.25 \text{ kg} + 2.4 \text{ kg} + 1 \text{ kg} + 0.75 \text{ kg} + 0.5 \text{ kg} = 5.9 \text{ kg}$

Answer: The total weight of the ingredients is 5.9 kg.

- e. Estimation: $\$25 + \$13 + \$2 = \$40$

Actual solution: $\$24.67 + \$13.43 + \$2.24 = \$40.34$

Answer: The extra cost you are paying is \$40.34.

- f. Estimation: $\$5 + \$2 + \$4 + 9 = \$20$

Actual solution: $\$4.86 + \$1.99 + \$3.99 + \$8.57 = \$19.41$

Answer: The groceries will cost \$19.41 so yes, Curtis has enough money.

4.6: Practice Questions

1. Find the sum.

	$\begin{array}{r} 3.67 \\ 12.55 \\ + 5.67 \\ \hline \end{array}$		$\begin{array}{r} 4.648 \\ 1.92 \\ + 86.9 \\ \hline \end{array}$
a.		c.	

b. $5.27 + 2.4 + 8 + 6.93 + 10.27 =$ d. $2.72 + 0.6 + 110 + 17/223 =$

2. Solve the following word problem.

- Gwen mailed Christmas presents to her family. She sent a 3.7 kg box to her daughter in Vancouver, a 2.145 kg package to her sister's family in Campbell River, a box to Prince George that was 1.06 kg, and a parcel to Kitwanga that was 4 kg. What was the combined weight of Gwen's parcels?
 - a. Estimate:
 - b. Actual Solution:
 - c. Answer:

4.6: Practice Answers

1. Find the sum.

a. 30.08	c. 94.743
b. 32.87	d. 130.543

2. Solve the following word problem.

- a. Estimation: $4 \text{ kg} + 2 \text{ kg} + 1 \text{ kg} + 4 \text{ kg} = 11 \text{ kg}$
- b. Actual Solution: $3.7 \text{ kg} + 2.145 \text{ kg} + 1.06 \text{ kg} + 4 \text{ kg} = 10.905 \text{ kg}$
- c. Answer: The combined weight of the parcels was 10.905 kg.

Attribution

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4.7: Subtracting Decimals

Line up the Decimals

To subtract decimals you must subtract each digit from the digit of the same place value:

- Subtract thousandths from thousandths.
- Subtract hundredths from hundredths.
- Subtract tenths from tenths.

The best way to do this is to **line up your decimals**.

How to Subtract Decimals

The same techniques that you used in adding decimals are helpful when you subtract decimals.

Step 1: Rewrite the problem. Write the first number. Put the amount you are subtracting underneath so the decimal points are in a straight column.

$$\begin{array}{r} 0.468 \quad - \quad 0.3 = \\ \quad \uparrow \quad \quad \uparrow \\ \text{starting number} \quad \text{subtracting this much} \\ \quad \quad \quad 0.468 \\ \quad \quad - \quad 0.3 \\ \hline \end{array}$$

Step 2: Put zeros at the end of the decimals so that all the decimals in the question have the **same number of decimal places**.

$$\begin{array}{r} 0.468 - 0.3 = \quad 0.468 \\ - \quad 0.300 \\ \hline \end{array}$$

Step 3: Subtract the numbers, keeping the decimal point in the answer directly beneath the other decimal points.

$$\begin{array}{r} 0.468 \\ - \quad 0.300 \\ \hline . \end{array}$$

Example A

$$2.536 - 0.59 =$$

Step 1: Rewrite the problem, lining up the decimals.

$$\begin{array}{r} 2.536 \\ - \quad 0.59 \\ \hline \end{array}$$

Step 2: Add zeros so that there are the same number of decimal places for each.

$$\begin{array}{r} 2.536 \\ - \quad 0.590 \\ \hline \end{array}$$

Step 3: Subtract the numbers. You will need to borrow.

$$\begin{array}{r}
 \begin{array}{ccccccc}
 & 1 & & 14 & 13 & & \\
 & \cancel{2} & . & \cancel{5} & \cancel{6} & & \\
 - & & 0 & . & 5 & 9 & 0 \\
 \hline
 & 1 & . & 9 & 4 & 6 &
 \end{array}
 \end{array}$$

Vocabulary review:

Write the definition.

$$8 - 5 = 3 \leftarrow \text{Difference}$$

Difference:

Exercise 1

Subtract to find the differences.

Examples:

$$\begin{array}{r} 2.75 \\ - 0.68 \\ \hline 2.07 \end{array}$$

$$\begin{array}{r} 9.64 \\ - 7.15 \\ \hline 2.49 \end{array}$$

$$\begin{array}{rcl} & 3.85 & 0.362 & 5.276 \\ \text{a. } & - 1.75 & \text{e. } - 0.177 & \text{i. } - 3.298 \\ & \hline & & & \hline \end{array}$$

$$\begin{array}{rcl} & 1.17 & 6.85 & 5.251 \\ \text{b. } & - 0.92 & \text{f. } - 1.28 & \text{j. } - 2.738 \\ & \hline & & & \hline \end{array}$$

$$\begin{array}{rcl} & 27.3 & 18.5 \\ \text{c. } & - 18.9 & \text{g. } - 7.9 \\ & \hline & & \hline \end{array}$$

$$\begin{array}{rcl} & 0.732 & 98.6 \\ \text{d. } & - 0.651 & \text{h. } - 45.8 \\ & \hline & & \hline \end{array}$$

Exercise 1 Answers

a. 2.10	e. 0.185	i. 1.978
b. 0.25	f. 5.57	j. 2.513
c. 8.4	g. 10.6	
d. 0.081	h. 52.8	

Subtracting a Decimal From a Whole Number

Follow these steps to subtract a decimal from a whole number:

Step 1: Put a decimal point after the whole number.

$$16. - 0.4 =$$

Step 2: Rewrite the problem. Write the first number. Put the amount you are subtracting underneath so the decimal points are in a straight column.

$$\begin{array}{r} 16. \\ - 0.4 \\ \hline \end{array}$$

Step 3: Put zeros after the decimal point as needed.

$$\begin{array}{r} 16.0 \\ - 0.4 \\ \hline \end{array}$$

Step 4: Do the subtraction as usual. See that you will need to borrow right away.

Example B

$$32 - 0.12 =$$

$$\begin{array}{r} 32.00 \\ - 0.12 \\ \hline \end{array}$$

Step 1: Rename the 2 in the ones place as 1 and 10 tenths.

$$\begin{array}{r}
 \begin{array}{cc} 1 & 10 \\ 3 & 2 \end{array} . \cancel{0} \cancel{0} \\
 - \quad \quad 0 . 12 \\
 \hline
 \end{array}$$

Step 2: Now rename the 10 tenths as 9 tenths and 10 hundredths. You are ready to subtract.

$$\begin{array}{r}
 \begin{array}{ccc} & 9 & \\ 1 & \cancel{10} & 10 \\ 3 & 2 & . \cancel{0} \cancel{0} \end{array} \\
 - \quad \quad 0 . 12 \\
 \hline
 31 . 88
 \end{array}$$

Example C

$$\$14 - \$3.49$$

$$\begin{array}{r}
 \$14.00 \\
 - \quad \$3.49 \\
 \hline
 \end{array}$$

$$\begin{array}{r}
 ^9 \\
 ^4 ^{10} ^{10} \\
 \$1\cancel{4}.\cancel{0}\cancel{0} \\
 - ^3.^4^9 \\
 \hline
 \$10.^5^1
 \end{array}$$

Exercise 2

Rewrite each question in columns and find the difference.

Example:

$$\begin{array}{r}
 6 - 3.42 = \\
 ^5 ^{10} ^{10} \\
 \cancel{6}.\cancel{0}\cancel{0} \\
 - ^3.^4^2 \\
 \hline
 ^2.^5^8
 \end{array}$$

a. $14 - 9.23 =$

$$\begin{array}{r} 14.00 \\ - 9.23 \\ \hline \end{array}$$

- b. $11 - 3.821 =$
 c. $2 - 1.98 =$
 d. $7 - 3.976 =$

Exercise 2 Answers

- a. 4.77
 b. 7.179

- c. 0.02
 d. 3.024

If you had problems with this, go over your subtraction method with your instructor before you continue.

Exercise 3

Rewrite each question in columns and find the difference.

Example:

$$\begin{array}{r}
 163.682 - 41.5 = \\
 163.682 \\
 - \quad 41.500 \\
 \hline
 122.182
 \end{array}$$

a. $\$60 - \$44.28 =$	c. $89.0309 - 6.3 =$
b. $\$260.06 - \$3 =$	d. $\$100 - \$13.75 =$

Exercise 3 Answers

a. \$15.72	c. 82.7309
b. \$257.06	d. \$86.25

Word Problems Using Subtraction of Decimals

Review keywords pointing to subtraction:

• difference	• decreased by
• balance	• reduced by
• minus	• taken away
• amount left	• less
• subtracted from	• compare

A math question may ask you to compare or find the difference between two amounts. Look for such words as “how much more” (or larger, taller, greater) or “how much less” (or smaller, shorter).

What are the savings? Subtract to find the answer.

Exercise 4

Use your skills in subtracting decimal fractions to do the following word problems. Underline keywords in the problems that will help you to recognize subtraction problems. Remember to draw a picture first!

- a. Brad is 1.8 m tall. He just did the best high jump of his life, clearing 1.89 m. How much less is his own height than the height he jumped? (add the information to the drawing in Figure 4.7.1)



Figure 4.7.1

- Estimation:

- Actual Solution:
- b. Susan's best track and field event is the long jump. She leapt 6.16 m. Her mom used to long jump in high school and jumped 5.52 m. How much farther did Susan jump than her mom?
 - Estimation (to tenths):
 - Actual Solution:
- c. Joe had a bank balance of \$438. He wrote a cheque for \$111.59 to pay for a phone bill. What is the balance in his bank account now?
 - Estimation:
 - Actual Solution:
- d. A plumber needs to replace 11.5 m of pipe in a home. She has 6.5 m in her truck. How much more pipe does she need?
 - Estimation:
 - Actual Solution:
- e. Lee is going to install baseboards in the bachelor suite he has built in his basement. The room is 5.8 metres square. The baseboard material is expensive, so he will be sure to deduct 1 m for each of the two doorways. How much baseboard material does he need to buy? (this question involves addition and subtraction)
 - Estimation (to tenths):
 - Actual Solution:

Exercise 4 Answers

- a. Estimation: $1.9 \text{ m} - 1.8 \approx 0.1 \text{ m}$
Actual Solution: $1.89 - 1.8 = 0.09 \text{ m}$
Answer: Brad's height is 0.09 m less than the height he jumped.
- b. Estimation: $6.2 - 5.5 \approx 0.7 \text{ m}$
Actual Solution: $6.16 - 5.52 = 0.64 \text{ m}$
Answer: Susan jumped 0.64 m farther than her mom.
- c. Estimation: $\$440 - \$100 \approx \$340$
Actual Solution:
 $\$438 - \$111.59 = \$326.41$
Answer: Joe's bank balance is now \$326.41.
- d. Estimation: $12 \text{ m} - 7 \text{ m} \approx 5 \text{ m}$
Actual Solution: $11.5 \text{ m} - 6.5 \text{ m} = 5 \text{ m}$
Answer: The plumber needs 5 m more of pipe.
- e. Estimation:
 $6 \text{ m} \times 4 = 24 \text{ m}, 24 \text{ m} - 2 \text{ m (doors)} \approx 22 \text{ m}$
Actual Solution:
 $5.8 \text{ m} \times 4 = 23.2 \text{ m}, 23.2 \text{ m} - 2 \text{ m (doors)} = 21.2 \text{ m}$
Answer: Lee will need to buy 21.2 m worth of base board material.

Design Your Own House Part 1: Drawing & Measuring

1. On a sheet of graph paper, use a ruler to draw a one-story house (bird's-eye-view). Use the scale of $1\text{ cm}^2 = 1\text{ m}^2$.
2. You may make your house any shape you choose (square, rectangle, L-shaped, etc.) but one wall must measure 15.5 metres (it will be 15.5 cm on your graph paper).
3. Clearly label the length of each wall and show your calculations! Be precise and careful with your sketching. Feel free to add colour or get creative.
4. Be sure to organize and label all your work!
 - a. What is the perimeter of the entire house? (perimeter = side + side + side + side)
5. Give your house two bedrooms. Clearly label the length of each wall.
6. Include a bathroom, a living room, and a kitchen. Label the length of each wall.
 - b. What is the perimeter of each room?
 - Bedroom 1:
 - Bedroom 2:
 - Bathroom:
 - Kitchen:
 - Living room:
7. On your sketch, give your house two exterior doors and four exterior windows, each 1m wide. Because it is a two-dimensional bird's eye view, you can't see the height of your doors and windows, but imagine that each door would be 2.3m tall and each window would be 1.1m tall. Sketch one window and one door here, and label the measurements.
 - c. Each window needs external trim and internal trim. How

much trim would you need to buy to have enough to go around all of the windows?

- d. Each door would also need external and internal trim. How much trim would you need to go around both doors? Keep in mind that the bottom of the door will not need trim.

When you have finished this project, put your graph paper somewhere safe because you will be using it again at the end of 4.10: Multiplying Decimals.

Marking Checklist of House Project Part 1

[May 9 – Do you want to keep this section and the one in 4.10? The unit pricing chapter, which has part 3, is not one of the chapters you wanted to include – K]

/50 marks

- **Sketch: /14**
 - A ruler was used for drawing lines. /2
 - The house has five rooms. /2
 - All wall lengths (internal and external) are marked clearly on the sketch. /5
 - Locations for doors and windows have been marked on the sketch. /2
 - One door and one window have been drawn with measurements marked. /2
 - The house has a yard and measurements have been clearly marked on the sketch. /2
- **Calculations /36**

- The perimeter of the house has been calculated correctly. /2
- The perimeter of each room has been calculated correctly. /10
- Bedroom 1 /2
- Bedroom 2 /2
- Bathroom /2
- Kitchen /2
- Living room /2
- Window and door locations have been marked on the graph paper sketch. /2
- One window and one door has been sketched and measurements labelled. /2
- The amount of trim for the windows has been calculated correctly. /5
- The amount of trim for the doors has been calculated correctly. /5

4.7: Practice Questions

1. Subtract.

a.
$$\begin{array}{r} 72.04 \\ - 13.98 \\ \hline \end{array}$$

c.
$$\begin{array}{r} 11.21 \\ - 3.875 \\ \hline \end{array}$$

b. $19.6 - 6.254 =$

d. $\$140 - \$102.73 =$

2. Solve the following word problem.

- Gail spent \$273.24 on her groceries. She had \$300 with her. How much of her money is left?

- a. Estimation:
- b. Actual Solution:

4.7: Practice Answers

1. Subtract.

a. 58.06	c. 7.335
b. 13.346	d. \$37.27

2. Solve the following word problem.

- a. Estimation: $\$300 - \$275 \approx \$25$
- b. Actual Solution: $\$300 - \$273.24 = \$26.76$
Gail had \$26.76 left after buying groceries.

Attribution

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4.8: Bookkeeping

One everyday use of adding and subtracting decimals is the bookkeeping we all must do with our money.

Here are some examples:

- Keeping track of online payments.
- Figuring out how much money to take on a trip.
- Stretching a pay cheque over two weeks.
- Organizing the household budget.
- Deciding how much you can spend on eating out.

What are some other examples of bookkeeping that you do?

The bookkeeping that most of us do is straightforward:

- Add money received or deposited to our accounts.
- Subtract money spent or paid out.

The result of the addition or subtraction is the **balance**.

Methods of Payment

There are many different methods of paying for purchases. Some of the most common methods are:

- cash
- debit card
- credit card
- cheque
- online payment (i.e., PayPal, e-transfer)

There are benefits to each payment method. Each person chooses

to do what works best for them, depending on the situation. Here is a list of some of the benefits and drawbacks of each method of payment.

Cash

Advantages:

- It is quick and easy to pull money out of your wallet.
- You can see how much money you have left.
- It is impossible to overspend.
- It is a great method to use if you do not have a bank account.

Disadvantages:

- It is easier to lose.
- You may need to continue to visit the bank to get money out.
- You might run out of cash while trying to pay at the till.

Debit Card

Advantages:

- It is quick at the till.
- You cannot spend more than you have in your bank account.
- It is safe.

Disadvantages:

- There is often a service fee when using the card to pay for shopping.
- You cannot always use it for online shopping.

Credit Card

Advantages:

- You only have to pay the company money once a month.
- It is quick at the till.
- It is easy to track your spending because the credit card company sends you a monthly statement.
- You can use a credit card for online shopping.

Disadvantages:

- It is really easy to overspend your budget because you do not have to pay any money upfront.
- There are often service charges once a year that are expensive.
- You can get into debt with a credit card, and it may be difficult to get out of that debt.

Cheque

Advantages:

- It can be convenient when you want to mail someone money.
- It is safe.
- The chequebook helps you keep a written record of your bank balance.

Disadvantages:

- Some stores do not accept cheques or require several pieces of identification, usually a driver's licence and a credit card.
- Most banks and credit unions have a small service charge for each cheque that you write.
- If your account is **overdrawn**, your cheque will be N.S.F. (Not Sufficient Funds), for which you are charged extra by the store and the bank. To avoid this, keep careful, up-to-date records so you always know your balance.

Online Payment

Advantages:

- It allows you to shop online securely.
- You can pay directly online with money from your bank account or credit card.
- You can easily send money to friends or family.

Disadvantages:

- You may not have the rights that regular banks give you when you use an online payment company.
- The company is not local, so any problems may be hard to resolve.
- There are fees charged to have an online payment account.

No matter what payment method you choose to use, it is very helpful to keep track of your money. You can use a record book to track when you spent money and when you were paid money. This will help with budgeting and planning.

Online Banking

Many people use online banking to keep track of their finances. When online banking, you will see a record of your **transactions** that looks something like Table 4.8.1.

Table 4.8.1: Example Online Banking Transactions

Date	Transactions	Debit	Credit	Running Balance
2023-01-29	Electronic Funds Transfer PAYCHEQUE	—	175.00	1439.66
2023-02-23	Internet Banking INTERNET BILL PAY TELUS COMMUNICATIONS	65.49	—	1374.06
2023-03-10	Electronic Funds Transfer DEPOSIT CANADA	125.00	—	1499.06
2023-03-16	Internet Banking E-TRANSFER Larissa	100.00	—	1599.06

The transactions are usually recorded in chronological order by date and time.

Here is a description of each column:

- **Date** — The date of the transaction. The examples given use this method: 2023-01-29. January 29 is the 29th day of the 1st month.
- **Debit** — This is money that has gone out of the account, either through a withdrawal, a bill payment, or a transfer out.
- **Credit** — This is money deposited into the account, either through a cash deposit, a paycheck, or a transfer in.
- **Running Balance** — The amount in the bank account after each transaction.

Exercise 1

Look carefully at this sample online banking transaction record in Table 4.8.2, and answer the questions that follow.

Table 4.8.2: Exercise 1 Transactions

Date	Transactions	Debit	Credit	Running Balance
—	Balance Forward	—	—	121.16
2022-03-29	Electronic Funds Transfer PAYCHEQUE	—	675.62	798.78
2022-03-30	Internet Banking INTERNET BILL PAY MCCARTHY GM	175.40	—	621.38
2022-03-30	Internet Banking INTERNET BILL PAY BC HYDRO	50.27	—	571.11
2022-04-05	CASH DEPOSIT	—	25.00	596.11
2022-04-08	Internet Banking INTERNET BILL PAY CITYWEST	19.80	—	576.31
2022-04-09	INTERAC PAYMENT MAVERICK FOODS	128.54	—	447.77
2022-04-09	CASH WITHDRAWAL	30.00	—	417.77

- How much money was in the account at the beginning of this transaction record (balance forward)?
- Name the month when the Citywest online bill payment happened.
- What is the amount of the interact payment to Maverick Foods?
- How much was the pay deposit?
- What was the balance after the B.C. Hydro transaction?

- f. How many deposits were made to the account?
- g. What is the total amount of the deposits?

Exercise 1 Answers

a. April	e. \$121.16
b. \$128.54	f. 2
c. \$675.62	g. \$700.62
d. \$571.11	

Exercise 2

Complete the transaction record in Table 4.8.3 using the information in Table 4.8.4 below.

Continue to fill out the “Running Balance” column as you add each credit or debit.

Table 4.8.3: Transaction Record to Fill In

D ate	Transactions	De bit	Cre dit	Runnin g Balance
—	Balance Forward	—	—	—
—	—	—	—	—
—	—	—	—	—
—	—	—	—	—
—	—	—	—	—
—	—	—	—	—
—	—	—	—	—
—	—	—	—	—
—	—	—	—	—
—	—	—	—	—
—	—	—	—	—
—	—	—	—	—

Information to use for question:

Table 4.8.4: Exercise 2 Transaction Details

Date	Details	Transaction
April 23, 2022	Balance forward	\$210.83
April 25, 2022	Cash withdrawal	\$45.00
April 28, 2022	Debit to Maverick Mart	\$99.95
April 30, 2022	Pay deposit	\$843.29
May 1, 2022	Online payment Mark Jones for rent	\$420.00
May 3, 2022	E-transfer to Kathy Smythe (for Facebook raffle)	\$25.00
May 6, 2022	Interac payment to Chevron gas	\$18.27
May 8, 2022	Cash withdrawal	\$110.00
May 10, 2022	Online bill payment for Mastercard	\$150.00
May 12, 2022	Deposit Child Care Tax Credit	\$66.48
May 13, 2022	Interac payment Maverick Mart	\$183.00
May 15, 2022	Pay deposit	\$792.18

Exercise 2 Answers

Table 4.8.5: Exercise 2 Answers

Date	Transactions	Debit	Credit	Running Balance
2022-04-23	Balance Forward	—	—	210.83
2022-04-25	Cash withdrawal	45.00	—	165.83
2022-04-28	INTERAC PAYMENT MAVERICK FOODS	99.95	—	65.88
2022-04-30	Electronic Funds Transfer PAYCHEQUE	—	843.29	909.17
2022-05-01	Internet Banking INTERNET BILL PAY MARK JONES	420.00	—	489.17
2022-05-03	Internet Banking E-TRANSFER KATHY SMYTHE	25.00	—	464.17
2022-05-06	INTERAC PAYMENT CHEVRON GAS	18.27	—	445.90
2022-05-08	Cash withdrawal	100.00	—	335.90
2022-05-10	Internet Banking INTERNET BILL PAY MASTERCARD	150.00	—	185.90
2022-05-12	DEPOSIT CANADA CHILD TAX CREDIT	—	66.48	252.38
2022-05-013	INTERAC PAYMENT MAVERICK MART	183.00	—	68.38
2022-05-15	Electronic Funds Transfer PAYCHEQUE	—	792.18	861.56

Cheque Writing

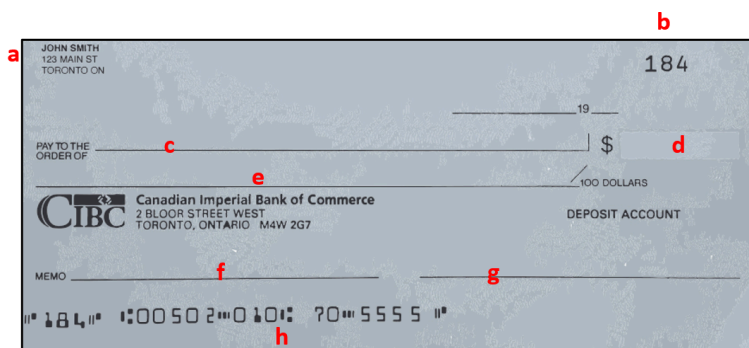


Figure 4.8.1

Figure 4.8.1 shows an example cheque with the following information:

- When you have a chequing account, you sometimes have cheques printed with your name, address and phone number, usually in the top left corner.
- The cheques are numbered in sequence (in order) to help you keep track of your cheques.
- This line is for the name of the person or institution that will be receiving the money.
- This line is for the amount of the cheque in numbers: \$22.98.
- This spot is for you to write, in words, the dollars to be paid. Write the cents as a fraction over 100. If you do not use the whole line, fill unused parts of the space with a straight line so that nothing can be added. \$22.98 becomes twenty-two and 98/100 dollars.
- The Memo spot is a place for details. It is useful if you want to use the cheque as a receipt, too. For example, you might list the invoice number for the bill you are paying.

- g. When you open a bank account, the bank will ask you for a sample signature for their files. Sign exactly as you plan to sign your cheques.
- h. Your bank account number and codes used at the bank will be printed on your cheques.

Any cheques you do not want to be cashed should have **VOID** or **SAMPLE ONLY** written on them.

As soon as you write a cheque, be sure to enter the details in your chequebook.

A chequebook is a simple accounts book or ledger. A ledger is a convenient way to record expenditures (money spent) and income.

Exercise 3

Use the information in Table 4.8.6 and Table 4.8.7 below to complete the chequing account ledger in Table 4.8.8.

Arrange the information in chronological order. That means put the information with the earliest date first, then the next date, and so on.

Withdrawals:

Table 4.8.6: Exercise 3 Withdrawals

Date	Cheque #	Details	Debit
2022-05-01	#122	Mortgage payment	\$375.00
2022-05-06	#123	Cable	\$32.17
2022-04-23	—	E-transfer Mike the Mechanic	\$45.82
2022-04-18	#121	B.C. Hydro (Feb & Mar)	\$62.53
2022-03-02	—	Cash withdrawal	\$75:00
2022-03-02	—	Debit charge	\$1.50
2022-05-04	—	Grocery Mart	\$111.95

Deposits:

Table 4.8.7: Exercise 3 Deposits

Date	Details	Credit
2022-04-30	Pay	\$596.27
2022-04-15	Birthday money	\$200.00
2022-04-20	Child Care Tax Refund	\$33.64

Chequing account ledger to complete:

Table 4.8.8: Exercise 3 Transaction Record to Fill In

D ate	Cheq ue #	Transacti on	Debit or Cheque Amount	Depos it Amount	Bala nce
—	—	Balance forward	—	—	—
—	—	—	—	—	—
—	—	—	—	—	—
—	—	—	—	—	—
—	—	—	—	—	—
—	—	—	—	—	—
—	—	—	—	—	—
—	—	—	—	—	—
—	—	—	—	—	—
—	—	—	—	—	—
—	—	—	—	—	—

Use the cheque blank in Figure 4.8.2 to write out cheque # 121 from Table 4.8.6. Use any name and address you want. Ask your instructor to check your work.

Your name _____		
Your address _____		
Your phone number _____	_____20_____	No. _____
PAY TO		
THE ORDER OF _____		\$ _____
_____		_____DOLLARS
		100
ABE Bank	Account # 456-789-0	SAMPLE ONLY
123 Any Street		
Our Town, BC		
MEMO _____		_____

Figure 4.8.2

Exercise 3 Answers

Table 4.8.9: Exercise 3 Answers

Date	Cheque #	Transaction	Debit or Cheque Amount	Deposit Amount	Balance
—	—	Balance forward	—	—	312.07
2022-03-02	—	Cash Withdrawal	75.00	—	237.07
2022-03-02	—	Debit charge	1.50	—	235.57
2022-04-15	—	Birthday money	—	200.00	435.57
2022-04-18	121	BC Hydro (Feb & Mar)	62.52	—	373.04
2022-04-20	—	Child Care Tax Refund	—	33.64	406.68
2022-04-23	—	Mike the Mechanic (fix shocks)	45.82	—	360.86
2022-04-30	—	Pay	—	596.27	957.13
2022-05-01	122	Mortgage Payment	375.00	—	582.13
2022-05-04	—	Grocery Mart	111.95	—	470.18
2022-05-06	123	Cable	32.17	—	438.01

Your name <u>John Smith</u>		
Your address <u>123-FourthStreet,Anywhere,BC</u>		
Your phone number <u>250-123-4567</u>	<u>April 18</u> 20 <u>21</u>	No. <u>121</u>
PAY TO THE ORDER OF <u>BCHydro</u>	\$ <u>62.53</u>	
----- <u>Sixty-Two and</u>		<u>53</u> /100 DOLLARS
ABE Bank 123 Any Street Our Town, BC	Account # 456-789-0	SAMPLE ONLY
MEMO <u>Feb.&Mar Payment</u>	<u>John Smith</u>	

Figure 4.8.3

Attribution

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4.9: Adding & Subtracting Decimals Review

1. Find the sum.

a. $4.87 + 3.91 + 0.33 + 5.68 =$ c. $5.38 + 4.7102 + 24.003 + 0.78 =$

b. $0.187 + 28.27 + 8.039 + 4.44 =$ d. $3.912 + 46 + 72.04 + 19.19 =$

2. Solve the following word problems.

- Cynthia spent \$4.53 on butter, \$10.97 on a big bag of flour and \$3.50 on eggs. How much did she spend?
- A seaplane pilot is finding the weight of the freight and passengers on her next flight. The freight weighs 800.25 kg. All of the passengers together weigh 452.5 kg. How much is the total weight on the next flight?
- Use a ruler or metre stick to measure the edges of the top of the table where you are sitting. Draw and label a diagram, then use your measurements to find the perimeter.

3. Find the difference.

a. $82.07 - 14.86 =$ c. $\$72.53 - \$51.30 =$

b. $89.506 - 16.039 =$ d. $\$7 - \$0.31 =$

4. Solve the following word problems.

- Babies often lose a bit of weight in the first few days after they are born. They then usually go on to grow quite quickly. When Oliver was first born, he weighed 3.36 kg. When he was two days old, he weighed 3.19 kg. How much weight did he lose?
- Jules is installing trim around a doorway. The perimeter of the door is 5.78 m. Jules will not be putting trim on the base of the doorway. If the base of the doorway measures

0.85 m, how much trim should she purchase?

- c. Carissa is helping her relatives calculate a budget for a memorial feast. Her family has \$575 saved up. Several friends and family members have made contributions totalling \$360. The expenses will include \$120 for a hall rental, \$85.50 for table and chair rental, \$50 for a microphone and sound system, \$45.68 for insurance coverage, and \$600 for gifts. How much money will Carissa’s family have left after the expenses?

5. Read the following transaction record in Table 4.9.1 and answer the questions.

Table 4.9.1: Transaction Record

Date	Transactions	Debit	Credit	Running Balance
—	Balance Forward	—	—	559.58
2022-04-20	CASH DEPOSIT	—	200.00	—
2022-04-21	INTERAC PAYMENT Kaien Island Optometry	74.53	—	—
2022-04-29	DEPOSIT CANADA CHILD TAX CREDIT	—	89.70	—
2022-05-01	Electric Funds Transfer PAYCHEQUE	—	609.74	—
2022-05-08	Internet Banking INTERNET BILL PAY BC HYDRO	52.46	—	—
2022-05-08	INTERAC PAYMENT FEES	1.75	—	—
2022-05-10	INTERAC PAYMENT Safeway	73.02	—	—
2022-05-10	INTERAC PAYMENT FEES	1.89	—	—
2022-05-12	CASH WITHDRAWAL	60.00	—	—

- a. Complete the running balance column. You may use a calculator.

- b. How much was the pay transfer?
- c. What amount is the child tax credit?
- d. What day was the cash withdrawal made?
- e. How much was the cash deposit?
- f. Why do you think the gas station and grocery store purchases have a fee?

4.9: Review Answers

1. Find the sum.

a. 14.79	c. 34.8732
b. 40.936	d. 141.142

2. Solve the following word problems.

a. \$19.00	b. 86.6m

3. Find the difference.

a. 67.21	c. \$21.23
b. 73.467	d. \$6.69

4. Solve the following word problems.

- a. Oliver lost 0.17 kg or 170 g.
- b. Jules should purchase 4.93 m of trim.
- c. Carissa's family will have \$33.82 left.

5. Read the following transaction record and answer the questions.

a. **Table 4.9.1: Question 5 Answers**

Date	Transactions	Debit	Credit	Running Balance
—	Balance Forward	—	—	559.58
2022-04-20	CASH DEPOSIT	—	200.00	685.05
2022-04-21	INTERAC PAYMENT Kaien Island Optometry	74.53	—	774.75
2022-04-29	DEPOSIT CANADA CHILD TAX CREDIT	—	89.70	1384.49
2022-05-01	Electric Funds Transfer PAYCHEQUE	—	609.74	734.49
2022-05-08	Internet Banking INTERNET BILL PAY BC HYDRO	52.46	—	682.03
2022-05-08	INTERAC PAYMENT FEES	1.75	—	680.28
2022-05-10	INTERAC PAYMENT Safeway	73.02	—	607.26
2022-05-10	INTERAC PAYMENT FEES	1.89	—	605.37
2022-05-12	CASH WITHDRAWAL	60.00	—	545.37

- b. \$609.74
- c. \$89.70
- d. May 12, 2010
- e. \$200.00
- f. Some banks charge people for using their debit cards at places other than the bank.

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4.10: Multiplying Decimals

Multiplying decimals uses the same method that you learned for multiplying whole numbers. Review multiplication of whole numbers.

Vocabulary review:

Review the diagram in Figure 4.10.1 below and try to write in the explanations of the mathematical terms. You may refer to the glossary, if you wish. For right now, it is mostly important to remember what factor means.

$$\begin{array}{ccc} & \text{Factors} & \\ \downarrow & & \downarrow \\ 3 \times 7 = 21 & \leftarrow & \text{Product} \end{array}$$

Figure 4.10.1

Product:

Factors:

How to Multiply Decimals

Multiplying decimals follows almost the same steps as multiplying whole numbers. This section will show you two ways to multiply decimals.

Method 1

One method is to **estimate** the **product** using whole numbers to determine where the decimal goes.

Example A

$$4.3 \times 5.7$$

$$\text{Estimate: } 4.3 \times 5.7 \approx 4 \times 6 = 24$$

This tells us that the correct answer will be around 24 (which is two whole number places). We know that the answer will not be around 2.4 or 240.

If we take the decimals out and just multiply the digits, the answer is **2451**.

$$\begin{array}{r}
 57 \\
 \times 43 \\
 \hline
 171 \\
 + 2280 \\
 \hline
 2451
 \end{array}$$

The estimate shows that the decimal point will come after two whole number places, so

$$4.3 \times 5.7 = 24.51.$$

$$\begin{array}{r}
 5.7 \\
 \times 4.3 \\
 \hline
 171 \\
 + 2280 \\
 \hline
 24.51
 \end{array}$$

Example B

$$23.24 \times 3.9$$

$$\text{Estimate: } 23.24 \times 3.9 \approx 23 \times 4 = 92$$

The answer will be around 92. It will not be around 9.2 or 920.

If we take the decimals out and just multiply the digits, the answer is 90636.

The estimate shows that the decimal point will come after two whole number places, so

$$23.24 \times 3.9 = 90.636$$

$$\begin{array}{r} 23.24 \\ \times 3.9 \\ \hline 20916 \\ + 69720 \\ \hline 90.636 \end{array}$$

If the whole numbers in the question are large, you can round to the nearest ten or hundred to help you decide where to put the decimal point. This is a quick estimate.

Example C

$$383.298 \times 213.87$$

$$383.298 \times 213.87 \approx 400 \times 200 = 80\,000$$

The answer will be around 80 000. It will not be around 8 000 or 800 000.

If we take the decimals out and just multiply the digits 383.298 by 213.87, the numerals in the product are 8 197 594 326.

The estimate shows that the whole number will go up to the ten-thousands place, which is five whole number places, so

$$383.298 \times 213.87 = 81\,975.94326$$

Whew!

$$\begin{array}{r} 383.298 \\ \times \quad 213.87 \\ \hline 81975.94326 \end{array}$$

Exercise 1

All the multiplying has been done already. Your task is to put the decimal point in the product by doing a whole number estimate of the question.

$$\begin{array}{r}
 2.8 \\
 \times 4.3 \\
 \hline
 84 \\
 + 1120 \\
 \hline
 12.04
 \end{array}
 \quad
 \begin{array}{r}
 \text{Estimate: } \times 3 \\
 \hline
 4 \\
 \hline
 12
 \end{array}$$

	$ \begin{array}{r} 56.9 \\ \times 12.3 \\ \hline 1707 \\ 11380 \\ 56900 \\ \hline 69987 \end{array} $	c.	$ \begin{array}{r} 28.7 \\ \times 64.97 \\ \hline 1864639 \\ 73.2 \\ \hline 11712 \end{array} $	f.	$ \begin{array}{r} 48.29 \\ \times 55.6 \\ \hline 2684924 \\ 91.2 \\ \hline 13680 \end{array} $
a.			d.		g.
	$ \begin{array}{r} 7.3 \\ \times 19.6 \\ \hline 14308 \end{array} $	e.	$ \begin{array}{r} 1.8 \\ \times 1.3 \\ \hline 234 \end{array} $		

Exercise 1 Answers

a. 699.87	d. 117.12	g. 136.80
b. 143.08	e. 2.34	
c. 1864.639	f. 2684.924	

Method 2

Another way of locating the decimal point in the product is to look at the number of decimal places in the decimals you are multiplying.

Example D

$$\begin{array}{r} 43.23 \quad 2 \text{ decimal places} \\ \times 19.6 \quad 1 \text{ decimal place} \\ \hline \end{array}$$

Then add the number of decimal places you counted above ($2 + 1 = 3$)

This is the number of decimal places you will have in your answer.

$$\begin{array}{r} 43.23 \quad 2 \text{ decimal places} \\ \times 19.6 \quad 1 \text{ decimal place} \\ \hline 847.308 \quad 3 \text{ decimal places} \end{array}$$

Example E

$$\begin{array}{r} 0.35 \quad 2 \text{ decimal places} \\ \times 0.47 \quad 2 \text{ decimal places} \\ \hline 0.1645 \quad 4 \text{ decimal places} \end{array}$$

Example F

$$\begin{array}{r} 13.452 \quad 3 \text{ decimal places} \\ \times 30 \quad 0 \text{ decimal places} \\ \hline 403.560 \quad 3 \text{ decimal places} \end{array}$$

Exercise 2

Again, the multiplying has been done. Use the method of multiplying the understood **denominators to put the decimal point in the product.**

Examples:

$$\begin{array}{r} 9.2 \text{ 1 decimal place} \\ \times 0.3 \text{ 1 decimal place} \\ \hline 2.76 \text{ 2 decimal places} \end{array}$$

$$\begin{array}{r} 0.27 \text{ 2 decimal places} \\ \times 1.8 \text{ 1 decimal place} \\ \hline 0.486 \text{ 3 decimal places} \end{array}$$

a.	$\begin{array}{r} 6.8 \\ \times 0.4 \\ \hline 272 \end{array}$	d.	$\begin{array}{r} 0.91 \\ \times 6.2 \\ \hline 5642 \end{array}$	g.	$\begin{array}{r} 7.3 \\ \times 3.8 \\ \hline 2744 \end{array}$
b.	$\begin{array}{r} 2.4 \\ \times 0.13 \\ \hline 0312 \end{array}$	e.	$\begin{array}{r} 0.29 \\ \times 1.2 \\ \hline 0348 \end{array}$	h.	$\begin{array}{r} 4.24 \\ \times 0.07 \\ \hline 02968 \end{array}$
c.	$\begin{array}{r} 240 \\ \times 0.05 \\ \hline 1200 \end{array}$	f.	$\begin{array}{r} 0.87 \\ \times 0.19 \\ \hline 01653 \end{array}$	i.	$\begin{array}{r} 0.042 \\ \times 60 \\ \hline 2520 \end{array}$

Exercise 2 Answers

- a. 2.72
b. 0.312
c. 12.00

- d. 5.642
e. 0.348
f. 0.1653

- g. 27.74
h. 0.2968
i. 2.52

Exercise 3

If you had trouble with the first two exercises, then get help from your instructor. Here is extra practice if you want or need it.

Examples:

$$\begin{array}{r} 41 \quad 0 \text{ decimal places} \\ \times 0.061 \quad 3 \text{ decimal places} \\ \hline 2.501 \quad 3 \text{ decimal places} \end{array}$$

$$\begin{array}{r} 7.8 \quad 1 \text{ decimal place} \\ \times 0.5 \quad 1 \text{ decimal place} \\ \hline 3.90 \quad 2 \text{ decimal places total} \end{array}$$

$$\begin{array}{r} \text{a.} \quad \begin{array}{r} 59.275 \\ \times 0.08 \\ \hline 474200 \end{array} \quad \text{b.} \quad \begin{array}{r} 7.18 \\ \times 23.46 \\ \hline 1684428 \end{array} \quad \text{c.} \quad \begin{array}{r} 0.84 \\ \times 2.1 \\ \hline 1764 \end{array} \end{array}$$

$$\begin{array}{r}
 \text{d.} \quad \begin{array}{r} 18.6 \\ \times 0.3 \\ \hline 558 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{e.} \quad \begin{array}{r} 100 \\ \times 0.35 \\ \hline 3500 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{f.} \quad \begin{array}{r} 0.72 \\ \times 3.4 \\ \hline 2448 \end{array}
 \end{array}$$

Exercise 3 Answers

- a. 4.74200
b. 168.4428

- c. 1.764
d. 5.58

- e. 35.00
f. 2.448

Exercise 4

Multiply to find the product. Remember to put the decimal point in the correct place; you know two methods!

Example:

$$\begin{array}{r}
 13.8 \\
 \times 3.9 \\
 \hline
 1242 \\
 4140 \\
 \hline
 53.82
 \end{array}$$

a. $\begin{array}{r} 0.076 \\ \times 24 \\ \hline \end{array}$	c. $\begin{array}{r} 0.429 \\ \times 7.5 \\ \hline \end{array}$	e. $\begin{array}{r} 1.79 \\ \times 0.33 \\ \hline \end{array}$
b. $\begin{array}{r} 2.05 \\ \times 1.7 \\ \hline \end{array}$	d. $\begin{array}{r} 96 \\ \times 0.2 \\ \hline \end{array}$	f. $\begin{array}{r} 40.1 \\ \times 0.9 \\ \hline \end{array}$

Exercise 4 Answers

a. 1.824
b. 3.485

c. 3.2175
d. 19.2

e. 0.5907
f. 36.09

Prefixing Zeros

Remember this skill?

- $\frac{23}{1000} = 0.023$ – The **0** must be used after the decimal point in 0.023 to hold the tenths place. This makes it clear that the denominator is one thousand.
- $\frac{7}{100} = 0.07$ – The **0** must be used after the decimal point in 0.07 to hold the tenths and hundredths place. This makes it clear that the denominator is one thousand.

- $\frac{8}{1000} = 0.008$ – The **00** must be used after the decimal point in **0.008** to hold the tenths and hundredths place. This makes it clear that the denominator is one thousand.
- $\frac{19}{10000} = 0.0019$ – The **00** must be used after the decimal point in **0.0019** to hold the tenths and hundredths place. This makes it clear that the denominator is ten thousand.

When changing from a fraction to a decimal:

If there are not enough digits to fill all the decimal places, put zeros between the decimal point and the digits from the fraction – this is called prefixing zeros.

How does this apply to multiplying decimals? Look at the examples.

Example G

$$\begin{array}{rcl}
 0.07 & 2 \text{ decimal places} & \\
 \times 0.5 & 1 \text{ decimal place} & \\
 \hline
 35 & 3 \text{ decimal places} &
 \end{array}$$

Uh oh! There are not enough spots in the answer to make the decimals fit in!

$$\begin{array}{r}
 0.07 \\
 \times \quad 0.5 \\
 \hline
 \mathbf{0035} \quad \leftarrow \\
 3 \ 2 \ 1 \quad \text{count}
 \end{array}$$

Add zeros before your product. It is completely within the rules of math to do that. Then, put the decimal in the correct place.

$$\begin{array}{r}
 0.07 \\
 \times \quad 0.5 \\
 \hline
 \mathbf{0.035} \quad \leftarrow \\
 3 \ 2 \ 1 \quad \text{count}
 \end{array}$$

Example H

$$\begin{array}{r}
 0.07 \quad 2 \text{ decimal places} \\
 \times \quad 0.03 \quad +2 \text{ decimal places} \\
 \hline
 . \_ \_ 75 \quad 4 \text{ decimal places in product} \\
 4 \ 3 \ 2 \ 1 \quad \text{count}
 \end{array}$$

Example I

Look carefully at this one – it is tricky!

$$\begin{array}{r} 0.05 \quad 2 \text{ decimal places} \\ \times \quad 0.8 \quad +1 \text{ decimal place} \\ \hline .\_75 \quad 3 \text{ decimal places in product} \\ 3 \ 2 \ 1 \quad \text{count} \end{array}$$

Because the last digit, the zero, is the result of multiplying 8×5 , you must count it when working out the decimal places to put in the decimal point.

The product is forty thousandths:

$$\frac{40}{1000} = 0.004$$

which can now also be written as four hundredths:

$$\frac{40}{100} = 0.04$$

Note: If you had not counted that zero, you would have written 0.004, which is four thousandths and not correct.

Exercise 5

Find the products. Be certain to place all decimal points correctly.

Example:

$$\begin{array}{r} 0.35 \\ \times 0.13 \\ \hline 105 \\ 350 \\ \hline 0.0455 \end{array}$$

a. $\begin{array}{r} 1.8 \\ \times 0.05 \\ \hline \end{array}$ d. $\begin{array}{r} 500 \\ \times 0.073 \\ \hline \end{array}$ g. $\begin{array}{r} 0.025 \\ \times 0.25 \\ \hline \end{array}$

b. $\begin{array}{r} 300 \\ \times 0.04 \\ \hline \end{array}$ e. $\begin{array}{r} 0.603 \\ \times 0.4 \\ \hline \end{array}$ h. $\begin{array}{r} 3.5 \\ \times 0.018 \\ \hline \end{array}$

c. $\begin{array}{r} 1.3 \\ \times 0.027 \\ \hline \end{array}$ f. $\begin{array}{r} 38.2 \\ \times 1.9 \\ \hline \end{array}$ i. $\begin{array}{r} 0.043 \\ \times 3.3 \\ \hline \end{array}$

$$\begin{array}{r} \text{j.} \quad \begin{array}{r} 0.013 \\ \times \quad 1.4 \\ \hline \end{array} \quad \text{k.} \quad \begin{array}{r} 0.201 \\ \times \quad 2.1 \\ \hline \end{array} \end{array}$$

Exercise 5 Answers

a. 0.0455	e. 36.500	i. 0.063
b. 0.09	f. 0.2412	j. 0.1419
c. 12.00	g. 72.58	k. 0.0182
d. 0.0351	h. 0.00625	l. 0.4221

Multiplying by 10, 100, 1 000, 10 000...

You can see a pattern when we multiply a decimal number by 10, 100, 1 000, 10 000, and so on. Look at the following example and try to find the pattern:

- $45.9264 \times 10 = 459.264$
- $45.9264 \times 100 = 4592.64$
- $45.9264 \times 1000 = 45926.4$
- $45.9264 \times 10000 = 459264$

Do you see a pattern?

When **multiplying** by 10, 100, 1 000, 10 000, etc.,

count the zeros in the 10, 100, 1 000, etc. and move the decimal point that same number of places to the right.

Multiply by 10

When you multiply by 10, move the decimal point one place to the right. Remember that every whole number can have a decimal point at the right.

Example J

- a. $.37 \times 10 = 3.7$
- b. $13 \times 10 = 130.$

Exercise 6

a. $2.2 \times 10 =$

b. $6.67 \times 10 =$

c. $2.1 \times 100 =$

Exercise 6 Answers

a. 22

b. 66.7

c. 210

Multiply by 100

When you multiply by 100, move the decimal point two places to the right. Note that zeros may be needed at the end of the numeral.

Example K

a. $4.2 \times 100 = 420.$

b. $6 \times 100 = 600.$

Exercise 7

a. $5.67 \times 100 =$ c. $25 \times 100 =$

b. $92.737 \times 100 =$

Exercise 7 Answers

a. 567

b. 9 273.7

c. 2 500

Multiply by 1 000

To multiply by 1 000, move the decimal point three places to the right.

Example L

$$4.2 \times 1\,000 = 4\,200.$$

Exercise 7

- a. $6.721 \times 1\,000 =$ c. $0.7246 \times 1\,000 =$
b. $1.56 \times 1\,000 =$ d. $2.1 \times 1\,000 =$

Exercise 7 Answers

- a. 6 721 c. 724.6
b. 1 560 d. 2 100

Exercise 8

Write the products using the short method you now know.

a.	$0.4 \times 10 =$	e.	$0.62 \times 100 =$	i.	$1.934 \times 1\,000 =$
b.	$0.27 \times 10 =$	f.	$0.119 \times 100 =$	j.	$28.9 \times 100 =$
c.	$10 \times 0.926 =$	g.	$100 \times 2.03 =$		
d.	$10 \times 0.09 =$	h.	$100 \times 0.345 =$		

Exercise 8 Answers

a.	4	e.	62	i.	1 934
b.	2.7	f.	11.9	j.	2 890
c.	9.26	g.	203		
d.	0.9	h.	34.5		

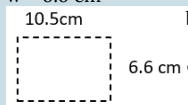
Multiplying Decimals to Determine Area

The area of an object is the measurement of the amount of space the object's surface covers. The area is described in square units.

Exercise 9

Find the area of the rectangles described below. The measures of the length (l) and width (w) have been given. You should draw and label a sketch for each.

- a. $l = 10.5 \text{ cm}$
 $w = 6.6 \text{ cm}$



$$A = l \times w$$

$$A = 10.5 \times 6.6 \text{ cm}$$

$$A =$$

- b. $l = 100.04 \text{ km}$

$$w = 70.2 \text{ km}$$

$$l = 15.5 \text{ mm}$$

$$w = 10.5 \text{ mm}$$

- d. $l = 9.75 \text{ cm}$

$$w = 3.5 \text{ cm}$$

- e. $l = 40.43 \text{ km}$

$$w = 10.9 \text{ km}$$

- f. $l = 19.6 \text{ cm}$

$$w = 2.8 \text{ cm}$$

Exercise 9 Answers

a. 69.3 cm^2

b. 7022.808 km^2

c. 162.75 mm^2

d. 34.125 cm^2

e. 440.687 km^2

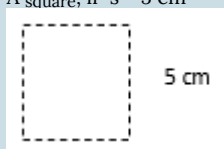
f. 54.88 cm^2

Exercise 10

Find the area of each square described in the questions below. Even though this is a simple square, it is still good practice to draw a picture.

Remember: All four sides of a square are the same length.

- a. A square, if $s = 5$ cm



- b. A square, if $s = 12.5$ km
c. A square, if $s = 1.4$ km
d. A square, if $s = 25.4$ cm

Exercise 10 Answers

- a. 25 cm^2
b. 156.25 km^2
c. 1.96 km^2
d. 645.16 cm^2

Word Problems Using Multiplication of Decimals

Multiplication problems usually give information about one thing and ask you to find a total amount for several of the same things.

Look for this pattern in the following problems. Also, look for keywords.

Review keywords pointing to multiplication:

- product
- total
- altogether
- of

Multiplication by a decimal or fraction often uses the word “of” in word problems.

“Of” usually means multiplying one number by another number.

Example M

Kathy spends 0.25 **of** her salary on rent. Her salary is \$1445 a month. How much is her rent?

$$0.25 \times \$1445 = \$361.25$$

She spends \$361.25 on rent every month.

Remember: It can be very useful to draw a picture to help yourself visualize the problem.

Exercise 11

Solve the following word problems. Estimate first.

- a. One crate of juice costs \$11.73. How much will five crates cost?
 - Estimation: \$ 11.73 $\approx$ \$12,
 $12 \times 5 = 60$
 - Actual Solution:
 - Answer:
- b. It takes Maria 0.75 hours to install a bathroom sink. The owner of the Ramada Hotel wants her to install 35 new sinks. How many hours of labour will Maria charge the hotel for this job?
 - Estimation:
 - Actual Solution:
 - Answer:
- c. Find the longest hallway in your learning centre.

Stand at one end and count how many steps you take until you reach the other end. Write that number here.

- i. How many steps would you take if you walked down your hallway 4.5 times?
 - ii. How many steps would you take if you walked down your hallway 33.68 times?
- d. Jessica's 4×4 truck averages 15.75 kilometres per litre of gas (km/L). How far could her truck travel with 52 L of gas?
 - Estimation:
 - Actual Solution:
 - Answer:
- e. The distance between Jackie's workplace and her home is 4.3 kilometres. She walks to work in the morning and then walks back to her home at the end of the workday, five days a week. How far does she walk each week?
 - Estimation:
 - Actual Solution:
 - Answer:
- f. A new subdivision is being built outside of Terrace. There are 21 new houses. Each house needs 25.2 metric tonnes of crushed rock for the driveway. How much crushed rock will be used to build all the driveways?
 - Estimation:
 - Actual Solution:

- Answer:

- g. Union workers who work on statutory holidays are paid double time and a half. This is 2.5 times their regular pay.
 - i. The employees at the coal terminal earn \$17.37 per hour. How much are they paid per hour if they work on a statutory holiday?
 - ii. Jesse worked a 7.5-hour shift at the coal terminal on Labour Day last year. What were his earnings for that day?
- h. Lucy's aunt in Hartley Bay has a large 50.5 m square vegetable garden. Lucy is going to help her aunt build a barbed wire fence to keep the deer out. The fence will be made with four strands of barbed wire. How much barbed wire should Lucy buy for her aunt? The fence will look like Figure 4.10.2 below.

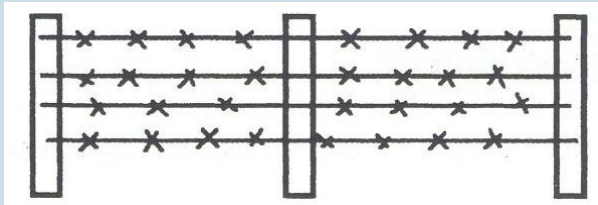


Figure 4.10.2

- Draw a picture:
- Estimation:
- Actual Solution:
- Answer:

- i. The directions on a bottle of carpet shampoo say one bottle will clean 50 m^2 of carpet. Joyce wants to clean the wall-to-wall carpet in three rooms. The living room is 6 m by 4.5 m, the bedroom is 3 m by 4 m, and the hall is 1.2 m by 5 m. How many bottles of carpet shampoo should she buy? (This is a two-part question: Calculate the area of each of the rooms and then find the total area to be cleaned. Remember, Joyce cannot buy a partial bottle!)
- Draw a picture:
 - Estimation:
 - Actual Solution:
 - Answer:

Exercise 11 Answers

- a. Estimation: $\$12 \times 5 \approx \$60$
 Actual Solution: $\$11.73 \times 5 = \$58.65$
 Five packages of juice will cost \$58.65
- b. Estimation: $1 \times 35 \approx 35$ hours
 Actual Solution: $0.75 \times 35 = 26.25$ hours
 It will take Maria 26.25 hours to install the sinks.
- c. Answers will vary. Please ask your instructor to check your work.
- d. Estimation: $16 \text{ km} \times 50 \text{ L} \approx 800 \text{ km}$
 Actual Solution:
 $15.75 \text{ km} \times 52 \text{ L} = 819 \text{ km}$
 The truck could travel 819 km.
- e. Estimation:

$4 \times 10 \text{ km}$ (5 trips there and 5 trips home) ≈ 44 seconds

Actual Solution: $4.3 \text{ km} \times 10 = 43 \text{ km}$

Jackie walks 43 km each week.

f. Estimation: $25 \text{ m} \times 20 \text{ m} \approx 500 \text{ m}$

Actual Solution: $25.2 \text{ m} \times 21 = 529.2 \text{ m}$

There will be 529.2 metric tonnes of crushed rock used for the driveways.

g. Union workers:

i. $\$17.37 \times 2.5 = \$43.425$ per hour

ii. $\$43.425 \times 7.5 \text{ hours} = \$325.69$

Jesse earned \$325.69 on Labour Day.

h. Estimation:

$50 + 50 + 50 + 50 \approx 200 \text{ m}$, $200 \text{ m} \times 4 \approx 800 \text{ m}$

Actual Solution:

$50.5 \text{ m} \times 4 = 202 \text{ m}$, $202 \text{ m} \times 4 = 808 \text{ m}$

Lucy should buy 808 m of barbed wire.

i. Estimation:

$6 \text{ m} \times 5 \text{ m} \approx 30\text{m}^2$

$3 \text{ m} \times 4 \text{ m} \approx 12\text{m}^2$

$1 \text{ m} \times 5 \text{ m} \approx 5\text{m}^2$

$30 \text{ m}^2 + 12 \text{ m}^2 + 5 \text{ m}^2 \approx 47\text{m}^2$

Actual Solution:

$6 \text{ m} \times 4.5 \text{ m} = 27 \text{ m}^2$

$3 \text{ m} \times 4 \text{ m} = 12 \text{ m}^2$

$1.2 \text{ m} \times 5 \text{ m} = 6 \text{ m}^2$

$27 \text{ m}^2 + 12 \text{ m}^2 + 6 \text{ m}^2 = 45 \text{ m}^2$

Joyce should buy just one bottle of carpet cleaner.

Design Your Own House Project Part 2: Roofing Your House

[May 9 – Do you want to keep this section and the one in 4.7? The unit pricing chapter, which has part 3, is not one of the chapters you wanted to include – K]

Mark: /16

Use the graph paper house sketch you made in the Design Your Own House Project Part 1 in 4.7: Subtracting Decimals to do this activity.

- a. What is the area of your entire house? (area = length $\times$ width)
/2
- b. The roof of a house overhangs (sticks out past) the edges of the house by 0.5 m. Add the overhang to the length and width of your house to find the area of the roof. /2
- c. Before covering a roof, builders put down tarpaper, which keeps water out. Look at a hardware store flyer or go online and check prices for tar paper. /1
 - Choose one type of tar paper and write down the cost per roll.
- d. Use your answer for question a to figure out how many rolls you will need to cover the entire roof (remember, you cannot buy part of a roll). /2
- e. Using your answers for questions c and d, calculate what it would cost to cover the entire roof with tar paper. /2
- f. Use a flyer or look at hardware stores online and choose a roofing material. /1
 - Write down your choice.
- g. Roofing shingles are often sold in **bundles** that will cover 3 m^2 .

Metal roofing materials are sold in sheets that will cover 50 square feet, which is the equivalent of 15 m^2 . What is the price per bundle or sheet for the material you chose in question f?

/2

- h. Using your answer from question a, calculate how many bundles or sheets of roofing material you will need to cover your roof. /2
- i. Using your answers from questions g and h, calculate the total cost of roofing your house. /2

When you have finished this project, put your graph paper somewhere safe, because you will be using it again at the end of Unit 5 Topic A.

Marking Checklist for House Project Part 2

Calculations:

- The area of the house has been calculated correctly.
- The area of the roof has been calculated correctly.
- The number of rolls of tar paper has been calculated correctly.
- The cost of covering the roof in tar paper has been calculated correctly.
- The number of bundles or sheets of roofing material has been calculated correctly.
- The total cost of roofing the house has been calculated correctly.

4.10: Practice Questions

1. Find the product.

$$\begin{array}{r} 66 \\ \times 0.7 \\ \hline \end{array}$$

$$\begin{array}{r} 6.5 \\ \times 0.6 \\ \hline \end{array}$$

$$\begin{array}{r} 7.25 \\ \times 12 \\ \hline \end{array}$$

$$\begin{array}{r} 19.5 \\ \times 0.07 \\ \hline \end{array}$$

2. Solve the following word problem.

- a. Rhonda bought a 3.4 kg roast at \$2.89 per kilogram. What was the cost of her roast? (Round to the nearest cent.)

- Estimation:
- Actual Solution:

4.10: Practice Answers

- a. 46.2
b. 87

- c. 3.9
d. 1.365

- a. Estimate: $3 \times 3 = \$9$
b. Actual solution: $3.4 \times 2.89 = 9.826$ (round to nearest cent: \$9.83).
Rhonda paid \$9.83 for her roast.

Attribution

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4.11: Multiplying Decimals Review

1. Put the decimals in the correct place in the answer.

$$\begin{array}{r} 66 \\ \times 0.9 \\ \hline 594 \end{array}$$

$$\begin{array}{r} 19.5 \\ \times 3.47 \\ \hline 67665 \end{array}$$

2. Find the product.

$$\begin{array}{r} 0.78 \\ \times 1.4 \\ \hline \end{array}$$

$$\begin{array}{r} 7.94 \\ \times 4.02 \\ \hline \end{array}$$

$$\begin{array}{r} 3.469 \\ \times 4.63 \\ \hline \end{array}$$

3. Multiply by 10, 100, or 1 000.

$$\begin{array}{ll} \text{a. } 0.53 \times 10 = & \text{c. } 0.472 \times 10 \overset{\text{e}}{=} 46.3214 \times 100 = \\ \text{b. } 7.89 \times 10 = & \text{d. } 3.7834 \times 100 \overset{\text{f}}{=} 42.42 \times 100 = \end{array}$$

4. Solve the following word problems.

- Mark rides his bike at a speed of 11.4 km an hour. How much distance will he cover in a 7.5-hour bike ride?
- Johnny spends 3.25 hours cleaning the civic centre twice a week. How many hours does he work each week?
- Joe and Kat's rectangular family tent measures 3.4 m by 2.9 m. How much area does the floor of the tent cover?

4.II: Review Answers

1. Put the decimals in the correct place in the answer.

a. 59.4

b. 67.665

2. Find the product.

a. a 1.092

c. 31.9188

b. 16.06147

3. Multiply by 10, 100, or 1 000.

a. 5.3

c. 4.72

e. 4 632.14

b. 78.9

d. 378.34

f. 4 242

4. Solve the following word problems

a. 85.5 km

c. 9.86 m sq.

b. 6.5 hours

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4.12: Dividing Decimals

Dividing decimals uses the same method that you learned for dividing whole numbers.

Vocabulary review:

$$\begin{array}{r} \text{quotient} \\ \text{divisor} \overline{) \text{dividend}} \end{array}$$

$$\text{dividend} \div \text{divisor} = \text{quotient}$$

Using the above diagram, write the definitions.

Divisor:

Dividend:

Quotient:

Remember to use zeros to hold the places in the **quotient** if there is no other digit.

$$\begin{array}{r}
 \downarrow \\
 104 \\
 9 \overline{)936} \\
 \underline{9 \downarrow} \\
 03 \\
 \underline{0 \downarrow} \\
 36 \\
 \underline{36} \\
 0
 \end{array}$$

Where Do You Place the Decimal Point?

Dividing decimals follows almost the same steps as dividing whole numbers. This section will show you two ways to figure out where to place the decimal point.

Method 1

One way is to estimate the quotient using rounded whole numbers.

Example A

$$17.7 \div 3$$

Estimate:

$$17.7 \div 3 \approx 18 \div 3 = 6$$

This tells us that the correct answer will be around 6 (which is one whole number place).

We know that the answer will not be around 0.6 or 60.

If we take the decimals out and just divide the digits, the answer is 59.

$$\begin{array}{r} 59 \\ 3 \overline{)177} \end{array}$$

The estimate shows that the decimal point will come after one whole number.

$$17.7 \div 3 = 5.9$$

Example B

$$137.88 \div 18$$

Estimate:

$$137.88 \div 18 \approx 140 \div 20 = 7$$

The answer will be around 7. It will not be around 0.7, 70, or 700.

If we take the decimals out and just divide the digits, the answer is 7.66.

$$\begin{array}{r} 7.66 \\ 18 \overline{)137.88} \end{array}$$

The estimate shows that the decimal point will come after one whole number.

$$137.88 \div 18 = 7.66$$

To check the accuracy of your division, multiply the quotient by the **divisor**.

$$\text{dividend} \div \text{divisor} = \text{quotient}$$

$$\text{quotient} \times \text{divisor} = \text{dividend}$$

The **product** will equal the **dividend** if your arithmetic is correct.

$$\begin{array}{r} 23.72 \div 8 = 2.965 \\ \times \quad 8 \\ \hline 23.720 \end{array}$$

Exercise 1

The division has been done. Your task is to put the decimal point in the quotient by doing a whole number estimate of the question.

Examples:

$$\begin{array}{r} 10.8 \\ 2 \overline{)21.6} \end{array} \left(\begin{array}{r} 10 \\ 2 \overline{)20} \end{array} \right) \quad \begin{array}{r} 1.3 \\ 8 \overline{)10.4} \end{array} \left(\begin{array}{r} 1 \\ 8 \overline{)10} \end{array} \right)$$

a.	$\begin{array}{r} 236 \\ 6 \overline{)14.16} \end{array}$	e.	$\begin{array}{r} 864 \\ 7 \overline{)60.48} \end{array}$	i.	$\begin{array}{r} 243 \\ 18 \overline{)43.74} \end{array}$
b.	$\begin{array}{r} 53 \\ 7 \overline{)37.4} \end{array}$	f.	$\begin{array}{r} 182 \\ 6 \overline{)10.92} \end{array}$	j.	$\begin{array}{r} 43 \\ 21 \overline{)90.3} \end{array}$
c.	$\begin{array}{r} 434 \\ 4 \overline{)173.6} \end{array}$	g.	$\begin{array}{r} 3369 \\ 2 \overline{)6.738} \end{array}$		
d.	$\begin{array}{r} 345 \\ 5 \overline{)17.25} \end{array}$	h.	$\begin{array}{r} 18 \\ 37 \overline{)66.6} \end{array}$		

Exercise 1 Answers

- a. 2.36
- b. 5.3
- c. 43.4
- d. 3.45

- e. 8.64
- f. 1.82
- g. 3.369
- h. 1.8

- i. 2.43
- j. 4.3

Method 2 (If the Divisor Is a Whole Number)

Have you found the shortcut?

If the divisor is a whole number, put the decimal point in the quotient right above the decimal point in the dividend.

Then just go ahead and divide, ignoring the decimal point altogether.

Example C

$$18.45 \div 9 =$$

$$\begin{array}{r} . \\ 9 \overline{)18.45} \end{array}$$

```

\begin{array}{l}\hspace{0.7cm}2.05\\9
\enclose{longdiv}{18.45}\hspace{0.1cm}-18\
\downarrow\\\hline\
\hspace{0.7cm}0\,,\,\mathbf{4}\hspace{0.7cm}-0\
\downarrow\\\hline\hspace{1cm}4\,,\,\mathbf{5}\
\\\hspace{0.7cm}-45\ \ \ \hline\hspace{1.3cm}0\
\end{array}

```

Example D

$$420.75 \div 25 =$$

$$\begin{array}{r} \cdot \\ 25 \overline{)420.75} \end{array}$$

```

\begin{array}{l}\hspace{0.7cm}16.83\\25
\enclose{longdiv}{420.75}\hspace{0.5cm}-25\
\downarrow\\\hline\
\hspace{0.8cm}17\,,\,\mathbf{0}\hspace{0.5cm}-150\
\downarrow\\\hline\hspace{1.1cm}20\,,\,\mathbf{7}\
\\\hspace{0.8cm}-200\downarrow\ \ \ \hline\
\hspace{1.6cm}7\,,\,\mathbf{5}\ \ \ \hspace{1.3cm}-75\
\\\hline\hspace{1.9cm}0\ \end{array}

```

Exercise 2

Find the quotients. Check the answer by multiplying the quotient by the divisor.

$$\begin{array}{r}
 0.24 \\
 60 \overline{)14.40} \\
 \underline{12 \downarrow} \\
 24 \downarrow \\
 \underline{240} \\
 0
 \end{array}
 \quad \text{Check:} \quad
 \begin{array}{r}
 0.24 \\
 \times \quad 60 \\
 \hline
 14.40
 \end{array}$$

a. $91 \overline{)263.9}$

b. $25 \overline{)207.5}$

c. $57 \overline{)96.9}$

d. $63 \overline{)176.4}$

Exercise 2 Answers

a. 2.9

b. 8.3

c. 1.7

d. 2.8

Method 3 (If the Divisor Is a Decimal)

The past section taught us that if the divisor is a whole number, we put the decimal point in the quotient right above the decimal point in the dividend. Then we just go ahead and divide.

But what if the divisor has a decimal in it? A decimal divisor must be changed to a whole number before we can divide.

Remember: Multiplying by 10, 100 or 1 000, and so on, moves the decimal point as many places to the right as there are zeros in the 10, 100, 1 000.

- $46 \times 10 = 460, 4.6 \times 10 = 46.$

The decimal moved over one spot because of the one zero in 10

- $46 \times 100 = 4\,600, 4.6 \times 100 = 460$

The decimal moved over two spots because of the two zeros in 100

When the divisor and dividend are multiplied by the same number, the quotient is not changed.

Example E

$$0.05 \overline{)1.255}$$

If the divisor has a decimal, do this:

Step 1: Move the decimal point in the divisor as many places to the right as needed to make a whole number. In this example, the decimal is moved over two places, which is like multiplying by 100.

$$005. \overline{)1.255}$$

Step 2: Now move the decimal point in the dividend the same number of places to the right (like multiplying by 100).

$$005. \overline{)125.5}$$

Step 3: Put the decimal point in the quotient directly above the new place in the dividend. Now divide.

$$\begin{array}{r} 25.1 \\ 5 \overline{)125.5} \end{array}$$

$$1.255 \div 0.05 = 25.1$$

Note: Zeros may have to be put at the end of the dividend when you move the decimal point.

Example F

$$48.6 \div 0.24 = 0.24 \overline{)48.6} \text{ changes to } 24 \overline{)4860}.$$

There is nothing here, so we must add a zero.

Remember: If the dividend is a whole number, put a decimal to the right of it first, and then move the decimal as needed to match what you did to the divisor. You will need to add zeros to hold the places.

Example G

$$3.6 \div 1.8 =$$

Step 1: Put a decimal to the right of the dividend.

$$1.8 \overline{)36.}$$

Step 2: Move the decimals for both numbers one place to the left. This is like multiplying both numbers by ten. Add a zero to the dividend to hold the tens place.

$$18. \overline{)360.}$$

Step 3: Put the decimal directly above the decimal in the dividend.

$$18. \overline{)360.}$$

Exercise 3

Find the quotients.

a. $3.4 \overline{)3.808}$

c. $0.04 \overline{)15.2}$

b. $6.6 \overline{)19.14}$

d. $0.67 \overline{)6.164}$

Exercise 3 Answers

a. 1.12

b. 2.9

c. 380

d. 9.2

If you are having any difficulty with this exercise, ask your instructor for help before you go any further.

Exercise 4

Now, try these:

a. $0.5 \overline{)0.2635}$

b. $1.7 \overline{)15.47}$

c. $0.003 \overline{)42.12}$

d. $0.33 \overline{)0.1452}$

Exercise 4 Answers

a. 0.527

b. 9.1

c. 14 040

d. 0.44

Exercise 5

Set the question up for long division and find the quotient. Check your answers by multiplying the quotient with the divisor. The product should equal the dividend.

Example: $0.2448 \div 0.008 =$

$$\begin{array}{r} 30.6 \\ 0.008 \overline{)0.224.8} \\ \underline{24 \downarrow \downarrow} \\ 04 \downarrow \\ \underline{0 \downarrow} \\ 48 \\ \underline{48} \\ 0 \end{array}$$

Check:

$$\begin{array}{r} 30.6 \\ \times 0.008 \\ \hline 0.2448 \end{array}$$

a. $2.3412 \div 0.6 =$

b. $25.6 \div 0.008 =$

c. $2.176 \div 3.4 =$

d. $205 \div 4.1 =$

e. $2.1122 \div 59 =$

Exercise 5 Answers

a. 3.902

b. 3 200

c. 0.64

d. 50

e. 0.0358

What About Remainders?

The questions that you have been practicing all work out evenly. But, as you know, the world is seldom perfect, and division questions often have **remainders**! For everyday uses of mathematics, answers

to the hundredths or thousandths decimal places are accurate enough.

If the division problem does not work out evenly:

1. Do the long division until you have worked out three or four decimal places in your quotient. Add zeros to the decimal in the dividend as necessary.
2. Round the quotient to the nearest tenth, hundredth, or thousandth as you are asked or as you need for your own use. Review rounding if you need to. (In this course, round to the nearest thousandth unless you are asked otherwise.)
3. When planning to round the quotient, do the long division only to one decimal place past where you will round. It is not necessary to divide any further.
 - If rounding the quotient to the nearest tenths, divide to the hundredths place (2 decimal places).
 - If rounding the quotient to the nearest hundredths, divide to the thousandths place (3 decimal places).
 - If rounding the quotient to the nearest thousandths, divide to the ten-thousandths place (4 decimal places).

Example H

$$422 \div 1.7 =$$

```

\begin{array}{l}\hspace{0.7cm}248.2352\1.7\\
\enclose{longdiv}{422.00000}\hspace{0.4cm}-34\\
\downarrow\\\hline\hspace{1cm}8\,\,\mathbf{2}\hspace{0.8cm}-68\\
\downarrow\\\hline\hspace{1.2cm}14\,\,\mathbf{0}\\\hspace{1cm}-136\downarrow\\\hline\hspace{1.8cm}4\,\,\mathbf{0}\hspace{1.6cm}-34\\
\downarrow\\\hline\hspace{2.2cm}6\,\,\mathbf{0}\\\hspace{2cm}-51\downarrow\\\hline\hspace{2.6cm}9\,\,\mathbf{0}\hspace{2.4cm}-85\\
\downarrow\\\hline\hspace{3cm}5\,\,\mathbf{0}\\\hspace{2.8cm}-34\\\hline\hspace{3.2cm}16\\
\end{array}

```

The quotient 248.2352 will round this way:

- To the nearest thousandth: 248.235
- To the nearest hundredth: 248.24
- To the nearest tenth: 248.2

Example I

$$12.5 \div 7 =$$

```

\begin{array}{l}\hspace{0.7cm}1.7857\\7\\
\enclose{longdiv}{12.5000}\hspace{0.4cm}-7\\
\downarrow\\\hline\hspace{0.8cm}5\,,\,\mathbf{5}\hspace{0.6cm}-49\\
\downarrow\\\hline\hspace{1.1cm}6\,,\,\mathbf{0}\\
\hspace{0.8cm}-56\downarrow\\\hline\hspace{1.4cm}4\,,\,\mathbf{0}\hspace{1.2cm}-35\\
\downarrow\\\hline\hspace{1.8cm}5\,,\,\mathbf{0}\\
\hspace{1.6cm}-49\\\hline\hspace{2.2cm}1\\
\end{array}

```

The quotient 1.7857 will round this way:

- To the nearest thousandth: 1.786
- To the nearest hundredth: 1.79
- To the nearest tenth: 1.8

Always round money to the nearest cent.

Example J

$$\$47.26 \div 3 =$$

$$\begin{array}{l} \\ \enclose{longdiv}{47.260} \\ \downarrow \\ \hspace{0.7cm}1\,,\,\mathrm{\bf 7} \\ \downarrow \\ \hspace{0.9cm}-2\downarrow \\ \hspace{1.5cm}1\,,\,\mathrm{\bf 6} \\ \hspace{1.7cm}10\,,\end{array}$$
$$\$15.753 \approx \$15.75$$

Sometimes numbers repeat when you divide. This will go on forever — to **infinity**.

$$100 \div 3 =$$

```

\begin{array}{l}\hspace{0.7cm}33.333\3\\
\enclose{longdiv}{100.000}\hspace{0.3cm}-9\downarrow\\\hline\hspace{0.7cm}1\,\mathbf{0}\hspace{0.7cm}-9\downarrow\\\hline\hspace{1.1cm}1\,\mathbf{0}\hspace{1.1cm}-0\\\hline\hspace{1.5cm}10\end{array}

```

To show that the 3 keeps repeating as a decimal fraction, put a $\cdot$ (dot) or a $\bar{}$ (bar) above the repeating decimal digit.

$33.\dot{3}$ or $33.\bar{3}$

Sometimes, a group of digits will repeat. Put a bar above the repeating decimal digits. For example:

$$2.341341341341 = 2.\overline{341}$$

Exercise 6

Use long division to find the quotient. Round the quotient to the nearest tenth.

a. $7.359 \div 1.3 =$	c. $4.93 \div 6 =$
b. $15.68 \div 2.2 =$	d. $59 \div 8 =$

Exercise 6 Answers

a. 5.7

b. 7.1

c. 0.8

d. 7.4

Exercise 7

Divide and round the quotient to the nearest hundredth.

a. $43.893 \div 1.1 =$

c. $0.4474 \div 0.7 =$

b. $1.9525 \div 0.6 =$

d. $10.48 \div 1.5 =$

Exercise 7 Answers

a. 39.90

b. 3.25

c. 0.64

d. 6.99

Dividing by 10, 100, 1 000, 10 000, Etc.

Multiplication and division are opposite operations. Multiplying by ten, hundred, etc., moves the decimal point the same number of decimal places to the **right** as there are zeros in the 10, 100, 1000, etc. Moving a decimal point to the right gives a larger number.

Therefore, dividing by ten, hundred, etc., must move the decimal point to the **left**. Remember that moving a decimal point to the left gives a smaller number.

Study the following examples:

- $4.6 \div 10 = .46 = 0.46$
- $29.6 \div 10 = 2.96 = 2.96$
- $27.4 \div 100 = .274 = 0.274$
- $185.4 \div 100 = 1.854 = 1.854$
- $325 \div 1\,000 = 325. \div 1\,000 = .325 = 0.325$
- $2567.3 \div 1\,000 = 2.5673 = 2.5673$

To divide by a 10, 100, 1 000, 10 000, etc., move the decimal point the same number of places to the **left** as there are zeros in the divisor.

You may need to prefix zeros. Look at these examples:

- $0.3 \div 10 = .0.3 = 0.03$
- $1.75 \div 100 = .\_1.75 = 0.0175$
- $0.5 \div 1\,000 = . < \_\_.5 = 0.0005$

Exercise 8

Write the quotient. Use the short method you have just learned.

a. $7 \div 10 =$	g. $183.75 \div 1\,000 =$	m. $0.1 \div 1\,000 =$
b. $14 \div 100 =$	h. $2374.5 \div 1\,000 =$	n. $100.1 \div 10 =$
c. $6.5 \div 10 =$	i. $0.63 \div 10 =$	o. $16.2 \div 100 =$
d. $74.35 \div 10 =$	j. $7.46 \div 100 =$	p. $5692.1 \div 10 =$
e. $43.2 \div 100 =$	k. $0.035 \div 10 =$	q. $0.025 \div 100 =$
f. $147.6 \div 100 =$	l. $366 \div 100 =$	r. $3.31 \div 10 =$

Exercise 8 Answers

a. 0.7	g. 0.18375	m. 0.0001
b. 0.14	h. 2.3745	n. 10.01
c. 0.65	i. 0.063	o. 0.162
d. 7.435	j. 0.0746	p. 569.21
e. 0.432	k. 0.0035	q. 0.00025
f. 1.476	l. 3.66	r. 0.331

Word Problems Using Division of Decimals

Division problems usually give information about groups of things and ask you to find the information for one thing.

Review keywords pointing to division:

- separated
- split
- cut
- shared
- What is the cost per...?
- unit price
- what is the distance per...?
- average (speed, cost, weight, time)

Exercise 9

Solve these division problems.

Look carefully for the pattern of the problems and underline any **keywords** which point to division. Do an estimation before you find the actual solution.

Example: Joanne's little car has a 44.5 L tank. She can drive 525 km on a tank of gas. What is the average distance she can travel per litre of gas? (The answer will be kilometres per litre, so you must divide the kilometres by the litres.)

- Estimation: $520 \text{ km} \div 40 \text{ L} \approx 13 \text{ km/L}$
- Actual Solution:

$$\begin{array}{r} 11.798 \\ 445 \overline{)5250.0} \end{array} \quad \leftarrow \text{move the decimal over one}$$

spot in both numbers and add the appropriate number of zeros.

$$525 \text{ km} \div 44.5 \text{ L} = 11.798 \text{ km/L}$$

- Answer: Joanne's car travels an average of 11.798 kilometres per litre of gas.

- a. Three villages are organizing a feast in Prince Rupert to celebrate the start of the All Native Basketball Tournament (ANT). The total cost for the feast will be \$8 972.43. How much will each community pay?
 - Estimation:
 - Actual Solution:
 - Answer:

- b. If you can solve 30 math questions in 1.5 hours, how long would it take to solve just one?

Hint: You want the average time per question. The answer will be less than one hour, so it may be easier to work out the problem using minutes. Change hours to minutes by multiplying the hours by 60 minutes. [1.5 hours $\times$ 60 minutes = number of minutes]

 - Estimation:
 - Actual Solution:
 - Answer:

- c. During her shift at the container port, Lucinda used a forklift to load 3 675.6 kilograms of crated goods onto four and a half pallets. How many kilograms did

she load onto each pallet?

- Estimation:
- Actual Solution:
- Answer:

- d. Murray buys a double-double from Tim Hortons before class every morning and another on his lunch break. On weekends, he only has one each day. He spends \$113.40 in four and a half weeks. How much does he pay for each coffee?

- Estimation:
- Actual Solution:
- Answer:

Exercise 9 Answers

- a. Estimation: $9\ 000 \div 3 \approx 3\ 000$

Actual Solution: $8972.43 \div 3 = 2\ 990.81$

Answer: Each village will pay \$2 990.81.

- b. Estimation:

$90\text{ minutes} \div 30\text{ questions} \approx 3\text{ minutes}$

Actual Solution:

$90\text{ minutes} \div 30\text{ questions} = 3$

Answer: It takes you 3 minutes to solve one math question.

- c. Estimation: $3\ 700\text{ kg} \div 5\text{ kg} \approx 740$

Actual Solution:

$3\ 675.6\text{ kg} \div 4.5\text{ kg} = 816.8\text{ kg}$

Answer: Lucinda loaded 816.8 kg onto each pallet.

d. Step 1:

How many coffees per week? 12

How many coffees in 4.5 weeks? 54

Step 2:

Estimation: $100 \div 50 = 2$

Actual Solution: $113.40 \div 54 = 2.10$

Answer: Murray pays \$2.10 for each coffee.

4.12: Practice Questions

1. Find the quotients.

a. $4 \overline{)17.6}$

b. $0.3 \overline{)396}$

2. Divide and round the quotient.

a. To the nearest tenth:

$$2.3 \overline{)10.4}$$

b. To the nearest hundredth:

$$0.12 \overline{)0.4739}$$

3. Find the quotients.

a. $51 \div 10 =$

c. $62.811 \div 1\,000 =$

b. $81.81 \div 100 =$

4. Solve the following word problems.

- a. A train travelled 252.5 km in 4.25 hours. What was its average speed in kilometres per hour?

- Estimation:
- Actual Solution:
- Answer:

- b. The office manager ordered t-shirts for the staff for Orange Shirt Day. The shirts cost \$181.30 in total. How much should the manager charge each of the fourteen staff members for a shirt?

- Estimation:
- Actual Solution:
- Answer:

4.12: Practice Answers

1. Find the quotients.

a. 4.4

b. 1 320

2. Divide and round the quotient.

a. 4.5

b. 3.95

3. Find the quotients.

a. 5.1

c. 0.062811

b. 0.8181

4. Solve the following word problems.

a. Answers.

- Estimation:

$$250 \text{ km} \div 5 \text{ hours} \approx 50 \text{ km/hr}$$

- Actual Solution:
 $252.5 \text{ km} \div 4.25 \text{ hours} = 59.411764 \text{ km/hr}$
- Answer: The average speed is 59.41 kilometres per hour.

b. Answers.

- Estimation: $\$180 \div 15 \approx \$12$
- Actual Solution: $\$181.30 \div 14 = \$12.95$
- Answer: The manager should charge each team member \$12.95.

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4.13: Dividing Decimals Review

1. Write the decimal in the correct place in the quotient.

a.
$$\begin{array}{r} 864 \\ 7 \overline{)60.48} \end{array}$$

c.
$$\begin{array}{r} 205 \\ 9 \overline{)18.45} \end{array}$$

b.
$$\begin{array}{r} 43 \\ 21 \overline{)90.3} \end{array}$$

d.
$$\begin{array}{r} 024 \\ 60 \overline{)14.4} \end{array}$$

2. Find the quotient.

a.
$$34 \overline{)179.146}$$

c.
$$57 \overline{)30.21}$$

b.
$$75 \overline{)6947.25}$$

d.
$$75 \overline{)61.5}$$

3. Find the quotient.

a.
$$3.4 \overline{)23.46}$$

c.
$$1.005 \overline{)0.25929}$$

b.
$$2.1 \overline{)0.1134}$$

d.
$$0.72 \overline{)2.58768}$$

4. Find the quotient. Round the quotient to the nearest hundredth.

a. $14.71 \div 3.1 =$

c. $4.93 \div 6 =$

b. $81.13 \div 12 =$

5. Divide. Round your answer to the nearest tenth.

a. $98.9 \div 1.3 =$

c. $0.64 \div 3 =$

b. $1.742 \div 3.2 =$

- a. $53.4 \div 10 =$ d. $1.22 \div 100 =$ ~~g~~ $62.911 \div 100 =$
b. $0.34 \div 100 =$ ~~e~~ $124.32 \div 100 =$ ~~h~~ $4.2 \div 1\,000 =$
c. $10\,000 \div 1\,000 =$ ~~f~~ $12.12 \div 1\,000 =$

- Len pays \$35.89 each month to pay off his interest-free loan of \$304.73. How many months will it take to pay off the loan?
- The Robinson family children want to adopt a dog from the BC SPCA. The four children will split the cost of the dog evenly (see Table 4.13.1). What will each child pay?

Cost	Details
\$395.36	Adoption cost
\$159.30	Vet care
\$67.49	Immunizations
\$38.99	First month of food
\$278.43	Extra gear a dog needs (collars, leash, toys, crate, and a bed)
\$30.00	Licence fee

4.13: Review Answers

- a. 8.64 c. 2.05
b. 4.3 d. 0.24

- 4.13: Dividing Decimals Review | 823

- a. 5.269
- b. 92.63
- c. 0.53
- d. 0.82

3. Find the quotient.

- a. 6.9
- b. 0.054
- c. 0.258
- d. 3.594

4. Find the quotient.

- a. 4.75
- b. 6.76
- c. 0.82

5. Divide.

- a. 76.1
- b. 0.5
- c. 0.2

6. Divide by 10, 100 or 1 000.

- a. 5.34
- b. 0.0034
- c. 10
- d. 0.0122
- e. 1.2432
- f. 0.01212
- g. 0.62911
- h. 0.0042

7. Solve the following word problems.

- a. It will take Len 8.49 months to pay off his loan (so really 9 months).
- b. The children will pay \$242.40 each.

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CHAPTER 5: RATIO, PROPORTION, & PERCENT

5.1: Writing Ratios

Chapter 5: Learning Outcomes

By the end of this chapter, you should be able to:

- Read, write, interpret and compare ratios.
- Read, write and identify proportions and use them to solve problems.
- Use ratio and proportion to solve problems involving:
 - Finding percent when part and whole are known
 - Finding part when percent and whole are known
 - Finding whole when part and percent are known

Introduction to Ratios

Ratio is a comparison of one number or quantity with another number or quantity. Ratios show the relationship between the quantities.

Ratio is pronounced “rā’shō” or it can be pronounced “rā’shēō.” Check out the following YouTube video to listen to someone pronounce the word:

How To Pronounce Ratio – Pronunciation Academy
[22 s] (Pronunciation Academy, 2015).

You often use ratios. For example:

- **Scores in games** are ratios. For example, the Penguins won 4 to 3, or the Canucks lost 1 to 5.
- **Directions for mixing** can be ratios. For example, use 1 egg to each cup of milk or mix 25 parts gas to 1 part oil for the motorcycle.
- **Betting odds** are given as ratios. For example, Black Jade is a 3 to 1 favourite or the heavyweight contender is only given a 2 to 5 chance to win.
- **The scale at the bottom of maps** is a ratio. For example, 1 centimetre represents 10 kilometres.
- **Prices** are often given as ratios. For example, 100 grams for \$0.79 or 2 cans for \$1.85.

For ratios to have meaning, you must know what is being compared and the units being used. Read these examples of ratios and the units used. A general ratio may say “parts” for the units.

- It rained for four days and was sunny for three days last week. The ratio of rainy days to sunny days was **4 : 3**. (**4 : 3** is properly read as “4 is compared to 3” but is often read as “4 to 3”).
- The class has 12 men and 15 women registered. The ratio of men to women in the class is **12 : 15**.

- At the barbeque, 36 hot dogs and 18 hamburgers were eaten. The ratio of hot dogs eaten to hamburgers eaten is **36 : 18**.
- The class spends 3 hours on English and 2 hours on math each day. The ratio of time spent on English compared to math is **3 : 2**.

Exercise 1

Write the ratios asked for in these questions using the : symbol (for example, **4 : 1**). Write the units and what is being compared beside the ratio.

Example: Powdered milk is mixed using 1 part of milk powder to 3 parts of water. Write a ratio to compare the milk powder to the water.

Answer: **1 : 3** – 1 part of milk powder to 3 parts of water.

- One kilogram of ground beef will make enough hamburger for 5 people. Write a ratio to express the amount of ground beef for hamburgers to the number of people.
- Seventy-five vehicles were checked by the police. Fifteen vehicles did not meet the safety standards, but 60 of them did. Write a ratio comparing the unsafe vehicles to the safe vehicles.

- c. The recipe says to roast a turkey according to its weight. For every kilogram, allow 40 minutes of cooking. Write a ratio comparing time to weight.
- d. The 4-litre pail of semi-transparent oil stain should cover 24 square metres of the house siding if the wood is smooth. Write the ratio comparing the quantity of stain to the smooth wood surface area.
- e. The same 4 L of stain will only cover 16 square metres of the house siding if the wood is rough. Write that ratio.

Exercise 1 Answers

- a. **1 : 5** 1 kg of beef to 5 people
- b. **15 : 60** 15 unsafe vehicles to 60 safe vehicles
- c. **40 : 1** 40 minutes to 1 kg of turkey
- d. **4 : 24** 4 L of stain to 24 m² of smooth wood
- e. **4 : 16** 4 L of stain to 16 m² of rough wood

The numbers you have been using to write the ratios are called the terms of the ratio.

The order that you use to write the terms is very important. Read a ratio from left to right; the order must match what the numbers mean. For example, 3 scoops of coffee to 12 cups of water must be written as **3 : 12** as a ratio because you are comparing the quantity of coffee to the amount of water.

If you wish to talk about the amount of water compared to the coffee you have, you would say, “Use 12 cups of water for every 3 scoops of coffee,” and the ratio would be written **12 : 3**.

Ratios can be written three different ways:

1. Using the $:$ symbol – $2 : 5$
2. As a common fraction – $\frac{2}{5}$
 - The first number in the ratio is the numerator; the second number is the denominator.
 - Ratios written as a common fraction are read as a ratio, not as a fraction. Say “2 to 5,” not “two-fifths.”
3. Using the word “to” – 2 to 5

Exercise 2

Using the ratios you wrote in Exercise 1, write the ratios in the three ratio forms:

- $:$
- common fraction
- to

Exercise 2 Answers

b. $1 : 5$ or $\frac{1}{5}$ or 1 to 5

c. $15 : 60$ or $\frac{15}{60}$ or 15 to 60

d. $40 : 1$ or $\frac{40}{1}$ or 40 to 1

e. $4 : 24$ or $\frac{4}{24}$ or 4 to 24

f. $4 : 16$ or $\frac{4}{16}$ or 4 to 16

Exercise 3

Use the diagram in Figure 5.1.1 to write a ratio comparing the quantity of each shape, as asked.

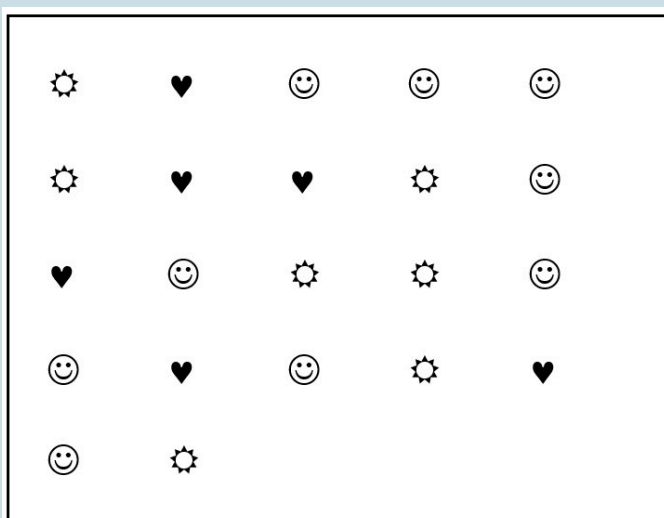


Figure 5.1.1

a.

😊 to ♥

b.

☀ to 😊

c.

♥ to 😊

Exercise 3 Answers

- a. $9 : 6$ or $\frac{9}{6}$ or 9 to 6
- b. $7 : 9$ or $\frac{7}{9}$ or 7 to 9
- c. $6 : 9$ or $\frac{6}{9}$ or 6 to 9

Equivalent Ratios

Like equivalent fractions, equivalent ratios are equal in value to each other.

$$10 : 100 = 1 : 10$$

Ratios can be written as common fractions. It is convenient to work with ratios in the common fraction form.

You can then easily find:

- equivalent ratios in higher terms
- equivalent ratios in lower terms
- a missing term

Example A

Express **4 : 5** in higher terms.

$$4 : 5 = \frac{4}{5} \longrightarrow \frac{4}{5} \times \left(\frac{2}{2}\right) \longrightarrow \left(\frac{4 \times 2}{5 \times 2}\right) \longrightarrow \frac{8}{10}$$

4 : 5 is equivalent to **8 : 10**

Example B

Express **3 : 6** in lower terms.

$$3 : 6 = \frac{3}{6} \longrightarrow \frac{3}{6} \div \left(\frac{3}{3}\right) \longrightarrow \left(\frac{3 \div 3}{6 \div 3}\right) \longrightarrow \frac{1}{2}$$

3 : 6 is equivalent to **1 : 2**

To find equivalent ratios in higher terms, multiply each term of the ratio by the same number.

To find equivalent ratios in lower terms, divide each term of the ratio by the same number.

Exercise 4

Write equivalent ratios in any higher term. You may want to write the ratio as a common fraction first. Ask your instructor to mark this exercise.

Example:

$$5 : 6 = \frac{5}{6} \times \left(\frac{3}{3}\right) = \left(\frac{5 \times 3}{6 \times 3}\right) = \frac{15}{18} = 15 : 18$$

a. $4 : 3$

b. $10 : 2$

c. $50 : 1$

d. $9 : 4$

e. $3 : 5$

Exercise 4 Answers

See your instructor.

Exercise 5

Write these ratios in lowest terms — that is, simplify the ratios.

Example:

$$4 : 12 = \frac{4}{12} \div \left(\frac{4}{4} \right) = \left(\frac{4 \div 4}{12 \div 4} \right) = \frac{1}{3} = 1 : 3$$

a. $10 : 5$

d. $6 : 14$

g. $16 : 8$

b. $7 : 21$

e. $2 : 4$

c. $20 : 5$

f. $6 : 3$

Exercise 5 Answers

Ratios written as a common fraction or using the word

“to” will also be correct in this exercise. The terms must be the same.

a. $1 : 3$

b. $2 : 1$

c. $1 : 3$

d. $4 : 1$

e. $3 : 7$

f. $1 : 2$

g. $2 : 1$

h. $2 : 1$

Exercise 6

Using a colon, write a ratio in the lowest terms for the information given.

- a. In the class of 25 students, only 5 are smokers. Write the ratio of smokers to non-smokers in the class. (**Note:** you must first calculate the number of non-smokers.)
- b. The police issued 12 roadside suspensions to drivers out of the 144 who were checked in the roadblock last Friday. Write the ratio of suspended drivers to the number checked.
- c. Twenty-seven students registered for the course and 24 completed it. Write a ratio showing the number of completions compared to the number enrolled.
- d. During an hour (60 minutes) of television viewing last night, there were 14 minutes of commercials, so

there were only 46 minutes of the actual program!

Write the ratio of commercial time to program time.

- e. For each pair of coins, write the ratio comparing the value. (Use cents.)
- i. a nickel to a dime
 - ii. a nickel to a quarter
 - iii. a nickel to a dollar
 - iv. a dime to a nickel
 - v. a dime to a quarter
 - vi. a dime to a dollar
 - vii. a dollar to a dime

Exercise 6 Answers

- a. **1 : 4**
 - b. **1 : 12**
 - c. **8 : 9**
 - d. **7 : 23**
-
- i. **1 : 2**
 - ii. **1 : 5**
 - iii. **1 : 20**
 - iv. **2 : 1**
 - v. **2 : 5**
 - vi. **1 : 10**
 - vii. **10 : 1**

5.1 Practice Questions

1. Write the definitions.
 - a. Ratio
 - b. Terms of the ratio
 - c. Equivalent ratios

2. Write the ratios asked for in the **lowest terms**. Use the colon style (e.g. **2 : 1**). Then, write the ratio as it is read, like this: two to one.
 - a. The campground had three vacant campsites and 47 occupied sites. Write the ratio of occupied sites to vacant sites.
Ratio:
Read: _____
 - b. For every ten dogs in the city, only 2 have current dog licences. Write the ratio of licensed dogs to unlicensed dogs. (Find the number of unlicensed dogs first).
Ratio:
Read: _____

3. Simplify these ratios.

a. 9 : 12	d. 2 : 9
b. 6 : 4	e. 35 : 15
c. 500 : 1000	

5.1 Practice Answers

1. Write the definitions.

- a. A **ratio** is a comparison of one number or quantity with another number or quantity. Ratios show the relationship between the quantities or amounts.
 - b. **Terms of a ratio** are the numbers used in the ratio (the parts of the ratio).
 - c. **Equivalent ratios** are ratios of equal value to each other.

2. Write the ratios asked for in lowest terms.
 - a. **47 : 3**
Read: “47 occupied sites to 3 vacant sites.”
 - b. **1 : 4**
Read: “1 licensed dog to 4 unlicensed dogs.”

3. Simplify these ratios.

a. 3 : 4	d. 2 : 9
b. 3 : 2	e. 7 : 3
c. 1 : 2	

References

Pronunciation Academy. (2015, March 25). How to pronounce ratio – Pronunciation academy [Video]. YouTube. https://youtu.be/tP_o0ogRagE?si=FKcuZZ2FAQf9nBeT.

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5.2: Rates

When a ratio is used to compare two different kinds of measure (e.g. apples and oranges or meters and hours), it is called a rate. The denominator must be 1.

Example A

A car can drive 725 km on 55 L of gas. What is the rate in km per L?

The ratio of this is $\frac{725 \text{ km}}{55 \text{ L}}$.

Step 1: Find the rate by making the denominator 1.

Step 2: Divide.

$$\frac{725}{55} \div \left(\frac{55}{55} \right) = \frac{725 \div 55}{55 \div 55} = \frac{13.18}{1} = 13.18$$

The rate is 13.18 km/L.

Example B

Sue bought 10 lb of oranges for \$4.99. What is the rate in cents per pound?

The ratio is $\frac{\$4.99}{10 \text{ lb}} = \frac{499 \text{ cents}}{10 \text{ lb}}$.

Step 1: Find the rate by making the denominator 1.

Step 2: Divide.

$$\frac{499}{10} \div \left(\frac{10}{10} \right) = \frac{499 \div 10}{10 \div 10} = \frac{49.9}{1} = 49.9$$

The rate is 49.9 ¢/lb.

When talking about rates, use the word 'per':

- In Example A, say: "The fuel economy of the car is 13.18 kilometres per litre."
- In Example B, say: "The oranges cost 49.9 cents per pound."

Example C

It takes 60 ounces of grass seed to plant 30 m^2 of lawn.
What is the rate in ounces per square metre (m^2)?

The ratio is $\frac{60 \text{ oz}}{30 \text{ m}^2}$.

Step 1: Find the rate by making the denominator 1.

Step 2: Divide.

$$\frac{60}{30} \div \left(\frac{30}{30} \right) = \frac{60 \div 30}{30 \div 30} = \frac{2}{1} = 2$$

The rate is 2 oz/m^2 , or 2 ounces per square metre.

Exercise 1

Write the following ratios as rates, comparing distance to time.

a. 120 km, 3 hours

b. 27 km, 9 hours

c. 203 km, 29 seconds

d. 444 km, 48 seconds

Exercise 1 Answers

a. 40 km/hour

b. 3 km/hour

c. 7 km/second

d. 9.25 km/second

Exercise 2

Write the following ratios as rates.

- a. A leaky faucet can lose 52 litres of water in a week.
What is the rate of litres lost per day? (round to two decimal places)
- b. The ratio of distance travelled to time is called speed. What is the rate (speed) in kilometres per hour (km/h)?
 - i. 45 km, 3 hours
 - ii. 129 km, 1.5 hours
 - iii. 65 km, 13 hours
- c. Vancouver Island has a population of 734 860 and a

land mass of 32 134 square kilometres. What is the rate of number of people per square kilometre? (This is called population density.) Round your answer to the nearest whole number.

- d. At rest, the heartbeat of a mouse is 30 000 beats per 60 minutes. What is the rate of beats per minute?

Exercise 2 Answers

- a. 7.43 L/day
- b. Answers
 - i. 15 km/hour
 - ii. 86 km/hour
 - iii. 5 km/hour
- c. 23 people/km²
- d. 500 beats/minute

5.2 Practice Questions

1. Write the definition.
 - a. Rate
2. Write the following ratios as rates. Round people to the nearest person.
 - a. 12 cups water, 3 cups sugar

- b. 72 metres, 24 seconds
- c. 1,365,000 people, 4,000 km²
- d. 5,000 cars on the road, 250 bikes on the road
- e. 12 cups of flour, 12 tsp. of baking powder
- f. 8 litres of gas, 2 litres of oil

5.2 Practice Answers

1. Write the definition.
 - a. A **rate** is used when a ratio compares two different kinds of measure, and when the denominator is 1.
2. Write the following ratios as rates.
 - a. 4 cups of water/cups of sugar
 - b. 3 m/second
 - c. 341 people km²
 - d. 20 cars/bike
 - e. 1 cup flour/tsp baking powder
 - f. 4 litres gas/litre oil

Attribution

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5.3: Proportion

A proportion is a statement that two ratios are equal or equivalent.
Table 5.3.1 some proportions.

Table 5.3.1: Proportion Examples

Proportion	Fraction Form	Read Like This
$1 : 2 = 2 : 4$	$\frac{1}{2} = \frac{2}{4}$	1 is to 2 as 2 is to 4
$1 : 4 = 25 : 100$	$\frac{1}{4} = \frac{25}{100}$	1 is to 4 as 25 is to 100
$18 : 9 = 10 : 5$	$\frac{18}{9} = \frac{10}{5}$	18 is to 9 as 10 is to 5
$15 : 20 = 3 : 4$	$\frac{15}{20} = \frac{3}{4}$	15 is to 20 as 3 is to 4

Proportions can be used to solve many math problems. You will soon learn to use proportions to solve problems involving percent. The techniques you practice in this section are important for that problem-solving work.

Problems often give incomplete information; that is, one of the terms is missing. To solve such problems, you first find the comparison or ratio that is given. It may be:

- a quantity of one thing that is mixed with a larger quantity of something else
- a scale of measurement given on a map, such as 1 cm on the map represents a 100 km distance on land
- cost for a certain number of items
- time to travel a certain distance

The problem will then give one term of the second ratio in the proportion. For example, if you have been told that 3 heads of lettuce cost \$1.49, you may be asked to find the cost of 7 heads of lettuce.

The missing term is the second cost. The proportion will be:

$$\frac{\text{number of heads of lettuce}}{\text{cost}} = \frac{\text{number of heads of lettuce}}{\text{cost}}$$

$$\frac{3}{\$1.49} = \frac{7}{?}$$

$$3 : \$1.49 = 7 : ?$$

The most important thing to remember is to keep the order of comparison the same in the first and second ratios in a proportion. If the first ratio compares time to distance, then the second ratio in the proportion must compare time to distance.

$$\frac{\text{time}}{\text{distance}} = \frac{\text{time}}{\text{distance}}$$

Or it could be:

$$\frac{\text{distance}}{\text{time}} = \frac{\text{distance}}{\text{time}}$$

Once you have decided on the order of comparison, it is a simple matter to write the proportion using the numbers given in the problem. Use a letter to stand for the missing term.

How would you find a missing term?

- You can use your skills with equivalent ratios (finding higher and lower terms).
- You can use your fraction skills of cross-multiplying and then dividing to find the missing term.

Using Equivalent Ratios to Solve Proportions

To solve a proportion problem using equivalent ratios:

1. Decide on the order of comparison and write a ratio that describes the information given in the problem. Write a proportion using words of the items being compared in fraction form.
2. Write two more ratios with the numbers matching the words in the first ratio. The missing term (number) can be given a letter (e.g. N).
3. Mentally set the ratio with words (the first ratio) aside.
4. Multiply or divide the complete ratio to find the missing term.

Example A

Use 1 teaspoon of baking powder for every 2 cups of flour. If a recipe uses 6 cups of flour, how much baking powder is needed

The missing term is the teaspoons of baking powder for 6 cups of flour. Call this term N.

Step 1:

Ratio is $\frac{\text{baking powder}}{\text{flour}}$

Step 2:

$$\frac{\text{baking powder}}{\text{flour}} = \frac{1}{2} = \frac{N}{6}$$

Step 3:

$$\frac{1}{2} = \frac{N}{6}$$

Step 4:

$$\frac{1}{2} \times \left(\frac{3}{3} \right) = \frac{1 \times 3}{2 \times 3} = \frac{3}{6}, \text{ so } \frac{1}{2} = \frac{3}{6}, \text{ so}$$
$$N = 3$$

Use 3 teaspoons of baking powder for 6 cups of flour.

Example B

Reports suggest that 3 out of 10 people will at some time miss work due to back pain. If a company has 1 000 employees, how many can be expected to miss work due to back pain?

The missing term is the number of people out of 1 000 who will miss work due to back pain. Call this term P .

Step 1:

Ratio is $\frac{\text{people who will miss work}}{\text{all people at work}}$

Step 2:

$$\frac{\text{people who will miss work}}{\text{all the people at work}} = \frac{3}{10} = \frac{P}{1000}$$

Step 3:

$$\frac{3}{10} = \frac{P}{1000}$$

Step 4:

$$\frac{3}{10} \times \left(\frac{100}{100} \right) = \frac{3 \times 100}{10 \times 100} = \frac{300}{1000}, \text{ so}$$
$$\frac{3}{10} = \frac{300}{1000}, \text{ so } P = 300$$

300 people out of 1 000 people may miss work due to back pain.

Exercise 1

Write the ratio of the words to describe the information given.

Example: Three cups of flour to one teaspoon of yeast.

$$\text{Answer: } \frac{\text{flour}}{\text{yeast}}$$

- a. Four parts oil, ten parts gasoline
- b. One centimetre represents 100 kilometres
- c. 100 grams for \$6.89
- d. 3 eggs for each cup of milk
- e. 5 men and 7 women

Exercise 1 Answers

$$\begin{array}{l} \text{a. } \frac{\text{oil}}{\text{gasoline}} \\ \text{b. } \frac{\text{centimetres}}{\text{kilometres}} \end{array}$$

$$\begin{array}{l} \text{c. } \frac{\text{grams}}{\text{dollars}} \\ \text{d. } \frac{\text{eggs}}{\text{milk}} \\ \text{e. } \frac{\text{men}}{\text{women}} \end{array}$$

Exercise 2

Use equivalent ratios to find the answers.

Example: One cup of sugar and four cups of water will make great hummingbird food. How much sugar do you need for 8 cups of water?

$$\frac{\text{sugar}}{\text{water}} \longrightarrow \frac{1}{4} = \frac{N}{8} \longrightarrow \frac{1}{4} \times \left(\frac{2}{2}\right) = \frac{1 \times 2}{4 \times 2} = \frac{2}{8} \longrightarrow N = 2$$

- Reports show that for every 100 vehicles checked by police, 20 vehicles do not meet the safety standard. If only 50 vehicles are checked, how many would not meet the safety standard?
- Four litres of paint covers 24 square metres of wall. How much paint is needed to cover 72 square metres?
- Powdered milk uses 1 part milk powder to 3 parts water. How much powder should be added to 9 parts water?

Exercise 2 Answers

- 10 cars would not meet the safety standards
- 12 litres of paint
- 3 parts milk powder

Exercise 3

Use equivalent ratios to find the missing term in these proportions.

a. $3 : 5 = Y : 15$

b. $1 : 2 = P : 8$

c. $5 : 7 = 10 : N$

d. $2 : 3 = 8 : W$

e. $4 : 7 = 16 : A$

f. $1 : 3 = 2 : N$

- g. The KX 250 motorcycle (see Figure 5.3.1) uses a mixture of one part oil to 30 parts of gasoline. How much oil must be added to 3,000 mL of gasoline?



Figure 5.3.1

Exercise 3 Answers

- a. $Y = 9$
- b. $P = 4$
- c. $N = 14$
- d. $W = 12$
- e. $A = 28$
- f. $N = 6$
- g. $N = 100\text{mL}$

Using Cross-Multiplication to Solve a Proportion

Review cross products:

Multiply the numerator of each fraction with the denominator of the other fraction.

$$\frac{2}{5} \times \frac{4}{10}$$

$$2 \times 10 = 5 \times 4$$

$$20 = 20$$

$$2 \times 10 = 20 \text{ and } 5 \times 4 = 20$$

Therefore:

$$\frac{2}{5} = \frac{4}{10}$$

Remember: When the cross products are the same, the fractions are equivalent.

When finding the missing terms in a proportion, cross-multiplication can be used. Follow the examples carefully.

Example D

$$\frac{2}{3} = \frac{N}{45}$$

Step 1: Cross-multiply.

$$\begin{aligned}2 \times 45 &= 3 \times N \\90 &= 3N\end{aligned}$$

The idea is to have the unknown term N by itself on one side of the equal sign. To do that, remember these things that you already know:

- Division and multiplication are opposite operations
- Whatever is done to one side of an equation or proportion must be done to the other side to keep the equation equal

Step 2: $3N$ means N is multiplied by 3.

To get rid of the 3, divide by 3.

You must also divide the other side of the equation by 3.

$$\frac{90}{3} = \frac{3N}{3}$$

Step 3: Solve by reducing the $\frac{3}{3}$ and dividing 90 by 3.

$$\begin{aligned}\frac{90}{3} &= \frac{3N}{3} \\ \frac{90}{3} &= \frac{1N}{1} \\ \frac{90}{3} &= N \\ 90 \div 3 &= N \\ 30 &= N\end{aligned}$$

Reducing the fraction $\frac{3N}{3}$ to $\frac{1N}{1}$ to N is also called **cancelling**.

In math, a fraction can be cancelled when the numerator and denominator are the same number. For example:

$$\frac{6P}{6} = \frac{1P}{1} = P$$

Example E

$$\frac{6}{7} = \frac{24}{N}$$

Step 1: Cross-multiply.

$$6 \times N = 7 \times 24$$

$$6N = 168$$

Step 2: Divide both sides by 6. The 6's with the N will cancel (reduce), and the N will be alone.

$$\begin{aligned} \frac{6N}{6} &= \frac{168}{6} \\ N &= 168 \div 6 \\ N &= 28 \end{aligned}$$

Check: Use cross-multiplying.

$$\text{Is } 6 \times 28 = 7 \times 24?$$

$$6 \times 28 = 168$$

$$7 \times 24 = 168$$

the cross-product $168 =$ the cross-product 168

$$\text{Yes - } 6 : 7 = 24 : 28$$

Example F

$$\frac{8}{10} = \frac{N}{80}$$

Step 1: Cross-multiply.

$$8 \times 80 = 10 \times N$$

$$640 = 10N$$

Step 2: Divide both sides by 10 so N will be alone.

$$\frac{640}{10} = \frac{10N}{10}$$

$$\frac{\cancel{64}^{\cancel{10}}}{\cancel{10}} = \frac{\cancel{10}^{\cancel{10}} N}{\cancel{10}}$$

$$64 = N$$

To solve a proportion problem using cross-multiplication:

1. Write the first ratio using the information given.
2. Write the proportion using a letter in place of the missing term. Be sure the **order of comparison is the same** in both the first and second ratios in your proportion.
3. Write the proportion in the fraction form. (Try to **simplify** the ratio **before** you do all the calculations).
4. Cross-multiply and set the cross-products equal to each other.
5. Divide both sides of the equation by the number with the unknown term.
6. Check by putting your answer back into the original proportion and cross-multiplying.

Exercise 4

Practise using cross-multiplying to find the missing term in these proportions.

$$\frac{5}{8} = \frac{N}{32}$$

$$5 \times 32 = 8 \times N$$

$$160 = 8N$$

Example:

$$\frac{160}{8} = \frac{8N}{8}$$

$$160 \div 8 = N$$

$$20 = N$$

a. $\frac{4}{N} = \frac{24}{30}$
b. $\frac{12}{4} = \frac{18}{x}$

c. $\frac{y}{6} = \frac{20}{12}$
d. $4 : 15 = 8 : N$
e. $W : 100 = 6 : 50$

Exercise 4 Answers

a. $N = 5$

b. $x = 6$

c. $y = 10$

d. $N = 30$

e. $W = 12$

The numbers in a ratio often are common fractions, decimals or mixed numbers.

Follow exactly the same steps that you have been using to solve whole number proportions. The calculations will use your skills with fractions.

Example G

$$2\frac{1}{4} : 3 = N : 7$$

Step 1: Rewrite the proportion.

$$\frac{2\frac{1}{4}}{3} = \frac{N}{7}$$

Step 2: Cross-multiply.

$$2\frac{1}{4} \times 7 = 3 \times N$$

$$\frac{9}{4} \times \frac{7}{1} = 3 \times N$$

$$\frac{63}{4} = 3N$$

$$\frac{63}{4} \div \frac{3}{1} = \frac{3 \times N}{3} \longrightarrow \frac{63}{4} \times \frac{1}{3} = N$$

$$\frac{63}{12} = N \longrightarrow 5\frac{3}{12} \longrightarrow 5\frac{1}{4} = N$$

Exercise 5

Practise using cross-multiplying to find the missing term in these proportions.

$$6.5 : 5 = 13 : A$$

$$\frac{6.5}{5} = \frac{13}{A}$$

Example:

$$6.5A = 65$$

$$A = 65 \div 6.5$$

$$A = 10$$

a. $3\frac{1}{2} : 2 = N : 8$

b. $9 : 6 = 4\frac{1}{2} : N$

c. $7.5 : B = 10 : 20$

d. $3.75 : 5 = 9x$

e. $4\frac{1}{8} : A = 3 : 6$

Exercise 5 Answers

a. $N = 14$

b. $N = 3$

c. $B = 15$

d. $x = 12$

e. $A = 8\frac{1}{4}$ or 8.25

Exercise 6

Solve the following word problems.

- a. Joanne can walk 18 km in 3 hours. How far can she walk at the same rate in $5\frac{1}{2}$ hours?
- b. The taxes on the property valued at \$300 000 are valued at \$5 000. At the same rate of taxation, what would the taxes be on the smaller lot down the street, which is valued at \$240 000?
- c. One B.C. road map has a scale of 0.5 centimetres, equal to 10 kilometres. Complete the chart in Table 5.3.2 by calculating actual driving distances in kilometres between some B.C. places. The proportions will be $\frac{0.5}{10} = \frac{\text{cm given in chart}}{\text{actual distance in km}}$

Table 5.3.2: Exercise 6c Complete the Chart

Places in B.C.	Number of cm Between Places on the Map	Actual Distance in Kilometres
Kelowna and Vernon	2.5 cm	—
Burns Lake and Vanderhoof	5.5 cm	—
TaTa Creek and Skookumchuk	0.75 cm	—
Kitimat and Terrace	3.3 cm	—

- d. The directions on the lawn fertilizer say to spread 1.7 kg over 100 m² of lawn.
- How much fertilizer is needed for a 130 m² lawn?
 - How much fertilizer for a 75 m² lawn?

Exercise 6 Answers

- 33 km
- \$4 000

c.

Table 5.3.3: Exercise 6c Answers

Places in B.C.	Number of cm Between Places on the Map	Actual Distance in Kilometres
Kelowna and Vernon	2.5 cm	50 km
Burns Lake and Vanderhoof	5.5 cm	110 km
TaTa Creek and Skookumchuk	0.75 cm	15 km
Kitimat and Terrace	3.3 cm	66 km

d. Answers

- i. 2.21 kg
- ii. 1.275 kg

5.3 Practice Questions

1. Solve the following proportions.

- a. $N : 14 = 28 : 56$
- b. $3 : 11 = N : 22$
- c. $50 : 45 = 10 : N$
- d. $4\frac{1}{5} : Y = 3 : 2$

2. Complete the following exercise on map scales.

- a. Get a map of BC, a map of Canada, and a map of your city

- or town.
- b. Find the scale on each map (usually at the bottom) and write down the ratio of map distance to the actual distance.
 - c. With another student or an instructor, calculate actual distances between places by measuring the distance on the map and working out the proportion according to the scale given. Do at least three distance calculations on each map.

Ask your instructor to mark your work.

5.2 Practice Answers

1. Solve these proportions.
 - a. $N = 7$
 - b. $N = 6$
 - c. $N = 9$
 - d. $Y = 2\frac{4}{5}$ or 2.8
2. See your instructor.

Attribution

All figures in this chapter are from Topic C: Proportion in Adult Literacy Fundamental Mathematics: Book 6 – 2nd Edition by Liz Girard and Wendy Tagami, via BCcampus.

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5.4: Ratio, Rate, & Proportion Review

1. Write the ratios asked for in the following word problems.
Reduce when needed.
 - a. It snowed for two days and was sunny for five days last week. Write a ratio to compare snowy days to sunny days.
 - b. The class spends 6 hours a week on math and 8 hours on English. Write the ratio of hours spent on math to English.
 - c. One kilogram of pork will make enough sausages for 7 people. Write a ratio to express the amount of pork for sausages to the number of people.
 - d. A jam recipe calls for 8 cups of blueberries to 3 cups of sugar. Write a ratio to express the amount of berries to sugar.

2. Write the ratios in the lowest terms.

a. $14 : 21$

b. $8 : 24$

c. $15 : 150$

d. $9 : 54$

3. Write the following ratios as rates.

- a. 16 cups of water to 4 cups of sugar
- b. 150 kilometres to 1.25 hours
- c. 465 000 people to 3 000 square kilometres
- d. \$3.99 to 10 pounds of apples

4. Use cross multiplication to solve the following proportions.

a. $\frac{5}{8} = \frac{P}{24}$

b. $\frac{S}{100} = \frac{8}{25}$

$$\begin{array}{l} \text{c. } \frac{4}{N} = \frac{8}{40} \\ \text{d. } \frac{4}{14} = \frac{F}{68} \end{array}$$

5. Use cross multiplication to solve the proportions.

$$\text{a. } \frac{2\frac{1}{4}}{5} = \frac{A}{30}$$

$$\text{c. } \frac{3\frac{3}{4}}{5} = \frac{8}{X}$$

$$\text{b. } \frac{11}{8} = \frac{4\frac{1}{8}}{P}$$

$$\text{d. } \frac{2\frac{2}{3}}{3} = \frac{A}{16}$$

5.4: Review Answers

1. Write the ratios asked for in the following word problems.

$$\text{a. } 2 : 5$$

$$\text{c. } 1 \text{ kg} : 7 \text{ people}$$

$$\text{b. } 6 : 8$$

$$\text{d. } 8 \text{ cups berries} : 3 \text{ cups sugar}$$

2. Write the ratios in the lowest terms.

$$\text{a. } 2 : 3$$

$$\text{c. } 1 : 10$$

$$\text{b. } 1 : 3$$

$$\text{d. } 1 : 6$$

3. Write the following ratios as rates.

$$\text{a. } 4 \text{ cups water per cup of sugar}$$

$$\text{b. } 120 \text{ km/hour}$$

$$\text{c. } 155 \text{ people per km}^2$$

$$\text{d. } \$0.40 \text{ per pound of apples}$$

4. Use cross multiplication to solve the following proportions.

a. $\frac{15}{32}$

b. $\frac{32}{15}$

c. $\frac{20}{19.43}$

d. $\frac{19.43}{20}$

5. Use cross multiplication to solve the following proportions.

a. $13\frac{1}{2}$

b. 3

c. $10\frac{2}{3}$

d. $14\frac{2}{9}$

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5.5: Percent

Percents are another form of fractions and are used in many everyday situations. Interest rates, credit card charges, taxes, pay deductions, increases and decreases are all calculated with percent. Percents are a convenient way to express part of the whole thing because the unwritten denominator is always 100.

- The denominator of 100 is not written or said.
- A percent sign % follows the number and is read as “percent.”
- The whole thing is 100%
- $100\% = 1$

Remember:

- **Cent** is the Latin word meaning a hundred or a hundredth.
- **per** is a Latin word meaning by or for each.

So, **percent** means for each hundred.

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5.6: Introducing Percent

Reading & Writing Percents

To write a percent:

- Write the number in the usual way
- Place the percent sign after the numerals
 - 50%
 - $5\frac{1}{2}\%$ or 5.5%
 - $\frac{3}{4}\%$ or 0.75%

To read a percent:

- Read the numbers in the usual way.
- Say “percent” after the number.
 - 16% – say, “sixteen percent”
 - $4\frac{1}{2}\%$ – say, “four and one-half percent”
 - 0.25% – say, “twenty-five hundredths percent,” or, “one-quarter percent,” or, “point two five percent”

Exercise 1

Write these percents using numerals and a percent sign.
Note that the mixed numbers may be expressed with common fractions or decimals.

Example: Thirty-four percent – 34%

a. Twelve percent d. Thirteen percent
b. Four-fifths percent e. Six and one-fifth percent
c. One hundred sixteen and f. Ninety-four and one-half
three-tenths percent percent

Exercise 1 Answers

a. 12% b. 0.8% or $\frac{4}{5}\%$ c. 116.3% d. $116\frac{3}{10}\%$ e. 13% f. $6\frac{1}{5}\%$ or 94.5% g. $94\frac{1}{2}\%$

Exercise 2

Write the percent as you would read it.

Example: 62% – sixty-two percent

a. $37\frac{1}{2}\%$	c. $\frac{3}{4}\%$	e. $14\frac{1}{2}\%$
b. 202%	d. 18.3%	f. $100\frac{1}{2}\%$

Exercise 2 Answers

a. Thirty-seven and one-half percent	e. Five-tenths percent or one-half percent or zero point five percent
b. Two hundred two percent	f. One hundred percent
c. Three-quarters percent	
d. Eighteen and three-tenths	

Changing Decimals to Percents

Writing equivalent fractions is an important math skill.

Equivalent common fractions, decimals, and percents all represent the same amount (see Table 5.6.1).

Table 5.6.1: Equivalent Fractions, Decimals, and Percentages

Fractions	Decimals	Percentages
$\frac{1}{2}$	0.5	50%
$\frac{3}{10}$	0.3	30%

You need the skill of writing equivalent fractions for working with percents.

To change any number to a percent, multiply the number by 100% and place a percent sign % after the product.

Remember this shortcut for multiplying by 100?

$$\begin{aligned} 4.27 \times 100 &= 427 \\ 0.287 \times 100 &= 28.7 \\ 53 \times 100 &= 5300 \end{aligned}$$

The shortcut is:

When multiplying by 100, move the decimal point two places to the right.

Example A

Change these numbers to a percent.

1	$1 \times 100\%$	=	100%
0.25	$0.25 \times 100\%$	=	25%
0.8	$0.8 \times 100\%$	=	80%
0.375	$0.375 \times 100\%$	=	37.5%

So:

To change a decimal to a percent, move the decimal point two places to the right and then write the percent sign after the number.

Example B

Change each decimal to a percent.

$$0.125 \xrightarrow{\quad} 0.125 = 12.5\%$$

$$1.375 \xrightarrow{\quad} 1.375 = 137.5\%$$

If the decimal point moves to the end of the number, it is not necessary to write the decimal point.

Remember: Zeros at the beginning of a number are also not necessary.

$$0.05 = 0.05 = \overset{\curvearrowright}{0.05} = 5\%$$

If the decimal is a tenth (one decimal place), it will be necessary to add a zero. If you are changing a whole number to a percent, add two zeros.

$$0.4 = 0.40 = \overset{\curvearrowright}{0.40} = 40\%$$

$$1.7 = 1.70 = \overset{\curvearrowright}{1.70} = 170\%$$

$$2 = 2.00 = \overset{\curvearrowright}{2.00} = 200\%$$

Exercise 3

Change these decimals to percents.

Example: $0.75 = 0.75 = \overset{\curvearrowright}{0.75} = 75\%$

- a. 0.33
- b. 0.1
- c. 0.0025

- d. 0.9
- e. 0.325
- f. 0.0625

- g. 3

Exercise 3 Answers

a. 33%

b. 10%

c. 0.25%

d. 90%

e. 32.5%

f. 6.25%

g. 300%

Changing Percents to Decimals

Review dividing by 100:

$$47.39 \div 100 = 0.4739$$

$$429 \div 100 = 4.29$$

$$3.824 \div 100 = 0.03824$$

To divide by 100, move the decimal point two places to the left.

Example C

Change each percent to a decimal or mixed number.

$$58\% = 58 \div 100 = .58 = 0.58$$

$$20\% = 20 \div 100 = .2 = 0.2$$

$$6\% = 6 \div 100 = .06 = 0.06$$

$$110\% = 110 \div 100 = 1.10$$

So:

To change a percent to a decimal, divide by 100 (move the decimal point two places to the **left**) and remove the percent sign.

Example D

Change each percent to a decimal.

$$\begin{array}{rcl}
 75\% & = & 75.0\% = 0.75 \\
 12\% & = & 12.0\% = 0.12 \\
 37.5\% & = & 37.5\% = 0.375 \\
 125\% & = & 125.0\% = 1.25 \\
 5\% & = & 5.0\% = 0.05 \\
 4.6\% & = & 4.6\% = 0.046
 \end{array}$$

If there is no decimal point in the percent, place the decimal point **after** the last numeral and **then** divide by 100.

$$24\% = 24.0\% = 0.24$$

It may be necessary to **prefix** zeros. (This means adding zeros in front of the number if needed).

$$6\% = 6.0\% = 0.06$$

A zero at the right of a decimal is not needed and may be left off.

$$40\% = 40.0\% = 0.40 = 0.4$$

Exercise 4

Change each percent to its decimal equivalent.

Example: $23\% = 23. = 0.23\%$



a. 1%	d. 36%
b. 112%	e. 147%
c. 10.3%	

Exercise 4 Answers

a. 0.01	d. 0.36
b. 1.12	e. 1.47
c. 0.103	

To change a percent containing a common fraction to a decimal:

1. Change the common fraction in the percent to a decimal in the percent.
2. Divide by 100 (move the decimal 2 places to the **left**).

Example E

$$\begin{array}{lll} 3\frac{1}{2}\% & = & 3.5\% \quad 3.5\% \div 100 = .035 = 0.035 \\ 37\frac{1}{2}\% & = & 37.5\% \quad 37.5\% \div 100 = .375 = 0.375 \\ \frac{1}{4}\% & = & 0.25\% \quad 0.25\% \div 100 = .0025 \\ 17\frac{1}{3}\% & = & 17.\bar{3}\% \quad 17.\bar{3}\% \div 100 = 0.17\bar{3} \end{array}$$

Exercise 5

Change each percent to its decimal equivalent.

Example: $8\frac{4}{5}\% = 8.8\% = 0.088$

a. $4\frac{1}{2}\% =$ b. $56\frac{3}{4}\% =$ c. $1\frac{3}{5}\% =$

$$\text{d. } 112\frac{1}{2}\% = \quad \text{e. } 2\frac{3}{8}\% = \quad \text{f. } 5\frac{1}{4}\% =$$

Exercise 5 Answers

$$\text{a. } 0.045$$

$$\text{c. } 0.016$$

$$\text{e. } 0.02375$$

$$\text{b. } 0.5675$$

$$\text{d. } 1.125$$

$$\text{f. } 0.0525$$

Changing Common Fractions to Percents

To change any number to a percent, multiply the number by 100% and place the percent sign % after the product.

There are two methods you can use to change a common fraction to a percent.

Method 1

To change a common fraction to an equivalent percent, multiply the common fraction by 100%.

Example F

Change these common fractions to an equivalent percent:

a. $\frac{3}{4} = \underline{\hspace{1cm}}\%$

b. $1\frac{1}{5} = \underline{\hspace{1cm}}\%$

a. $\frac{3}{4} = \underline{\hspace{1cm}}\%$

Step 1: Multiply by 100%.

$$\frac{3}{4} \times 100\%$$

Step 2: Convert 100% to a fraction.

$$\frac{3}{4} \times \frac{100}{1}\%$$

Step 3: Simplify the fractions by dividing the numerator and denominator by 4.

$$\frac{\cancel{3}}{\cancel{4}1} \times \frac{\cancel{100}25}{1}\%$$

Step 4: The 4 cancels and 100 is reduced to 25.

$$\frac{3}{1} \times \frac{25}{1} \%$$

Step 5: Multiply 3 by 25%.

$$3 \times 25\% \\ 75\%$$

b. $1\frac{1}{5} = \underline{\hspace{2cm}}\%$

Step 1: Convert number to a fraction.

$$\frac{6}{5}$$

Step 2: Multiply by 100%.

$$\frac{6}{5} \times 100\%$$

Step 3: Convert 100% to a fraction.

$$\frac{6}{5} \times \frac{100}{1} \%$$

Step 4: Simplify the fractions by dividing the denominator and numerator by 5.

$$\frac{\cancel{6}}{\cancel{5}1} \times \frac{\cancel{100}20}{1} \%$$

Step 5: The 5 cancels and 100 is reduced to 20.

$$\frac{6}{1} \times \frac{20}{1} \%$$

Step 6: Multiply 6 by 20%.

$$\begin{array}{r} 6 \times 20\% \\ 120\% \end{array}$$

Exercise 6

Multiply by 100% to change each common fraction to an equivalent percent.

Example: $\frac{4}{5} \times 100\% = 80\%$

a. $\frac{1}{5}\% =$	c. $1\frac{1}{2}\% =$	e. $3\frac{3}{4}\% =$
b. $\frac{9}{10}\% =$	d. $\frac{7}{10}\% =$	f. $\frac{1}{2}\% =$

Exercise 6 Answers

a. 20%	c. 150%	e. 375%
b. 90%	d. 70%	f. 50%

Method 2

To change a common fraction to an equivalent percent, first write the common fraction as a decimal. Then, multiply the decimal by 100% (move the decimal point two places to the **right**).

Example G

$$\frac{3}{8} = \underline{\hspace{1cm}}\%$$

Step 1: Use long division to write the fraction as a decimal.

$$\frac{3}{8} = 0.375$$

Step 2: Move the decimal two spots to the right.

$$\begin{array}{c} \curvearrowright \\ 0.375 = 37.5 \\ 37.5\% \end{array}$$

Example H

Convert $\frac{3}{8}$ to a percentage.

Step 1:

$$\begin{array}{r} 0.375 \\ 8 \overline{)3.000} \\ \underline{-24} \\ 60 \\ \underline{-56} \\ 40 \end{array}$$

$$\frac{3}{8} = 0.375$$

Step 2: Move the decimal point two places to the right and add the percent sign.

$$0.375 = 37.5$$

Example I

Convert $\frac{1}{3}$ to a percentage.

Step 1: Use long division to write fraction as a decimal.

$$\begin{array}{r} 0.33\overline{3} \\ 3 \overline{)1.000} \\ \frac{1}{3} = 0.33\overline{3} \end{array}$$

Step 2: Move the decimal point two places to the right and add the percent sign.

$$0.33\overline{3} = 33.\overline{3}\%$$

[illegible][illegible]

[illegible]

[illegible]

[illegible]

Convert $\frac{11}{12}$ to a percentage.

Step 1: Use long division to write fraction as a decimal.

$$\begin{array}{r} 0.916\bar{6} \\ \hline 12 \overline{) 11.000} \\ \underline{108} \\ 20 \\ \underline{168} \\ 32 \\ \underline{312} \\ 8 \\ \underline{72} \\ 8 \\ \underline{72} \\ 0 \end{array}$$

$$\frac{11}{12} = 0.91\bar{6}$$

Step 2: Move the decimal point two places to the right and add the percent sign.

$$0.91\bar{6} = 91.\bar{6}\%$$

Exercise 7

Convert the following fractions to percentages.

Example:

$$\frac{1}{12} = 0.08\bar{3} \quad 0.08\bar{3} \times 100\% = 8.\bar{3}\%$$

a. $\frac{1}{8}$

d. $\frac{2}{3}$

g. $\frac{4}{9}$

b. $\frac{5}{8}$

e. $\frac{5}{16}$

c. $\frac{7}{8}$

f. $\frac{5}{6}$

Exercise 7 Answers

a. 12.5%

d. 66. $\bar{6}$ %

g. 44. $\bar{4}$ %

b. 62.5%

e. 31.25%

c. 87.5%

f. 83. $\bar{3}$ %

The method you use to change a common fraction to a percent will depend on the numbers you are working with. Choose whichever method seems easier for the situation. You will also memorize many

equivalencies as you work with them. But you should definitely memorize:

$$\begin{aligned} \bullet \quad \frac{1}{3} &= 33\frac{1}{3}\% \\ \bullet \quad \frac{2}{3} &= 66\frac{2}{3}\% \end{aligned}$$

Changing Percents to Common Fractions

You know that percents are a form of fraction with an unwritten denominator of 100. A % sign is used.

To change any percent to a decimal, common fraction, or mixed number, divide by 100 and remove the percent sign.

To change a percent to a common fraction:

1. Write the numerals in the percent as the numerator.
2. Write 100 as the denominator.
 - **Remember:** The line in a fraction can be a division sign, so $58\% = \frac{58}{100}$ is the same as $58 \div 100$.)

3. Remove the % sign.
4. Simplify the fraction: $\frac{58}{100} = \frac{29}{50}$

Example K

Write each percent as a common fraction.

$$\text{a. } 38\% = \frac{38}{100} \div \left(\frac{2}{2}\right) = \frac{38 \div 2}{100 \div 2} = \frac{19}{50}$$

$$\text{b. } 25\% = \frac{25}{100} \div \left(\frac{25}{25}\right) = \frac{25 \div 25}{100 \div 25} = \frac{1}{4}$$

$$\text{c. } 3\% = \frac{3}{100}$$

Note: Percents greater than or equal to 100 become improper fractions which will be rewritten as mixed numbers.

$$110\% = \frac{110}{100} = 1\frac{10}{100} = 1\frac{1}{10}$$
$$\begin{array}{r} 100 \overline{)110} \\ \underline{100} \\ 10 \end{array}$$

$$120\% = \frac{120}{100} = 1\frac{20}{100} = 1\frac{1}{5}$$

$$\begin{array}{r} 1 \\ 100 \overline{)120} \\ \underline{100} \\ 20 \end{array}$$

Remember: 100% is the whole thing $100\% = 1$.

Exercise 8

Change each percent to a common fraction. Simplify to the lowest terms.

a. $31\% =$	d. $20\% =$	g. $750\% =$
b. $11\% =$	e. $75\% =$	
c. $2\% =$	f. $100\% =$	

Exercise 8 Answers

a. $\frac{31}{100}$	c. $\frac{1}{50}$	e. $\frac{3}{4}$
b. $\frac{11}{100}$	d. $\frac{1}{5}$	f. 1
		g. $7\frac{1}{2}$

Percents Less Than 1%

Sometimes, a percent **smaller than 1%** is used.

For example, you will hear amounts such as $\frac{1}{4}\%$ or $\frac{1}{8}\%$ or $\frac{1}{2}\%$ on the news about the Bank of Canada rate and the rise and fall of inflation.

These are small amounts. Sometimes, the expression “ $\frac{1}{2}$ of a percentage point” is used instead of “ $\frac{1}{2}\%$ ”.

What is $\frac{1}{4}\%$?

$$\begin{aligned}\frac{1}{4}\% \text{ is } \frac{1}{4} \text{ of } 1\% \\ 1\% = \frac{1}{100}, \text{ so } \frac{1}{4} \text{ of } 1\% = \frac{1}{4} \times \frac{1}{100} = \frac{1}{400} \\ \frac{1}{4}\% = 0.25\% = 0.0025\end{aligned}$$

What is $\frac{1}{2}\%$?

$$\begin{aligned}\frac{1}{2}\% = \frac{1}{2} \text{ of } 1\% = \frac{1}{2} \times \frac{1}{100} = \frac{1}{200} \\ \frac{1}{2}\% = 0.5\% = 0.005\end{aligned}$$

To work with percents less than 1%, change the percent to a decimal by dividing by 100 (move the decimal point two places to the left).

$$\begin{aligned}0.2\% &= 0.002 \\ 0.75\% &= 0.0075\end{aligned}$$

If the percent is expressed as a common fraction:

- Write the common fraction percent as a decimal percent.
- Divide by 100 (move the decimal point two places left).

Exercise 9

Change each percent to an equivalent decimal.

a. $\frac{1}{2}\% =$	d. $\frac{3}{5}\% =$	h. $\frac{1}{4}\% =$
b. $0.6\% =$	e. $0.75\% =$	i. $0.125\% =$
c. $\frac{3}{10}\% =$	f. $\frac{3}{4}\% =$	j. $\frac{5}{8}\% =$
	g. $0.5\% =$	

Exercise 9 Answers

a. 0.005	e. 0.0075	i. 0.00125
b. 0.006	f. 0.0075	j. 0.00625
c. 0.003	g. 0.005	
d. 0.006	h. 0.0025	

$16\frac{2}{3}\%$, $33\frac{1}{3}\%$, $66\frac{2}{3}\%$, $83\frac{1}{3}\%$...

These percents will become repeating decimals. For example:

$$33\frac{1}{3}\% = 33.\overline{3}\% = 0.33\overline{3}$$

$$66\frac{2}{3}\% = 66.\overline{6}\% = 0.66\overline{6}$$

It is usually more convenient to use the common fraction equivalent of these percents. Memorize them, or make a note on a special paper and post it near your workspace.

$$33\frac{1}{3}\% = 33\frac{1}{3} \div 100 = \frac{100}{3} \times \frac{1}{100} = \frac{1}{3}$$

$$66\frac{2}{3}\% = 66\frac{2}{3} \div 100 = \frac{200}{3} \times \frac{1}{100} = \frac{2}{3}$$

$$16\frac{2}{3}\% = \frac{1}{6}$$

$$33\frac{1}{3}\% = \frac{1}{3}$$

$$66\frac{2}{3}\% = \frac{2}{3}$$

$$83\frac{1}{3}\% = \frac{5}{6}$$

Review of Equivalent Common Fractions, Decimals, & Percents

Exercise 10

Complete the chart in Table 5.6.4. These are equivalents that you will often use, so use this chart for reference.

Memorize as many equivalents as you can. You may wish to put other equivalents on the chart.

Table 5.6.4: Exercise 10 Complete the chart

Common Fraction	Decimal	Percent
$\frac{1}{4}$	—	—
—	0.5	—
—	—	75%
$\frac{1}{8}$	—	—
—	0.375	—
—	—	62.5%

Exercise 10 Answers

Table 5.6.5: Exercise 10 Answers

Common Fraction	Decimal	Percent
$\frac{1}{4}$	0.25	25%
$\frac{1}{2}$	0.5	50%
$\frac{3}{4}$	0.75	75%
$\frac{1}{8}$	0.125	12.5%
$\frac{3}{8}$	0.375	37.5%
$\frac{5}{8}$	0.625	62.5%

5.6: Practice Questions

1. Write these percents in numeral form.
 - a. Sixty-two and one-half percent

- b. One hundred six and one-fifth percent
2. Write these percents in words.
 - a. 72%
 - b. $\frac{3}{4}\%$
3. Change these percents to decimal fractions.
 - a. 32%
 - b. 18.5%
 - c. 125%
 - d. $\frac{4}{5}\%$
 - e. $\frac{2}{3}\%$
4. Change these percents to common fractions in the lowest terms.
 - a. 16%
 - b. 20%
 - c. 106%
 - d. 75%
5. Change these common fractions to equivalent percents.
 - a. $\frac{4}{5}$
 - b. $\frac{3}{8}$

5.6: Practice Question Answers

1. Write these percents in numeral form.
- a. 62.5% or $62\frac{1}{2}\%$ b. 106.2% or $106\frac{1}{5}\%$

2. Write these percents in words.
 - a. Seventy-two percent
 - b. Three-quarters percent

3. Change these percents to decimal fractions.

a. 0.32	d. 0.008
b. 0.185	e. 0.006
c. 1.25	

4. Change these percents to common fractions in the lowest terms.

a. $\frac{4}{25}$	c. $1\frac{3}{50}$
b. $\frac{1}{5}$	d. $\frac{3}{4}$

5. Change these common fractions to equivalent percents.

a. 80%	b. 37.5%

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5.7: Percent Review

1. Change these decimals to percents.

a. 0.75	d. 0.0035	g. 3.45
b. 0.156	e. 0.625	
c. 0.03	f. 0.048	

2. Change each percent to a decimal.

a. 59%	d. $3\frac{1}{2}\%$	f. $4\frac{3}{5}\%$
b. 39.5%		
c. 1%	e. $1\frac{4}{5}\%$	

3. Change each fraction to an equivalent percent.

a. $1\frac{3}{4}$	d. $\frac{9}{20}$
b. $\frac{1}{5}$	e. $2\frac{1}{3}$
c. $4\frac{3}{20}$	

4. Change each percent to a common fraction. Simplify your answer.

a. 16%	f. 284%	i. 0.125%
b. 25%	g. $\frac{1}{4}\%$	j. $\frac{5}{8}\%$
c. 110%		
d. 95%	h. $\frac{1}{8}\%$	k. $\frac{3}{4}\%$
e. 650%		

5. Change each percent to a common fraction. These ones should be memorized.

a. $33\frac{1}{3}\%$

c. $16\frac{2}{3}\%$

b. $66\frac{2}{3}\%$

d. $83\frac{1}{3}\%$

5.7: Review Answers

1. Change these decimals to percents.

a. 75%

d. 0.35%

g. 345%

b. 15.6%

e. 62.5%

c. 3%

f. 4.8%

2. Change each percent to a decimal.

a. 0.59

c. 0.01

e. 0.018

b. 0.395

d. 0.035

f. 0.046

3. Change each fraction to an equivalent percent.

a. 175%

d. 45%

b. 20%

e. 233.3%

c. 415%

4. Change each percent to a common fraction.

a. $\frac{4}{25}$

b. $\frac{1}{4}$

c. $1\frac{1}{10}$

$$\text{d. } \frac{19}{20}$$

$$\text{e. } 6\frac{1}{2}$$

$$\text{f. } 2\frac{21}{25}$$

$$\text{g. } \frac{1}{400}$$

$$\text{h. } \frac{1}{800}$$

$$\text{i. } \frac{0.125}{100} \text{ or } \frac{1}{800}$$

$$\text{j. } \frac{0.7}{100} \text{ or } \frac{7}{1000}$$

$$\text{k. } \frac{3}{400}$$

5. Change each percent to a common fraction.

$$\text{a. } \frac{1}{3}$$

$$\text{b. } \frac{2}{3}$$

$$\text{c. } \frac{1}{6}$$

$$\text{d. } \frac{5}{6}$$

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5.8: Working With Percent

Each type of percent problem can be solved using the following proportion:

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}$$

Both ratios in this proportion use the same order of comparison because in the ratio $\frac{\%}{100}$, the % represents a part, and 100 is the whole. That is, the % is a part of the whole.

Percent problems involve knowing three pieces of information:

1. the part (the “is” part)
2. the whole (the “of” part)
3. the percent

You will be given two pieces of information to find the third. That is, the problems will give two terms of the proportion, and you will solve for the missing term. Because these are problems of percent, the 100 is always known to you and will always be in the same position in the proportion.

Remember how to use cross multiplication to solve a proportion:

$$\frac{N}{4} = \frac{6}{8} \rightarrow 4 \times 6 = 8 \times N \rightarrow 24 = 8N \rightarrow \frac{24}{8} = \frac{8N}{8} \rightarrow \frac{24}{8} = \frac{\cancel{8}N}{\cancel{8}} \rightarrow 3 = N$$

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5.9: Finding a Percent of a Number

$$\frac{\text{is(part)}}{\text{of(whole)}} = \frac{\%}{100}$$

In problems in which you find a percent of a number, the **missing term** is the **part**. You will be given the %, which is always 100.

Example A

What is 25% of 40?

- part (is) = N
- whole (of) = 40
- % = 25

$$\frac{\text{is}}{\text{of}} = \frac{\%}{100} \longrightarrow \frac{N}{40} = \frac{25}{100}$$

Solve the proportion:

Step 1: Simplify if possible.

$$\frac{N}{40} = \frac{25}{100}$$

$$\frac{N}{40} = \frac{\cancel{25}^1}{\cancel{100}_4}$$

$$\frac{N}{40} = \frac{1}{4}$$

Step 2: Cross-multiply.

$$N \times 4 = 40 \times 1$$

$$4N = 40$$

Step 3: Divide.

$$N = 40 \div 4 = 10$$

$$25\% \text{ of } 40 = 10$$

Example B

What is **20%** of **18**?

- part (is) = N
- whole (of) = 18
- % = 20

$$\frac{\text{is}}{\text{of}} = \frac{\%}{100} \longrightarrow \frac{N}{18} = \frac{20}{100}$$

Solve the proportion:

$$\frac{N}{18} = \frac{20}{100}$$

$$\frac{N}{18} = \frac{\cancel{20} 1}{\cancel{100} 5}$$

$$\frac{N}{18} = \frac{1}{5}$$

$$5N = 18$$

$$N = 18 \div 5 = 3\frac{3}{5}$$

$$20\% \text{ of } 18 = 3\frac{3}{5}$$

The following examples all ask you to find a percent of a number. The missing term is the **part (the “is” part)**. Look at the examples carefully so you’ll recognize the wording.

- What is **14%** of **60**?

$$\circ \quad \frac{\text{is}}{\text{of}} = \frac{\%}{100} \longrightarrow \frac{N}{60} = \frac{14}{100}$$

- Find **10%** of **27**.

$$\circ \quad \frac{\text{is}}{\text{of}} = \frac{\%}{100} \longrightarrow \frac{P}{27} = \frac{10}{100}$$

- **5%** of **15** is _____.

- $\frac{\%}{100} = \frac{\text{is}}{\text{of}} \longrightarrow \frac{5}{100} = \frac{X}{15}$
- 75% of 12 = _____.
- $\frac{\%}{100} = \frac{\text{is}}{\text{of}} \longrightarrow \frac{75}{100} = \frac{K}{12}$

In percent problems, the number after the word **of** usually represents the **whole**.

Exercise 1

Solve each problem by setting up the proportion

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}.$$

- a. 20% of 18 = e. 3% of 33 = i. What is
 b. 19% of 200 = f. 30% of 64 = 72% of \$425
 c. 25% of 44 = g. 50% of 60 is? j. What is
 d. 6% of 110 = h. 30% of 40 is? 20% of 85?

Exercise 1 Answers

- a. $3\frac{3}{5}$ or 3.6 b. 38 d. $6\frac{3}{5}$ or 6.6
 c. 11

$$\begin{array}{llll} \text{e. } \frac{99}{100} \text{ or } 0.99 & \text{f. } 19\frac{1}{5} \text{ or } 19.2 & \text{h. } \frac{12}{17} & \text{i. } \$306 \\ & \text{g. } 30 & \text{j. } 17 & \end{array}$$

Percents Greater Than or Equal to 100%

Remember: $100\% = 1$.

100% of anything is the whole thing. If you spend 100% of your pay cheque, you spend the whole thing. If you get 100% on a test, you have the whole thing correct.

If you have **more than 100%**, you have **more than the whole thing**.

If you spend 110% of your paycheque, you spent more than you earned, and you may be in trouble! It is hard to get more than 100% on a test unless the instructor has given bonus marks for extra questions. You may hear of percents more than 100% in increases, such as costs of housing or inflation.

For example, “The Browns just sold their house and made a 200% profit.” This means they got back what they paid and two times more!

If a percent is less than ($<$) 100, it is less than the whole thing.

$$120\% \text{ of } 50 = 60$$

If a percent is 100, it equals the whole thing.

$$90\% \text{ of } 50 = 45$$

If a percent is more than ($>$) 100, it is more than the whole thing.

$$100\% \text{ of } 50 = 50$$

Exercise 2

Look at the percent. Is it 100? Circle the correct answer for each question. **Do not solve the problems.**

a. **200% of 10** is:

- i. equal to 10
- ii. less than 10
- iii. greater than 10

c. **90% of 75** is:

- i. equal to 75
- ii. less than 75
- iii. greater than 75

b. **50% of 0.25** is:

- i. equal to 0.25
- ii. less than 0.25
- iii. greater than 0.25

d. **$33\frac{1}{3}\%$ of 15** is:

- i. equal to 15
 - ii. less than 15
 - iii. greater than 15
- e. **100% of 100** is:
- i. equal to 100
 - ii. less than 100
- f. **127% of 936** is:
- i. equal to 936
 - ii. less than 936
 - iii. greater than 936

Exercise 2 Answers

- a. iii) greater than 10
- b. ii) less than 0.25
- c. ii) less than 75
- d. ii) less than 15
- e. i) equal to 100
- f. iii) greater than 936

Exercise 3

Use the proportion method to solve these questions.

- a. $16\frac{2}{3}\%$ of 12 = ? e. What is $37\frac{1}{2}\%$ of 80? h. **260% of 45** is?
- b. What is **60% of 15?** i. What is **109% of 200**?
- c. **75% of 144** is? f. **25% of 52** is? j. **98.75% of 50** = ?
- d. **30% of 90** = **8.2% of 300**

Exercise 3 Answers

- a. 2
- b. 9
- c. 108
- d. 27

- e. 30
- f. 13
- g. 24.6
- h. 117

- i. 218
- j. 49.375

Taxes

The amount of tax to be paid is calculated by finding a percent of a number. The tax rate is usually given as a percent.

The basic proportion for these problems is:

$$\frac{\text{tax (part)}}{\text{taxable amount (whole)}} = \frac{\% \text{tax}}{100}$$

Please note that the tax rates used in the questions in this book are for the year 2010 and are subject to change.

The British Columbia Harmonized Sales Tax (HST) is 12%. In B.C., the provincial portion of the harmonized sales tax does not have to be paid on children's clothes, food, books, gasoline and diesel fuel, and other special items.

Example C

How much HST (12%) will be charged on a new kitchen table that cost \$125?

Use proportion:

$$\frac{\text{HST}}{\$125} = \frac{12}{100}$$

$$100\text{HST} = 12 \times 125 = \$1,500$$

$$\text{HST} = \$1,500 \div 100 = \$15.00$$

HST on a \$125 table is \$15.00.

Exercise 4

Find the HST (12%) tax and total cost of each item.

a. Clothes: \$130	c. New Car: \$10 000
b. Washing Machine: \$589	d. Shoes: \$59.99

Exercise 4 Answers

a.	HST: \$15.60 Total Cost: \$145.60	c.	HST: \$1 200 Total Cost: \$11 200
b.	HST: \$70.68 Total Cost: \$659.68	d.	HST: \$7.20 Total Cost: \$67.19

Income tax is charged at different percentages according to the amount of a person's taxable income. The first \$28 000 of taxable income is taxed at 17%.

Note: Other tax rules and charges may apply in real situations.

Example D

If a person's taxable income for the year is \$23 400, what amount of income tax will that person pay?

Step 1: To use the proportion method, do this:

$$\frac{\text{tax}}{\text{income}} \rightarrow \frac{\$23\,400}{= \frac{17}{100}}$$

The tax is the part. The income is the whole.

Step 2: Solve for \$3 978.

The income tax on \$23 400 is \$3 978.

Exercise 5

Calculate the income tax for the annual taxable earnings listed. These amounts are all under \$28,000, so the tax rate is 17%.

a. \$18 500	c. \$15 365
b. \$27 620	d. \$25 900

Exercise 5 Answers

a. \$3 145	c. \$2 612.05
b. \$4 695.40	d. \$4 403

Cross-Border Shopping

The Canadian (CAN) and American (US) dollars are not equal in value. The **exchange rate** (the value of one Canadian dollar compared to a dollar from another country) changes often; the current rate is usually available from banks, on the news, in the newspapers and on a website.

In the winter of 2010, the Canadian dollar was around \$0.92 of an American dollar (the ratio is **\$1.00 CAN : \$0.92 US**), so CAN money was valued at 92% of US money.

To find the value of one US dollar in Canadian funds, use this proportion:

$$\frac{\$1 \text{ CAN}}{\$0.92 \text{ US}} = \frac{N \text{ CAN}}{\$1 \text{ US}}$$

$N = \$1 \div \$0.92 = \$1.086$, so US money was valued at 109% of CAN money.

Note: The proportion changes as the exchange rate changes.

What if you buy in the United States?

- Change the US cost to the Canadian equivalent (multiply by 109%).
- If you have more than the purchases allowed (call the **Canada Border Service Agency** for information), the Canadian Customs charge **duty** on the Canadian value of your purchases. The percent of the duty (the rate) varies according to what the item is, where it was made, and the duty rates of the day. For example, duty on poultry is 12.5%, on non-US cotton 25%, and

on liquor 110%!

- Duty is gradually being eliminated under the Canada-US Free Trade Agreement. If an item is made in North America, there is no duty charged because of NAFTA (North American Free Trade Agreement)
- HST (12%) is charged on the duty and the Canadian value of the purchases (which includes any US sales taxes).

Look at this example (assume \$1.00 CAN = \$0.92 US):

Men's leather shoes, US price	\$64.80

US sales tax 6%	3.89

US total cost	68.69

Equivalent cost in CAN funds (US cost $\times$ 109%)

$$\frac{N}{68.69} = \frac{109}{100} \quad \$74.87$$

Duty on leather shoes is 22.8%	\$17.07

HST (12%) on CAN value plus duty

12% of \$74.19 and 16.92 together	\$11.03

The total cost of a pair of leather shoes priced at \$64.80 in the United States will be the American price in Canadian funds + duty + HST = \$102.97 CAN.

Exercise 6

For each item, do the calculations using the duty and tax rates given. Assume \$1.00 US is \$1.09 CAN.

Groceries, US price \$75.

a. 3% US sales tax	d. Duty at 10.2% (No HST on food)
b. US total cost	e. Total cost in Canadian funds
c. Equivalent value in CAN funds	

Exercise 6 Answers

a. \$2.25	d. \$8.59
b. \$77.25	e. \$92.79
c. \$84.20	

Increases & Decreases, Discounts & Mark-Ups

Increases (amount changing to more) and decreases (amount changing to less) are often given as a percent. For example:

- The Insurance Corporation of B.C. increased car insurance rates by 3.3% in March 2007.
- The number of acute care beds at the local hospital has

decreased by 28% in the last year.

- The new work contract provides a 4% wage increase in the first year and a $2\frac{1}{2}\%$ increase in the second year.

The amount of an increase or decrease is calculated by **finding a percent of a number**. When the percent of an increase or decrease is given, the proportion is:

$$\frac{\text{amount of increase or decrease}}{\text{whole amount}} = \frac{\text{increase or decrease \%}}{100}$$

Discounts are a form of decrease. The discount is the amount taken off a price; it is the price reduction.

Sale prices (discounted prices) may be advertised as:

- “All items 20% off.”
- “Everything in stock reduced by 25% to 50%.”
- “ $33\frac{1}{3}\%$ savings!”
- “2% discount for cash.”

Decrease and discount problems may need to be solved in several steps. Sometimes, the problems ask for:

- the amount of the decrease (may be called the “saving”) (1 step)
- the amount left **after** the decrease (2 steps)

Example E

The sign says, “All winter coats 40% off.” How much

money will you **save** on a coat originally priced at \$128.99?
 What is 40% of \$128.99?

1 Step Problem

$$\frac{\text{decrease}}{\text{original price}} \longrightarrow \frac{D}{\$128.99} = \frac{40}{100}$$

$$100D = 40 \times \$128.99$$

$$D = \$5\,129.60 \div 100 = \$51.296$$

$$\text{round to nearest cent} = \$51.30$$

You will save \$51.30.

Example F

The couch and chair are advertised in a $33\frac{1}{3}\%$ price reduction sale. How much will you **pay** for a couch and chair originally priced at \$798?

2 Step Problem

Step 1: Find the amount of savings (the decrease).

Recall from p. 70 that it's best to substitute $\frac{1}{3}$ for $33\frac{1}{3}\%$.

$$\frac{\text{saving}}{\text{full cost}} \longrightarrow \frac{S}{798} = \frac{1}{3}$$

Step 2: Subtract the savings from the original amount.

$$\text{original amount} - \text{savings} = \text{sale price}$$

Find the amount of savings:

$$\frac{S}{798} = \frac{1}{3}$$

$$798 \times \frac{1}{3} = S$$

$$\frac{798}{1} \times \frac{1}{3} = S$$

$$266 = S$$

Find the sale price:

$$\begin{array}{rcl} \text{original amount} - \text{savings (decrease)} & = & \text{sale price} \\ \$798 - \$266 & & = \$532 \end{array}$$

The couch and chair will cost \$532 on sale (plus HST, of course, but you do not have to calculate tax for this problem).

Exercise 7

Solve the following problems. Round all answers to the nearest cent.

- a. The employees agreed to take a 5% pay cut (reduction), so no one will be laid off. If the pay rate was \$15.50 per hour, how much less per hour would the workers earn?
- b. “All shoes 25% off,” says the sign. What will be the sale price of a pair of dress shoes originally priced at \$69.98?
- c. The workforce at the factory has to be reduced by $16\frac{2}{3}\%$ over the next two years. Early retirements, attrition (not replacing people who leave) and some lay-offs will be used. The workforce is 3 000 people right now. What is the planned size of the workforce in two years?

Exercise 7 Answers

- a. \$0.78
- b. \$52.48
- c. 2 500 people

Increases and mark-ups are calculated in the same way as decreases

and discounts. However, an increase or mark-up is **added** to the original amount.

Example G

The auto insurance rate increased by 19%. The basic insurance rate for Don's car was \$550 before the increase. What is the basic insurance after the increase?

Step 1: Calculate the amount of the increase.

Find 19% of \$550.

$$\frac{\text{amount of increase}}{\text{present cost}} \longrightarrow \frac{N}{550} = \frac{19}{100}$$

$$\begin{aligned} 100N &= 550 \times 19 \\ N &= 104.5 \end{aligned}$$

Step 2: Add the amount of increase to the original amount.

$$\begin{aligned} \text{original amount} + \text{increase} &= \text{new insurance cost} \\ \$550 + \$104.50 &= \$654.50 \end{aligned}$$

Don's new basic insurance is \$654.50.

Mark-ups are the amount added to the cost price before an item is resold. Many factors must be considered when businesses decide on the percent of the mark-up:

- all costs of operating a business
- the profit wanted
- the community the business is in
- the competition the business has

For example, the mark-up on leather shoes may be 45%, but on running shoes, it may be 60%. Kitchen appliances might have a 42% mark-up, while lawnmowers might have a 55% mark-up.

Example H

A shoe seller pays \$40.00 per pair of running shoes from the factory. The shoe seller makes the mark-up 75%. What is the selling price of the shoes?

Step 1: What is 75% of \$40.00?

$$\frac{\text{mark-up cost}}{\text{original cost}} \longrightarrow \frac{N}{40} = \frac{75}{100}$$

$$100N = 3,000$$

$$N = 30$$

75% of \$40.00 is \$30.00.

Step 2: Add the mark up to the original cost to get the selling price of the shoes.

$$\$40 + \$30 = \$70.00.$$

Having a **wage increase** at work is always a good thing! Often, the raise will be given as a percentage. That means that everyone will see more money on their paycheque, but they will each have a different amount because they all get paid a different amount to start with.

Example I

The boss at A-1 House Painting will give a 1.5% wage increase to the 10 employees.

- a. 3 staff are paid the minimum wage of \$13.85 an hour.
- b. The other 7 staff are paid \$21.00 an hour.

a. What is 1.5% of \$13.85?

$$\frac{\text{increase}}{\text{present wage}} \longrightarrow \frac{I}{13.85} = \frac{1.5}{100}$$

$$100I = 13.85 \times 1.5$$

$$I = \$0.21$$

The new wage will be the old wage plus the increase:

$$\$13.85 + \$0.21 = \$14.06 \text{ per hour}$$

b. What is 1.5% of \$21.00?

$$\frac{\text{increase}}{\text{present wage}} \longrightarrow \frac{I}{21} = \frac{1.5}{100}$$

$$100I = 21 \times 1.5$$

$$I = \$0.32$$

The new wage will be the old wage plus the increase:

$$\$21.00 + \$0.32 = \$21.32 \text{ per hour}$$

Exercise 8

Solve the following problems. Round money to the nearest cent.

- a. If the mark-up on the craft supplies was set at 75%, calculate the mark-up and selling price for these items.

Example: Silk Flowers: \$1.48

$$\text{Mark-up (75\%): } \frac{N}{1.48} = \frac{75}{100} = \$1.11$$

$$\text{Selling Price: } \$1.48 + \$1.11 = \$2.59$$

- i. Stuffing: \$4.50/bag
- ii. Beads: \$3.20/dozen

- b. The population of the town has increased by 30% since the pulp mill was built. The population before the pulp mill was 8 436 people. What is the population now?
(Round to the nearest person.)
- c. The wage contract gave the workers a 4% increase in the first year and a $2\frac{1}{2}\%$ increase in the second year. If the hourly rate of pay was \$12.45 before the new contract, calculate the following:
- the amount of the increase per hour in the first year.
 - the hourly pay rate in the first year.
(old pay + increase = first year rate)
 - the amount of pay increases in the second year.
(**Note:** Use the new hourly pay rate from the first year to calculate the increase for the second year.)
 - the hourly pay rate in the second year.
(first year rate + increase = second year rate)

Exercise 8 Answers

- a. Answers
- Mark-up: \$3.38
Selling Price: \$7.88
 - Mark-up: \$2.40
Selling Price: \$5.60

b. 10 967 people

c. Answers

i. \$0.50

ii. \$12.95

iii. \$0.32

iv. \$13.27

Commission & Tips

Salespeople may receive a **commission** as part or all of their pay. The business owner pays the salesperson an agreed-upon percent of the selling price of the product.

- Real estate agents are paid by commission on their sales.
- Car and truck salespeople may be paid a small salary per month, but their main income is the commission on the vehicles they sell.

Tips are appreciation payments for service. The customer gives tips directly to the worker. Taxi drivers, waiters, bellhops and chambermaids in hotels often receive a minimal hourly wage. A large part of their earnings is from tips. In restaurants, expect to leave at least a 15% tip for adequate service.

To calculate the amount of a commission (or a tip), find the percentage of the total amount using the proportion:

$$\frac{\text{commission } \square}{\text{total amount}} = \frac{\text{commission } \%}{100}$$

The commission is the part. The total amount is the whole.

Commission problems often have several steps. You may have to:

- Add together several items to find the total of the sales.
- Subtract a base amount for which salespeople do not receive a commission.
- Add the amount of commission to the basic wage to find out how much the person earned.

Example J

The bill for the excellent dinner at the restaurant was \$56.40. The service had been good, and the waiter was very pleasant, so Bill and Diane wanted to leave at least a 15% tip.

To calculate the tip, find 15% of \$56.40.

The tip is the part. The bill is the whole.

$$\frac{\text{tip}}{\text{bill}} \longrightarrow \frac{N}{56.40} = \frac{15}{100}$$

$$100N = 56.40 \times 15$$

$$N = \$8.46$$

Diane and Bill will round this amount to \$8.50

How much will he pay for the meal and the tip?

cost of dinner	\$56.40
+tip	\$8.50
	\$64.90

In a **real** situation, we would probably round the amount of the bill to the nearest dollar and then calculate the tip.

Example K

The salespeople at XW Ford receive a monthly salary of \$1 000. They also receive a 12% commission on any sales over \$35 000 in a month. This means they are expected to sell \$35 000 worth of vehicles every month to earn the \$1 000 salary.

If a saleswoman made \$54 000 in sales one month, what would her gross earnings be?

You are asked to find the gross monthly earnings. What do you know?

- She earns \$1 000 per month.
- She earns a 12% commission on sales over \$35 000.
- She had \$54 000 in sales.

Step 1: Find the amount of commissionable sales.

Subtract the base amount for which she will not earn a commission from her total sales.

$$\begin{aligned}\text{total sales} - \text{base amount} &= \text{amount of sales that commission will be paid for} \\ \$54\,000 - \$35\,000 &= \$19\,000 \text{ in commissionable sales}\end{aligned}$$

Step 2: Calculate the commission.

What is 12% of \$19 000?

$$\frac{\text{commission}}{\text{commissionable sales}} \longrightarrow \frac{X}{19,000} = \frac{12}{100}$$

$$100X = 19\,000 \times 12$$

$$100X = 228\,000$$

$$X = \$2\,280$$

Step 3: Add the salary and commission to find gross earnings.

$$\$1\,000 + \$2\,280 = \$3\,280.$$

The saleswoman earned \$3 280.

Exercise 9

Solve the following problems.

- a. A clerk sold \$18 000 worth of clothes last year. He was paid a 15% commission. How much was his

commission?

- b. Mr. Green receives a weekly salary of \$325 plus a commission of 10% on all sales he makes over \$1 500. Last week, Mr. Green sold \$3 500 worth of merchandise. How much money did he earn last week?
- c. The final bill at the restaurant is \$160, and you want to leave a 15% tip.
 - i. What amount of tip should you leave?
 - ii. What is the total cost (bill and tip)?

Exercise 9 Answers

- a. \$2 700
- b. \$525
- c. Answers
 - i. \$24.00
 - ii. \$184.00

Exercise 10

Solve the following problems.

- a. To successfully pass the course, the student must get at least 80% on the test. The test is out of 125. What mark will give the student 80%?
- b. George made a $12\frac{1}{2}\%$ down payment on a new car, which cost \$3,200. How much was the down payment?
- c. In B.C., employers are required to pay 6% holiday pay to all employees. Holiday pay is added to their regular salary if a paid vacation is not taken. The young grocery clerk who earns \$8.00 an hour worked 25 hours last week.
 - i. What amount of holiday pay is he eligible for?
 - ii. His employer pays holiday pay on each cheque to part-time employees. What are his total earnings for the week? (salary + holiday pay)
- d. Nutrition experts recommend that no more than 30% of the food calories a person consumes should be from fats. Foods such as fatty meat, dairy products with lots of butter fat, cooking oils, margarine, some salad dressings, and nuts contain a high percentage of fat. If a person's daily intake of calories is 2,560,

what is the most number of calories that should be from fat?

Exercise 10 Answers

- a. 100 marks
- b. \$400
- c. Answers
 - i. \$12.00
 - ii. \$212.00
- d. 768 calories

5.9: Practice Questions

1. Answer the following.
 - a. 6% of 30 =
 - b. $\frac{1}{4}\%$ of 48 is?
 - c. Find 131% of 400.
 - d. Write the general proportion that can be used to solve percent problems.
2. Solve the following word problems.
 - a. The \$125 000 house is insured for 85% of its value against

- fire damage. How much money should the owner receive if the house is destroyed by fire?
- b. Charlotte sells leisure clothes in her small town for a large national company. She receives \$500 a month, and a 20% commission on all monthly sales over \$1 000. What are her monthly earnings if her total sales are \$2 300?
 - c. The barbeque was originally priced at \$599 but Jack bought it during a 35% off, end-of-season sale.
 - i. What was the sale price?
 - ii. Calculate the harmonized sales tax (12%).
 - iii. Give the total cost of Jack's barbeque.

5.9: Practice Answers

1. Answer the following.
 - a. 1.8
 - b. 0.12
 - c. 524
 - d. $\frac{\text{part}}{\text{whole}} = \frac{\%}{100}$ or $\frac{\text{is}}{\text{of}} = \frac{\%}{100}$
2. Solve the following word problems.
 - a. \$106 250
 - b. \$760.00
 - c. Answers
 - i. \$389.35
 - ii. \$46.72
 - iii. \$436.07

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5.10: Working With Percent Review

1. Solve each problem by setting up a proportion.

- a. 150% of 7 d. 18% of 40 f. 0.5% of 122
b. 12.5% of 48 e. $\frac{1}{4}$ % of 20
c. 20% of 16

2. Use the proportion method to solve these questions.

- a. 75% of 12 c. 16% of 34 f. $16\frac{2}{3}$ % of 163
b. $3\frac{1}{4}$ % of 200 d. 90% of 75
e. 10% of 27

3. Find the total cost of each item using 12% HST.

Example: T-shirt: \$14.99

$$\text{HST (12\%): } \frac{X}{14.99} = \frac{12}{100}$$
$$X = \$1.80$$

$$\text{Total Cost: } \$1.80 + \$14.99 = \$16.79$$

- a. 40" flat screen TV: \$699.00
b. Tent: \$168.79
c. Drill and bit set: \$248.99
d. Shoes: \$79.98
4. Solve the following percent problems.
- a. "All pants are 25% off," said the sign in the window. What will be the sale price of a pair of pants originally priced at \$59.78?
- b. 30% of the forest will be cut down in the next three years. The forest is 150 acres large. How many acres will be left

after the cut?

- c. All winter clothes are on sale for 15% off. A set of gloves cost \$18.00. How much will you **save**?
- d. Rent will increase by 10% on Oliver's apartment on January 1. His rent was \$460.00 per month. What will his new rent be?
- e. The French Bread Bakery staff will get a one-time 15% pay increase on their Christmas paycheck. If Manon works 63 hours at \$12.78/hour for that pay check, how much will she get paid? (Figure out the regular pay first and then add o the 15% increase.)
- f. The bill at the restaurant was \$38.00. Add a 20% tip. What did the customer pay in total?
- g. The mark up on a pair of fancy running shoes is 45%. The shoes cost the buyer \$38.00. How much will they be sold for after mark-up?

5.10: Review Answers

1. Solve each problem by setting up a proportion.

a. 10.5	c. 3.2	e. 0.05
b. 6	d. 7.2	f. 0.61

2. Use the proportion method to solve these questions.

a. 9	c. 5.44	e. 2.7
b. 6.5	d. 67.5	f. 27.16

3. Find the total cost of each item, using 12% HST.

a. HST: \$83.88	c. HST: \$29.88
Total Cost: \$782.88	Total Cost: \$278.87
b. HST: \$20.25	d. HST: \$9.60
Total Cost: \$189.04	Total Cost: \$89.58

4. Solve the following percent problems.

a. \$44.83	d. \$506.00	g. \$55.10
b. 105 acres	e. \$925.91	
c. \$2.70	f. \$45.60	

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5.11: Finding What Percent One Number Is of Another

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}$$

In problems where you must find what percent one number is of another, the **missing term** is the **percent**. You will be told the part (is) and the whole (of), you know the 100, and you solve for the missing percent.

Example A

4 is what percent of 5?

- 4 is the part (is)
- 5 is the whole (of)
- % is unknown (call it P)

Step 1: Write the proportion.

$$\frac{4}{5} = \frac{P}{100}$$

Step 2: Cross-multiply to solve.

$$4 \times 100 = 5 \times P$$

$$400 = 5P$$

$$400 \div 5 = P$$

$$80\% = P$$

Be sure to write the percent sign (%).

In percent problems, the number after “of” **usually** is the whole. The number close to “is” **usually** is the part. You may find it helpful to think “is over of.”

Like this:

$$\frac{\text{is}}{\text{of}}$$

An equal to sign (=) can substitute for “is.”

12 is what percent of 15?

$$\frac{\text{is}}{\text{of}} \text{ will help to find } \frac{\text{part}}{\text{whole}}$$
$$\frac{12 \text{ (is)}}{15 \text{ (of)}} = \frac{N}{100}$$

Example B

What percent of 85 is 60?

- 60 = the part (close to “is”)
- 85 = the whole (after “of”)
- % = the unknown

Step 1: Set up the proportion: $\frac{\text{is}}{\text{of}}$

$$\frac{60}{85} = \frac{P}{100}$$

Step 2: Simplify.

$$\frac{\cancel{60} 12}{\cancel{85} 17} = \frac{P}{100}$$

$$\frac{12}{17} = \frac{P}{100}$$

Step 3: Cross-multiply to solve.

$$12 \times 100 = 1\,200, \text{ and } 17 \times P = 17P$$

$$1\,200 = 17P$$

$$1\,200 \div 17 = P$$

$$70.588\% = P$$

Round to 70.6%.

Exercise 1

The following examples ask you to find what percent one number is of another. The **missing term** is the **percent**.

Look carefully at the wording and decide which number is the part (close to “is”) and which number is the whole thing (after “of”). Write the proportion but **do not** solve the problem.

- a. 3 is what % of 6? c. What % of 27 is 9? 25?
 b. 12 is _____% of 5? d. What % of $\frac{1}{2}$ is $\frac{1}{4}$? f. _____% of 64 = 48
 e. _____% of 50 is 48

Exercise 1 Answers

- a. $\frac{3}{6} = \frac{N}{100}$ c. $\frac{9}{27} = \frac{X}{100}$ e. $\frac{X}{100} = \frac{25}{50}$
 b. $\frac{F}{100} = \frac{12}{5}$ d. $\frac{\frac{1}{4}}{\frac{1}{2}} = \frac{P}{100}$ f. $\frac{48}{64} = \frac{N}{100}$

Exercise 2

Solve each question by first setting up the proportion

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}.$$

a. 3 is what percent of 60?	e. What percent of 350 is 42?	i. 2 is _____% of 125.
b. 3 is what percent of 4?	f. 15 is _____% of 12.	j. 46 is _____% of 40.
c. 1 is what percent of 3?	g. 14 is _____% of 700.	
d. What % of 50 is 35?	h. What percent of 96	

Exercise 2 Answers

a. 5%	e. 12%	i. 1.6%
b. 75%	f. 125%	j. 115%
c. $33.\bar{3}\%$	g. 2%	
d. 70%	h. 12.5%	

Exercise 3

Solve the following questions by setting up the proportion.

a. 16 is _____ % of 64.	e. 1 is _____ % of 200.	27. _____ % of 48 = 18.
b. 17 is _____ % of 85.	f. 36 = _____ % of 12.	125 = _____ % of 75.
c. What % of 52 is 13?	g. _____ % of 72 = 12.	What % of 18 is 24?
d. What percent of 125 is 12?		

Exercise 3 Answers

a. 25%	f. 300%	$166\frac{2}{3}\%$	
b. 20%	g. 37.5%	j. $133.\overline{3}$	or
c. 25%	h. 37.5%	or $133\frac{1}{3}\%$	
d. 60%	i. $166.\overline{6}$		
e. 0.5%			

Finding the Percent of an Increase or Decrease

You learned to find the amount of an increase (gain) or decrease (loss) when given the percent of the increase or decrease.

Now, you are going to find the percent of the increase or decrease when you are given the amounts. This is called the rate of increase or decrease.

Problems which ask you to find the percent of increase or decrease often involve two steps:

1. Find the amount of change (either increase or decrease) by finding the difference between the two amounts given. Subtract to find the difference.
2. Find the percentage of increase/decrease. Always compare the change (amount of increase or decrease) to the amount before the change (the original amount) using this proportion.

$$\frac{\text{amount of increase or decrease}}{\text{original amount}} = \frac{P}{100}$$

Example C

The rent went from \$375 a month to \$427.50 a month. What is the percent of the increase?

Step 1: Find the change (the amount of increase) by finding the difference between the amounts.

$$\$427.50 - 375 = \$52.50$$

The amount of increase is \$52.50.

Step 2: Find the % of increase.

- The amount of increase is \$52.50.

- The original amount (the amount before the increase) is \$375.
- What % of \$375 is 52.50?

$$\begin{aligned}\frac{X}{100} &= \frac{52.50}{375} \\ 375X &= 5250 \\ X &= \frac{5250}{375} \\ X &= 14\%\end{aligned}$$

The rent increase is 14%.

Example D

The hours of operation at the college were reduced from 35 hours a week to 30 hours a week. What is the percent of this cut in operations?

Step 1: Find the amount change (a decrease) by finding the difference between the amounts.

$$35 \text{ hours} - 30 \text{ hours} = 5 \text{ hours}$$

The amount of decrease is 5 hours.

Step 2: Find the % of increase.

- Decrease is 5 hours.
- Original amount is 35 hours.
- What percent is 5 of 35?

$$\begin{aligned}\frac{5}{35} &= \frac{P}{100} \\ 500 &= 35P \\ \frac{500}{35} &= P \\ 14\frac{2}{7}\% &= P\end{aligned}$$

The hours of operation at the college were cut $14\frac{2}{7}\%$.

Exercise 4

Solve the following problems.

- Ms. Lister's bi-weekly unemployment cheque increased from \$405 to \$435. What percent increase is this?
- Joan weighed 72 kg before she went on a program of strict exercise and careful eating. She now weighs

- 60 kg. What is the percent of her weight loss?
- c. The car dealership gives a special deal if the customer does not have a trade-in and pays cash. The dealers will only charge \$10 650 for a car listed at \$12 000. What is the percent savings in this deal?
- d. A regular toilet uses 20 litres of water per flush. By purchasing a new low-flow toilet, the water use is 6 litres per flush. What is the percent savings of water per flush if the new tank is used?

Exercise 4 Answers

a. 7.4% increase	c. 11.25% savings
b. 16.6% decrease	d. 70% savings

Other Problems

Many situations compare one number to another:

- 24 out of 25 on the test.
- 6 out of 10 people are overweight.
- The government spends 27¢ of every federal tax dollar on the national debt.

These numbers are often more easily thought about if written as a percent.

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}$$

The following problems ask you to find what percent one number is of another. Often, several steps are involved to calculate the part or to calculate the whole (as in question e).

You may be asked to use the % after you find it.

Remember: The whole thing = 100%.

Exercise 5

Solve the following word problems.

- a. The Doal family's net income is \$2 300 per month. Their mortgage payment is \$750 each month. What percent is the mortgage payment of their monthly income?
- b. Jean played on the college volleyball team and missed a lot of classes when she travelled to tournaments. She missed nine of the 42 English classes last semester.
 - i. What percent of her English classes did she miss?
 - ii. What percent of her English classes did she attend?
- c. Four women and six men serve on the Village Council. What percent of the council members are

- women?
- d. Gail bought a \$500 G.I.C. (Guaranteed Investment Certificate) one year ago. She was delighted to receive her annual interest cheque of \$52.50 today. What percent interest did Gail's G.I.C. pay for that year?
 - e. If a math book had 320 pages and you still had 110 pages left to do, what percent of the book had you finished? (2 steps)

Exercise 5 Answers

- a. 32.6% of monthly income
- b. Answers
 - i. 21.4% of English classes
 - ii. 78.6% of English classes
- c. 40% of members are women
- d. 10.5% interest
- e. 65.625% or 65 $\frac{5}{8}$ % of the book

School Grades

When looking at test results, the mark shows how you did on the test.

If you get $\frac{7}{10}$ on a test, you know you got 7 answers right and 3 answers wrong.

Sometimes, it is also helpful to see your mark as a percentage.

Example E

By solving for N, the percentage can be found:

$$\begin{aligned}\frac{7}{10} &= \frac{N}{100} \\ 7 \times 100 &= N \times 10 \\ \frac{\cancel{700} 70}{\cancel{10}} &= \frac{\cancel{10} N}{\cancel{10}} \\ \frac{70}{1} &= \frac{N}{1} \\ 70 &= N\end{aligned}$$

$$\text{So, } \frac{7}{10} = 70\%.$$

Now, you can see that the test mark of $\frac{7}{10}$ equals.

Example F

The test result was $\frac{15}{43}$, what was the percent on the test?

$$\frac{15}{43} = \frac{N}{100}$$
$$15 \times 100 = N \times 43$$

$$\frac{1500}{43} = \frac{\cancel{43} N}{\cancel{43}}$$

$$\frac{1500}{43} = \frac{N}{1}$$

$$34.88\% = N$$

If you round, $N = 35\%$.

Not such a great mark!

Example G

Find the percent of the following grade: $\frac{89}{97}$.

$$\frac{89}{97} = \frac{P}{100}$$

$$89 \times 100 = P \times 97$$

$$\frac{8900}{97} = \frac{\cancel{97} P}{\cancel{97}}$$

$$\frac{8900}{97} = \frac{P}{1}$$

$$91.75\% = P$$

$$P = 91.75\% \text{ or } 92\%.$$

Exercise 6

Find the percents for the following test grades. Round your answer to the nearest percent.

a. $\frac{33}{42}$

b. $\frac{24}{40}$

c. $\frac{90}{120}$

d. $\frac{100}{110}$

e. $\frac{10}{20}$

Exercise 6 Answers

a. 79%

b. 60%

c. 75%

d. 91%

e. 50%

5.II: Practice Questions

1. Solve to find the missing percents.

a. 12 is _____% of 60.

b. 15 is _____% of 75.

c. $8.2 =$ _____% of 32.8.

d. What percent of 64 is 48?

2. Solve the following word problems.
 - a. The \$140 jacket was on sale for \$126. What percent is the savings?
 - b. The rent on the apartment went from \$320 a month to \$400 dollars a month. What percent is this rent increase?
3. Find the percents for the following grades on tests. Round your answer to the nearest whole percent.

a. $\frac{12}{15}$
 b. $\frac{10}{19}$

c. $\frac{71}{92}$
 d. $\frac{132}{140}$

5.11: Practice Answers

1. Solve to find the missing percents.
 - a. 20%
 - b. 20%
 - c. 25%
 - d. 75%
2. Solve the following word problems.
 - a. 10% savings
 - b. 25% increase
3. Find the percents for the following grades on tests.
 - a. 80%
 - b. 53%
 - c. 77%
 - d. 94%

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5.12: Finding a Number When a Percent of It Is Given

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}$$

In problems where a certain percentage of a number is given, the **missing term is the whole**. You will be told the % and the part (is) and asked to find the whole (of), which is 100%.

Example A

20% of what number is 14?

- part = 14
- whole (“what number”) = unknown (call it N)
- percent = 20%

Step 1: Write the proportion.

$$\frac{14}{N} = \frac{20}{100}$$

Solve the proportion:

Step 2: Simplify.

$$\frac{14}{N} = \frac{\cancel{20}^1}{\cancel{100}_5}$$
$$\frac{14}{N} = \frac{1}{5}$$

Step 3: Cross-multiply to solve.

$$14 \times 5 = N \times 1$$

$$70 = N$$

$$20\% \text{ of } 70 = 14$$

Check: Find 20% of 70.

The answer should be 14.

$$\frac{20}{100} = \frac{N}{70}$$
$$140 = 100N$$
$$14 = N$$

Example B

$33\frac{1}{3}\%$ of _____ is 60.

- part = 60
- whole = N
- percent = $33\frac{1}{3}\% = \frac{1}{3}$

Set up proportion:

$$\begin{aligned}\frac{60}{N} &= \frac{1}{3} \\ 3 \times 60 &= N \times 1 \\ 180 &= N\end{aligned}$$

$33\frac{1}{3}\%$ of 180 is 60.

Check: Find $33\frac{1}{3}\%$ of 180.

The answer should be 60.

Exercise 1

Set up the proportion. **Do not** solve the question.

- a. 18 is 50% of what number?
- b. 24 is 15% of what number?
- c. 200% of what number is 86?
- d. $66\frac{2}{3}\%$ of what number = 500?

Exercise 1 Answers

a. $\frac{18}{P} = \frac{50}{100}$

b. $\frac{24}{N} = \frac{15}{100}$

c. $\frac{86}{P} = \frac{200}{100}$

d. $\frac{500}{P} = \frac{66\frac{2}{3}}{100}$ or $\frac{2}{3}$

Exercise 2

Solve the following. Check your answers to see if you set up the proportion correctly.

- a. 60 is 75% of what number?
- b. 950 is 95% of:
- c. $125 = 33\frac{1}{3}\%$ of what number?
- d. 120% of _____ is 6.
- e. 270 is 100% of what number?

Exercise 2 Answers

- a. $\frac{60}{X} = \frac{75}{100}$, 80
- b. $\frac{950}{P} = \frac{95}{100}$, 1,000
- c. $\frac{125}{N} = \frac{33\frac{1}{3}}{100}$, 375
- d. $\frac{120}{100} = \frac{6}{J}$, 5
- e. $\frac{100}{100} = \frac{270}{N}$, 270

Exercise 3

Solve the questions.

- a. 480 is $66\frac{2}{3}\%$ of _____.
- b. 40% of _____ is 50.
- c. $33\frac{1}{3}\%$ of what number is 99?
- d. 3 is 150% of what number?
- e. 122 is 80% of _____.

Exercise 3 Answers

a. 720	d. 2
b. 125	e. 152.5
c. 297	

Solving Problems When the Percent of a Number Is Given

Read the problems carefully. More than one step may be needed. Look at the wording so you will recognize problems missing the whole and be able to tell them from problems missing the part.

Exercise 4

Solve the following problems. Round money to the nearest cent.

- a. When the business was declared bankrupt, all the creditors (people owed money by the business) were

paid 55% of the money owed them.

- i. If a creditor received \$8 000 from the bankrupt business, how much money had he really been owed?
 - ii. How much money did the creditor lose on this business?
- b. In telethons and other fund-raising events, records show that about 80% of the money pledged is actually collected. The local telethon organizers need to raise \$12 000. To actually raise \$12 000, their goal for pledges should be what amount?
- c. The shoe store sold all merchandise at 25% off in a huge clearance sale. They took in \$3 500 the first day of the sale. If the same shoes had been sold at the regular price, how much money would they have taken in?
(**Note:** this problem has 2 steps. The merchandise was 25% off, so it sold for $100\% - 25\% = 75\%$ of the original price.)
- d. A common rate of commission earned by real estate agents is $3\frac{1}{2}\%$. If an agent had a gross income of \$63 000 from commissions in one year, what was the value of the houses sold?

Exercise 4 Answers

- a. Answers
- i. \$14 545.45

- ii. \$6 545.45
- b. \$15 000.00
- c. \$4 666.67
- d. \$1 800 000

5.12: Practice Questions

1. Solve these questions.

- a. 2.5% of what number is 160?
- b. 5 is 4.5% of _____.
- c. 180 is 90% of what number?
- d. 28 is 35% of what number?

2. Solve the following word problems.

- a. If a bank insists that new house buyers have a cash down payment of 12%, what house price can a couple afford if they have saved a \$15,000 down payment?
- b. Jim has really cut down on his smoking. He now smokes 7 cigarettes a day, which he says is only 20% of what he used to smoke. How many cigarettes a day did Jim smoke before he started cutting down?

5.12: Practice Answers

1. Solve these questions.

- a. 6 400
- b. 625
- c. 200
- d. 80

2. Solve the following word problems.

a. \$125 000

b. 35 cigarettes per day

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5.13: More Working With Percent Review

You have been practising three types of percent problems. You have learned that one proportion can be used to solve all the problems:

$$\frac{\text{is (part)}}{\text{of (whole)}} = \frac{\%}{100}$$

1. Finding a Percent of a Number

- You are given the percent and the whole.

- The missing term is the part (call it N).
$$\frac{N}{\text{whole}} = \frac{\%}{100}$$

2. Finding What Percent One Number is of Another

- You are told the part and the whole.
- The missing term is the percent (call it P).

$$\frac{\text{part}}{\text{whole}} = \frac{P}{100}$$

3. Finding a Number When a Percent of it is Given

- You are given the part and the percent.
- The missing term is the whole (call it W).

$$\frac{\text{part}}{W} = \frac{\%}{100}$$

Real-life situations and real math problems often require several steps to collect and organize all the information. Look for those extra steps in the problems that follow.

When you read the problems, look for the part, the whole and the percent. Decide which term is missing. Once you know which

term is missing, the problem can be solved by using the proportion

$$\frac{\text{part}}{\text{whole}} = \frac{\%}{100} \text{ or the appropriate short method.}$$

More Working With Percent Review

Solve these problems using proportion. **Write all your work with the problem so your instructor can help you should you have any difficulty.** Remember to check that the answer makes sense and to write a sentence answer. For these problems, round your answers this way:

- Percents to the nearest tenth of a percent
 - Money to the nearest cent
 - Decimal fractions to the nearest thousandth
1. You need a minimum of 80% on the test, which means you must get at least 36 marks. What are the total possible marks on the test?
 2. Ann and Joe bought their home twelve years ago and have paid 45% of the principal amount of their mortgage. They have paid \$18 000 towards the principal of the mortgage. What was the principal amount of the mortgage to start with?
 3. The waiters at the restaurant must contribute money to be shared among the cocktail servers and kitchen staff. Each waiter contributes 4% of his or her total food and drink sales. Craig's total sales were \$645 in his 5-hour shift.
 - a. How much money must he contribute to the kitchen and cocktail servers?
 - b. Craig made 12% in tips on his sales tonight. What amount were his tips?

4. The cost of hydroelectric power for our home last year was 210% of what it was six years ago. Last year, our power bill totalled \$960. How much was it six years ago?
5. 16 $\frac{2}{3}$ % of the tickets for the rock concert were sold in the first hour the telephone order lines were open. In that hour, 2 500 tickets were sold. What was the total number of tickets available for the concert?
6. The total student enrolment in the school district has increased from 18 506 to 19 724 students in the last year. What is the percent of this increase in student enrolment?
7. Al bought his secondhand off-road motorcycle for \$1 500 and sold it three years later for \$1 175. By what percent did his motorcycle depreciate (decrease in value)?
8. Pat operates a street vendor's cart selling hot dogs, sausages on a bun and soft drinks. The basic pay is \$100 per week with a 28% commission on all sales over \$450 in a week. Pat sold \$1 244 of food and soft drinks last week. Calculate Pat's earnings from the street-vending cart for the week.
9. The 1 500 blouses purchased by the large retail chain of ladies' clothing stores cost the company a total of \$24 000. The blouses were then priced to sell at \$45 each. What is the percent of the mark-up on these blouses?
(**Hint:** First, calculate the company's cost price for each blouse.)
10. Maureen was happy to see that she got 27 out of 30 on her English essay. What percent did she get?
11. The ski jackets were on the summer clearance rack marked "45% off".
 - a. What is the sale price of a jacket priced regularly at \$229.95?
 - b. What is the total cost of this jacket with H.S.T (12%)?
12. Tuition fees at the university have increased from \$49 per credit hour to \$62 per credit hour in the last three years. What

is the percent of the tuition increase?

13. Jill bought 3 bottles of liquor in the US. The bill, including US sales tax, was \$28.50. Assume the American dollar was \$1.08 Canadian. Calculate:

- the value of the liquor in Canadian funds.
- duty at 110%.
- HST at 12% (on Canadian value + duty).
- total cost in Canadian funds.

14. The consignment store will sell your good used women's clothing for you. The store owners take a percentage of the selling price as their fee for service. The consignment charges (the % the store owners keep) are as follows:

Coats	45%
Dresses and skirts	$33\frac{1}{3}\%$
Bridal and evening gowns	50%
Blouses, jeans, and slacks	25%

Lisa and her daughters did a huge closet clean-out and had the following items sold at the consignment store. For each category, calculate the amount for the store fee and the amount Lisa and the girls received. Add your answers to Table 5.13.1.

Table 5.13.1: Question 14 Complete the Chart

Question Number	Item	Selling Price	Store Fee	Amount for Lisa & Her Daughters
a	Wedding dress	\$275	—	—
b	3 dresses at \$40 each	3 @ \$40 = \$120	—	—
c	Lisa's winter coat at \$120	\$120	—	—
d	4 pairs of outgrown jeans at \$10 each	4 @ \$10 = \$40	—	—

5.13: Review Answers

1. 45 marks
2. \$40 000
3. Restaurant:
 - a. \$25.80
 - b. \$77.40
4. \$457.14
5. 15 000 tickets
6. 6.6% increase
7. 21.7% depreciation
8. \$322.32
9. 181.3% mark-up
10. 90% correct
11. Ski jackets:
 - a. \$126.47
 - b. \$141.65
12. 26.5% increase
13. Tax:
 - a. \$30.78
 - b. \$33.86
 - c. \$7.76
 - d. \$72.40

14.

Table 5.13.2: Question 14 Answers

Question Number	Item	Selling Price	Store Fee	Amount for Lisa & Her Daughters
a	Wedding dress	\$275	\$137.50	\$137.50
b	3 dresses at \$40 each	3 @ \$40 = \$120	\$40.00	\$80.00
c	Lisa's winter coat at \$120	\$120	\$54.00	\$66.00
d	4 pairs of outgrown jeans at \$10 each	4 @ \$10 = \$40	\$10.00	\$30.00

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CHAPTER 6: MEASUREMENT

6.1: Systems of Measurement

Chapter 6: Learning Outcomes

By the end of this chapter, you should be able to:

- Use the common metric units for temperature, length, area, volume/capacity, and mass.
- Use the common imperial for temperature, length, area, volume/capacity, and force.
- Convert between and within metric and imperial units using tables and/or calculators.
- Use proportional reasoning for conversions.

Make Unit Conversions in the Imperial System

There are two systems of measurement commonly used around the world. Most countries use the metric system. Canada uses the metric system for measuring, while the United States uses the imperial system. However, people in Canada often use imperial measurements as well. We will look at the imperial system first.

The imperial system of measurement uses the following units:

- **Length** — inch, foot, yard, and mile
- **Weight** — pound and ton
- **Capacity** — cup, pint, quart, and gallon

The imperial and metric systems both measure time in seconds, minutes, and hours.

The equivalencies of measurements are shown in the lists below. The lists also show, in parentheses, the common abbreviations for each measurement.

Length:

- 1 foot (ft.) = 12 inches (in.)
- 1 yard (yd.) = 3 feet (ft.)
- 1 mile (mi.) = 5 280 feet (ft.)

Weight:

- 1 pound (lb.) = 16 ounces (oz.)
- 1 ton = 2 000 pounds (lb.)

Volume:

- 3 teaspoons (t) = 1 tablespoon (T)
- 16 tablespoons (T) = 1 cup (C)
- 1 cup (C) = 8 fluid ounces (fl. oz.)
- 1 pint (pt.) = 2 cups (C)
- 1 quart (qt.) = 2 pints (pt.)
- 1 gallon (gal.) = 4 quarts (qt.)

Time:

- 1 minute (min) = 60 seconds (sec)
- 1 hour (hr) = 60 minutes (min)
- 1 day = 24 hours (hr)
- 1 week (wk) = 7 days
- 1 year (yr) = 365 days

In many real-life applications, we need to convert between units of measurement, such as feet and yards, minutes and seconds, and quarts and gallons. We will use the identity property of multiplication to do these conversions. We will restate the identity property of multiplication here for easy reference.

Identity property of multiplication:

For any real number a

$$a \cdot 1 = a$$

$$1 \cdot a = a$$

1 is the **multiplicative identity**.

To use the identity property of multiplication, we write 1 in a form that will help us convert the units. For example, suppose we want to change inches to feet. We know that 1 foot is equal to 12 inches, so we will write 1 as the fraction $\frac{1 \text{ foot}}{12 \text{ inches}}$. When we multiply by this fraction, we do not change the value; instead, we just change the units.

However, $\frac{12 \text{ inches}}{1 \text{ foot}}$ also equals 1. How do we decide whether to multiply by $\frac{1 \text{ foot}}{12 \text{ inches}}$ or $\frac{12 \text{ inches}}{1 \text{ foot}}$?

We choose the fraction that will make the units we want to convert *from* divide out. Treat the unit words like factors and “divide out” common units like we do common factors. If we want to convert 66 inches to feet, which multiplication will eliminate the inches?

$$66 \text{ inches} \cdot \frac{1 \text{ foot}}{12 \text{ inches}} \quad \text{or}$$

$$\cancel{66 \text{ inches}} \cdot \frac{12 \cancel{\text{inches}}}{1 \text{ foot}}$$

The first form works because $\cancel{66 \text{ inches}} \cdot \frac{1 \text{ foot}}{12 \cancel{\text{inches}}}$

The inches divide out and leave only feet. The second form does not have any units that will divide out, so it will not help us.

Example A

MaryAnne is 66 inches tall. Convert her height into feet.

Step 1: Multiply the measurement to be converted by 1. Write 1 as a fraction relating the units given and the units needed.

Multiply 66 inches by 1, writing 1 as a fraction relating inches and feet. We need inches in the denominator so that the inches will divide out.

$$66 \text{ inches} \times 1$$

$$66 \text{ inches} \times \frac{1 \text{ foot}}{12 \text{ inches}}$$

Step 2: Multiply.

Think of 66 inches as $\frac{66 \text{ inches}}{1}$

$$\frac{66 \text{ inches} \times 1 \text{ foot}}{12 \text{ inches}}$$

Step 3: Simplify the fraction.

$$\frac{66 \cancel{\text{ inches}} \times 1 \text{ foot}}{12 \cancel{\text{ inches}}}$$

Notice how the inches divide out.

$$\frac{66 \text{ feet}}{12}$$

Step 4: Simplify.

Divide 66 by 12.

5.5 feet

Exercise 1

Lexie is 30 inches tall. Convert her height to feet.

Exercise 1 Answer

2.5 feet

Exercise 2

Rene bought a hose that is 18 yards long. Convert the length to feet.

Exercise 2 Answer

54 feet

To make unit conversions:

1. Multiply the measurement to be converted by 1;

write 1 as a fraction relating the units given and the units needed.

2. Multiply.
3. Simplify the fraction.
4. Simplify.

When we use the identity property of multiplication to convert units, we need to make sure the units we want to change from will divide out. Usually, this means we want the conversion fraction to have those units in the denominator.

Example B

A female orca in the Salish Sea weighs almost 3.2 tons. Convert her weight to pounds.

We will convert 3.2 tons into pounds. We will use the identity property of multiplication, writing 1 as the fraction

$$\frac{2000 \text{ pounds}}{1 \text{ ton}}.$$

Step 1: Multiply the measurement to be converted, by 1.

$$3.2 \text{ tons} \cdot 1$$

Step 2: Write 1 as a fraction relating tons and pounds.

$$3.2 \text{ tons} \cdot \frac{2,000 \text{ pounds}}{1 \text{ ton}}$$

Step 3: Simplify.

$$\frac{3.2 \cancel{\text{ tons}} \cdot 2\,000 \text{ pounds}}{1 \cancel{\text{ ton}}}$$

Step 4: Multiply.

6 400 pounds

The female orca weighs almost 6 400 pounds.

Exercise 3

Arnold's SUV weighs about 4.3 tons. Convert the weight to pounds.

Exercise 3 Answer

8 600 pounds

Exercise 4

The Carnival *Destiny* cruise ship weighs 51 000 tons.
Convert the weight to pounds.

Exercise 4 Answer

102 000 000 pounds

Sometimes, to convert from one unit to another, we may need to use several other units in between, so we will need to multiply several fractions.

Example C

Juliet is going with her family to their summer home. She will be away from her boyfriend for 9 weeks. Convert the time to minutes.

To convert weeks into minutes, we will convert weeks into days, days into hours, and then hours into minutes. To do this, we will multiply by conversion factors of 1.

Step 1: Write 1 as

$$\frac{7 \text{ days}}{1 \text{ week}}, \frac{24 \text{ hours}}{1 \text{ day}}, \text{ and } \frac{60 \text{ minutes}}{1 \text{ hour}}.$$

$$\frac{9 \text{ wk}}{1} \cdot \frac{7 \text{ days}}{1 \text{ wk}} \cdot \frac{24 \text{ hr}}{1 \text{ day}} \cdot \frac{60 \text{ min}}{1 \text{ hr}}$$

Step 2: Divide out the common units.

$$\frac{9 \cancel{\text{ wk}}}{1} \cdot \frac{7 \cancel{\text{ days}}}{1 \cancel{\text{ wk}}} \cdot \frac{24 \cancel{\text{ hr}}}{1 \cancel{\text{ day}}} \cdot \frac{60 \text{ min}}{1 \cancel{\text{ hr}}}$$

Step 3: Multiply.

$$\frac{9 \cdot 7 \cdot 24 \cdot 60 \text{ min}}{1 \cdot 1 \cdot 1 \cdot 1}$$

Step 4: Multiply again.

$$90\,720 \text{ min}$$

Juliet and her boyfriend will be apart for 90 720 minutes (although it may seem like an eternity!).

Exercise 5

The distance between the earth and the moon is about 250 000 miles. Convert this length to yards.

Exercise 5 Answer

440 000 000 yards

Exercise 6

The astronauts of Expedition 28 on the International Space Station spent 15 weeks in space. Convert the time to minutes.

Exercise 6 Answer

151 200 minutes

Example D

How many ounces are in 1 gallon?

We will convert gallons to ounces by multiplying by several conversion factors. Refer to the Imperial Systems of Measurement listed above.

Step 1: Multiply the measurement to be converted by 1.

$$\frac{1 \text{ gallon}}{1} \cdot \frac{4 \text{ quarts}}{1 \text{ gallon}} \cdot \frac{2 \text{ pints}}{1 \text{ quart}} \cdot \frac{2 \text{ cups}}{1 \text{ pint}} \cdot \frac{8 \text{ ounces}}{1 \text{ cup}}$$

Step 2: Use conversion factors to get to the right unit.

Simplify.

$$\frac{\cancel{1 \text{ gallon}}}{1} \cdot \frac{\cancel{4 \text{ quarts}}}{1 \cancel{\text{ gallon}}} \cdot \frac{\cancel{2 \text{ pints}}}{1 \cancel{\text{ quart}}} \cdot \frac{\cancel{2 \text{ cups}}}{1 \cancel{\text{ pint}}} \cdot \frac{8 \text{ ounces}}{1 \cancel{\text{ cup}}}$$

Step 3: Multiply.

$$\frac{1 \cdot 4 \cdot 2 \cdot 2 \cdot 8 \text{ ounces}}{1 \cdot 1 \cdot 1 \cdot 1 \cdot 1}$$

Step 4: Simplify.

128 ounces

There are 128 ounces in a gallon.

Exercise 7

How many cups are in 1 gallon?

Exercise 7 Answer

16 cups

Exercise 8

How many teaspoons are in 1 cup?

Exercise 8 Answer

48 teaspoons

Use Mixed Units of Measurement in the Imperial System

We often use mixed units of measurement in everyday situations. Suppose Joe is 5 feet 10 inches tall, stays at work for 7 hours and 45 minutes, and then eats a 1-pound 2-ounce steak for dinner — all these measurements have mixed units.

Performing arithmetic operations on measurements with mixed units of measures requires care. Be sure to add or subtract like units!

Example E

Seymour bought three steaks for a barbecue. Their weights were 14 ounces; 1 pound, 2 ounces; and 1 pound, 6 ounces. How many total pounds of steak did he buy?

We will add the weights of the steaks to find the total weight of the steaks.

Step 1: Add the ounces. Then, add the pounds.

$$\begin{array}{t}{\text{rlr}}\text{&}&14\text{ ounces} \\ \text{&}&1\text{ pound} \text{ & } 2\text{ ounces} \\ \text{&}&1\text{ pound} \text{ & } 6\text{ ounces} \\ \hline \text{&}&22\text{ ounces} \end{array}$$

Step 2: Convert 22 ounces to pounds and ounces.

1 pound, 6 ounces

Step 3: Add the pounds and ounces.

2 pound + 1 pound + 6 ounces

3 pounds, 6 ounces

Seymour bought 3 pounds, 6 ounces of steak.

Exercise 9

Laura gave birth to triplets weighing 3 pounds 3 ounces, 3 pounds 3 ounces, and 2 pounds 9 ounces. What was the total birth weight of the three babies?

Exercise 9 Answer

9 lbs. 8 oz

Exercise 10

Stan cut two pieces of crown moulding for his family room that were 8 feet 7 inches and 12 feet 11 inches. What was the total length of the moulding?

Exercise 10 Answer

21 ft. 6 in.

Example F

Anthony bought four planks of wood that were each 6 feet 4 inches long. What is the total length of the wood he purchased?

We will multiply the length of one plank to find the total length.

Step 1: Multiply the inches and then the feet.

$$\begin{array}{r} 6 \text{ feet} \quad 4 \text{ inches} \\ \times \hspace{10em} 4 \\ \hline 24 \text{ feet} \quad 16 \text{ inches} \end{array}$$

Step 2: Convert the 16 inches to feet.

$$16 \text{ inches} = 1 \text{ foot}, 4 \text{ inches}$$

Step 3: Add the feet.

24 feet + 1 foot 4 inches

25 feet, 4 inches

Anthony bought 25 feet and 4 inches of wood.

Exercise 11

Henri wants to triple his vegan spaghetti sauce recipe that uses 1 pound 8 ounces of black beans. How many pounds of black beans will he need?

Exercise 11 Answer

4 lbs. 8 oz.

Exercise 12

Joellen wants to double a solution of 5 gallons 3 quarts.
How many gallons of solution will she have in all?

Exercise 12 Answer

11 gallons 2 qt.

Make Unit Conversions in the Metric System

In the metric system, units are related by powers of 10. The root words of their names reflect this relation. For example, the basic unit for measuring length is a metre. One kilometre is 1 000 metres; the prefix *kilo* means *thousand*. One centimetre is $\frac{1}{100}$ of a metre, just like one cent is $\frac{1}{100}$ of one dollar.

The equivalencies of measurements in the metric system are shown in the lists below. The common abbreviations for each measurement are given in parentheses.

Length:

- 1 kilometre (km) = 1 000 m
- 1 hectometre (hm) = 100 m
- 1 dekametre (dam) = 10 m
- 1 metre (m) = 1 m
- 1 decimetre (dm) = 0.1 m
- 1 centimetre (cm) = 0.01 m
- 1 millimetre (mm) = 0.001 m
- 1 metre = 100 centimetres
- 1 metre = 1 000 millimetres

Mass:

- 1 kilogram (kg) = 1 000 g
- 1 hectogram (hg) = 100 g
- 1 dekagram (dag) = 10 g
- 1 gram (g) = 1 g
- 1 decigram (dg) = 0.1 g
- 1 centigram (cg) = 0.01 g
- 1 milligram (mg) = 0.001 g
- 1 gram = 100 centigrams
- 1 gram = 1 000 milligrams

Capacity:

- 1 kilolitre (kL) = 1 000 L
- 1 hectolitre (hL) = 100 L
- 1 dekalitre (daL) = 10 L
- 1 litre (L) = 1 L
- 1 decilitre (dL) = 0.1 L
- 1 centilitre (cL) = 0.01 L
- 1 millilitre (mL) = 0.001 L
- 1 litre = 100 centilitres
- 1 litre = 1 000 millilitres

To make conversions in the metric system, we will use the same technique we did in the Imperial system. Using the identity property

of multiplication, we will multiply by a conversion factor of one to get to the correct units.

Have you ever run a 5K or 10K race? The length of those races are measured in kilometres. The metric system is commonly used in Canada when talking about the length of a race.

Example G

Nick ran a 10K race. How many metres did he run?

We will convert kilometres to metres using the identity property of multiplication.

Step 1: Multiply the measurement to be converted by 1.

$$10 \text{ kilometres} \times 1$$

Step 2: Write 1 as a fraction relating kilometres and metres.

$$10 \text{ kilometres} \times \frac{1,000 \text{ metres}}{1 \text{ kilometres}}$$

Step 3: Simplify.

$$\cancel{10 \text{ kilometres}} \times \frac{1,000 \text{ metres}}{\cancel{1 \text{ kilometres}}}$$

Step 4: Multiply.

$$10\,000 \text{ metres}$$

Nick ran 10,000 metres.

Exercise 13

Sandy completed her first 5K race! How many metres did she run?

Exercise 13 Answer

5 000 metres

Exercise 14

Herman bought a rug 2.5 metres in length. How many centimetres is the length?

Exercise 14 Answer

250 centimetres

Example H

Eleanor's newborn baby weighed 3 200 grams. How many kilograms did the baby weigh?

We will convert grams into kilograms.

Step 1: Multiply the measurement to be converted by 1.

$$3\,200 \text{ grams} \cdot 1$$

Step 2: Write 1 as a function relating kilograms and grams.

$$3\,200 \text{ grams} \cdot \frac{1 \text{ kg}}{1\,000 \text{ grams}}$$

Step 3: Simplify.

$$3\,200 \cancel{\text{ grams}} \cdot \frac{1 \text{ kg}}{1\,000 \cancel{\text{ grams}}}$$

Step 4: Multiply.

$$\begin{array}{r} 3\,200 \text{ kilograms} \\ \hline 1\,000 \end{array}$$

Step 5: Divide.

3.2 kilograms

The baby weighed 3.2 kilograms.

Exercise 15

Kari's newborn baby weighed 2 800 grams. How many kilograms did the baby weigh?

Exercise 15 Answer

2.8 kilograms

Exercise 16

Anderson received a package that was marked 4 500 grams. How many kilograms did this package weigh?

Exercise 16 Answer

4.5 kilograms

As you become familiar with the metric system, you may see a pattern. Since the system is based on multiples of ten, the calculations involve multiplying by multiples of ten. We have learned how to simplify these calculations by just moving the decimal.

To multiply by 10, 100, or 1 000, we move the decimal to the right one, two, or three places, respectively. To multiply by 0.1, 0.01, or 0.001, we move the decimal to the left one, two, or three places, respectively.

We can apply this pattern when we make measurement conversions in the metric system. In Example 8, we changed 3 200 grams to kilograms by multiplying by $\frac{1}{1\,000}$ (or 0.001). This is the same as moving the decimal three places to the left (see Figure 6.1.1).

$$3,200 \cdot \frac{1}{1,000} \qquad 3,200.$$

$$3.2 \qquad 3.2$$

Figure 6.1.1

Example I

Convert:

- a. 350 L to kilolitres
- b. 4.1 L to millilitres

a. 350 L to kilolitres

We will convert litres to kilolitres. In the Metric System of Measurement listed above, we see that 1 kL = 1 000 litres.

Step 1: Multiply by 1, writing 1 as a fraction relating litres to kilolitres.

$$350 \text{ L} \cdot \frac{1 \text{ kL}}{1\,000 \text{ L}}$$

Step 2: Simplify.

$$350 \cancel{\text{L}} \cdot \frac{1 \text{ kL}}{1,000 \cancel{\text{L}}}$$

Step 3: Move the decimal 3 units to the left.

$$0.35 \text{ kL}$$

b. 4.1 L to millilitres

We will convert litres to millilitres. In the Metric System of Measurement listed above, we see that 1 litre = 1 000 millilitres.

Step 1: Multiply by 1, writing 1 as a fraction relating litres to millilitres.

$$4.1 \text{ L} \cdot \frac{1\,000 \text{ mL}}{1 \text{ L}}$$

Step 2: Simplify.

$$4.1 \cancel{\text{L}} \cdot \frac{1\,000 \text{ mL}}{1 \cancel{\text{L}}}$$

Step 3: Move the decimal 3 units to the right.

$$4\,100 \text{ mL}$$

Exercise 17

Convert:

- a. 725 L to kilolitres b. 6.3 L to millilitres

Exercise 17 Answers

- a. 7250 kilolitres b. 6300 millilitres

Exercise 18

Convert:

- a. 350 hL to litres b. 4.1 L to centilitres

Exercise 18 Answers

a. 35 000 litres

b. 410 centilitres

Use Mixed Units of Measurement in the Imperial System

Performing arithmetic operations on measurements with mixed units of measures in the imperial system requires the same care we used in the Canadian system. Make sure to add or subtract like units.

Example J

Ryland is 1.6 metres tall. His younger brother is 85 centimetres tall. How much taller is Ryland than his younger brother?

Step 1: Convert both measurements to either centimetres

or metres. Since metres is the larger unit, we will subtract the lengths in metres.

Step 2: Convert 85 centimetres to metres by moving the decimal 2 places to the left to get 0.85 m.

$$\begin{array}{r} 1.60 \text{ m} \\ - 0.85 \text{ m} \\ \hline 0.75 \text{ m} \end{array}$$

Ryland is 0.75 m taller than his brother.

Exercise 19

Mariella is 1.58 metres tall. Her daughter is 75 centimetres tall. How much taller is Mariella than her daughter? Write the answer in centimetres.

Exercise 19 Answer

83 centimetres

Exercise 20

The fence around Hank's yard is 2 metres high. Hank is 96 centimetres tall. How much shorter than the fence is Hank? Write the answer in metres.

Exercise 20 Answer

1.04 metres

Example K

Dena's recipe for lentil soup calls for 150 millilitres of olive oil. Dena wants to triple the recipe. How many litres of olive oil will she need?

We will find the amount of olive oil in millilitres and then convert it to litres.

Step 1: Translate to algebra.

$$3 \cdot 150 \text{ mL}$$

Step 2: Multiply.

$$450 \text{ mL}$$

Step 3: Convert to litres.

$$450 \text{ mL} \cdot \frac{0.001 \text{ L}}{1 \text{ mL}}$$

Step 4: Simplify.

$$450 \cancel{\text{ mL}} \cdot \frac{0.001 \text{ L}}{1 \cancel{\text{ mL}}} = 0.45 \text{ L}$$

Dena needs 0.45 litres of olive oil.

Exercise 21

A recipe for Alfredo sauce calls for 250 millilitres of milk. Renata is making pasta with Alfredo sauce for a big party and needs to multiply the recipe amounts by 8. How many litres of milk will she need?

Exercise 21 Answer

2 litres

Exercise 22

To make one pan of baklava, Dorothea needs 400 grams of filo pastry. If Dorothea plans to make 6 pans of baklava, how many kilograms of filo pastry will she need?

Exercise 22 Answer

2.4 kilograms

Convert Between the Imperial & Metric Systems of Measurement

Many measurements in Canada are made in metric units. Our soda may come in 2-litre bottles, our calcium may come in 500-mg capsules, and we may run a 5K race. To work easily in both systems, we need to be able to convert between the two systems.

The lists below shows some of the most common conversions.

Length:

- 1 in. = 2.54 cm
- 1 ft. = 0.305 m
- 1 yd. = 0.914 m
- 1 mi. = 1.61 km
- 1 m = 3.28 ft.

Mass:

- 1 lb. = 0.45 kg
- 1 oz. = 28 g
- 1 kg = 2.2 lb.

Capacity:

- 1 qt. = 0.95 L
- 1 fl. oz. = 30 mL
- 1 L = 1.06 qt.

Figure 6.1.2 shows how inches and centimetres are related on a ruler.



Figure 6.1.2

Figure 6.1.3 shows the ounce and millilitre markings on a measuring cup.

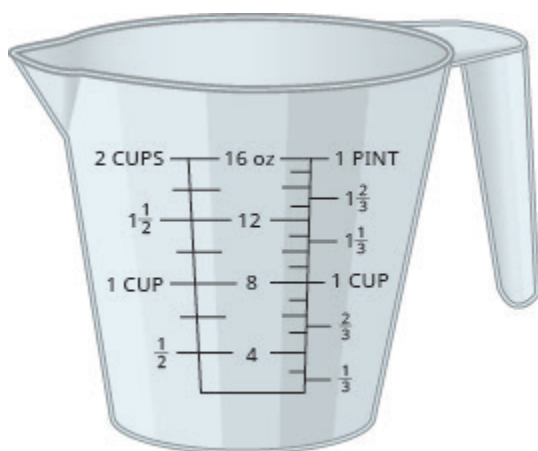


Figure 6.1.3

Figure 6.1.4 shows how pounds and kilograms marked on a bathroom scale.



Figure 6.1.4

We make conversions between the systems just as we do within the systems — by multiplying by unit conversion factors.

Example L

Lee's water bottle holds 500 mL of water. How many ounces are in the bottle? Round to the nearest tenth of an ounce.

Step 1: Multiply by a unit conversion factor relating to mL and ounces.

$$500 \text{ millilitres} \cdot \frac{1 \text{ ounce}}{30 \text{ millilitres}}$$

Step 2: Simplify.

$$\frac{50 \text{ ounce}}{30}$$

Step 3: Divide.

16.7 ounces

The water bottle has 16.7 ounces.

Exercise 23

How many quarts of soda are in a 2-L bottle?

Exercise 23 Answer

2.12 quarts

Exercise 24

How many litres are in 4 quarts of milk?

Exercise 24 Answer

3.8 litres

Example M

Soleil was on a road trip and saw a sign that said the next rest stop was in 100 kilometres. How many miles until the next rest stop?

Step 1: Multiply by a unit conversion factor relating km and mi.

$$100 \text{ kilometres} \cdot \frac{1 \text{ mile}}{1.61 \text{ kilometre}}$$

Step 2: Simplify.

$$\frac{100 \text{ miles}}{1.61}$$

Step 3: Divide.

62 ounces

Soleil will travel 62 miles.

Exercise 25

The height of Mount Kilimanjaro is 5 895 metres. Convert the height to feet.

Exercise 25 Answer

19 335.6 feet

Exercise 26

The flight distance from Toronto to Vancouver is 3 364 kilometres. Convert the distance to miles.

Exercise 26 Answer

2 090 miles

Convert Between Fahrenheit & Celsius Temperatures

Have you ever been in a foreign country and heard the weather forecast? If the forecast is for $71^{\circ} F$, what does that mean?

The Canadian and imperial systems use different scales to measure temperature. The Canadian system uses degrees Celsius, written as $^{\circ}C$. The imperial system uses degrees Fahrenheit, written as $^{\circ}F$. Figure 6.1.5 shows the relationship between the two systems.

Figure 6.1.5 shows normal body temperature, along with the freezing and boiling temperatures of water in degrees Fahrenheit and degrees Celsius.

Celsius (°C)

Fahrenheit (°F)

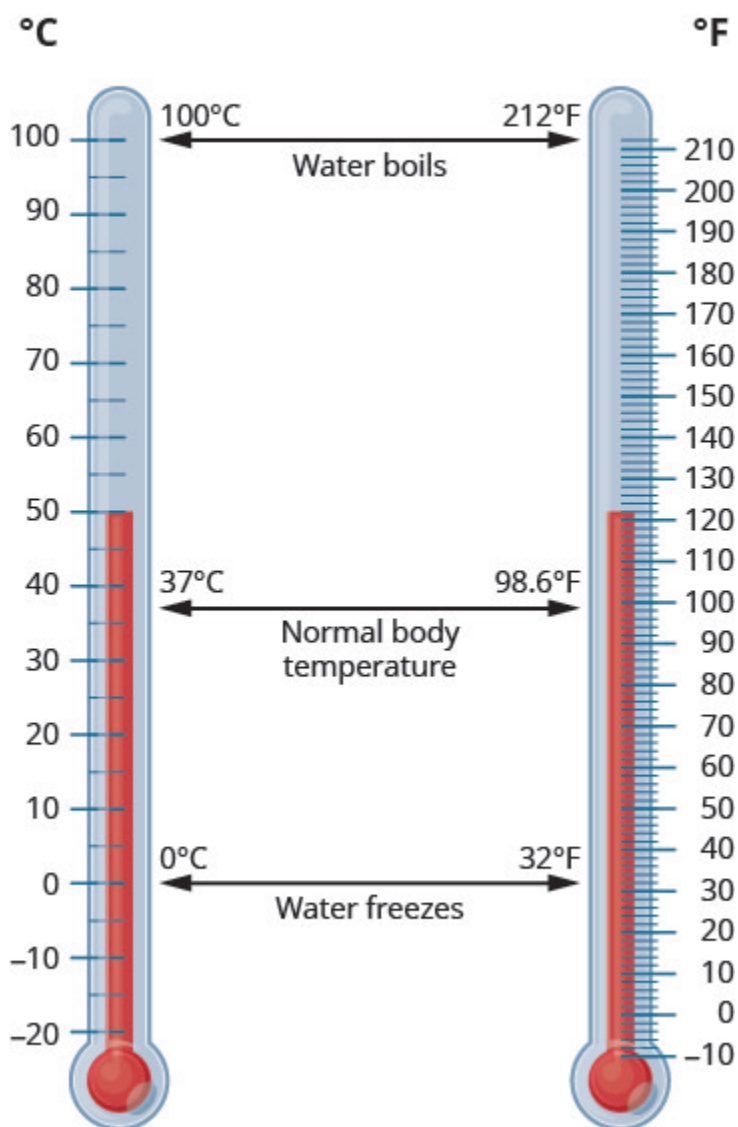


Figure 6.1.5

Temperature conversions:

To convert from Fahrenheit temperature, F , to Celsius temperature, C , use the formula:

$$C = \frac{5}{9}(F - 32).$$

To convert from Celsius temperature, C , to Fahrenheit temperature, F , use the formula:

$$F = \frac{9}{5}C + 32.$$

Example N

Convert 50° Fahrenheit into degrees Celsius.

We will substitute 50°F into the formula to find C .

Step 1: Substitute 50 for F .

$$C = \frac{5}{9}(50 - 32)$$

Step 2: Simplify in parentheses.

$$C = \frac{5}{9}(18)$$

Step 3: Multiply.

$$C = 10$$

50°F is equivalent to 10°C.

Exercise 27

Convert the Fahrenheit temperature to degrees Celsius:

59° Fahrenheit.

Exercise 27 Answer

15°C

Exercise 28

Convert the Fahrenheit temperature to degrees Celsius:
41° Fahrenheit.

Exercise 28 Answer

5°C

Example O

While visiting Paris, Woody saw the temperature was 20° Celsius. Convert the temperature to degrees Fahrenheit.

We will substitute 20°C into the formula to find F.

Step 1: Substitute 20 for C.

$$F = \frac{9}{5}(20) + 32$$

Step 2: Multiply.

$$F = 36 + 32$$

Step 3: Add.

$$F = 68$$

20°C is equivalent to 68°F.

Exercise 29

Convert the Celsius temperature to degrees Fahrenheit:
The temperature in Helsinki, Finland, was 15° Celsius.

Exercise 29 Answer

59°F

Exercise 30

Convert the Celsius temperature to degrees Fahrenheit:
The temperature in Sydney, Australia, was 10° Celsius.

Exercise 30 Answer

50° F

Key Concepts

Metric System of Measurement

Length

- 1 kilometre (km) = 1 000 m
- 1 hectometre (hm) = 100 m
- 1 dekatmetre (dam) = 10 m
- 1 metre (m) = 1 m
- 1 decimetre (dm) = 0.1 m

- 1 centimetre (cm) = 0.01 m
- 1 millimetre (mm) = 0.001 m
- 1 metre = 100 centimetres
- 1 metre = 1 000 millimetres

Mass

- 1 kilogram (kg) = 1 000 g
- 1 hectogram (hg) = 100 g
- 1 dekagram (dag) = 10 g
- 1 gram (g) = 1 g
- 1 decigram (dg) = 0.1 g
- 1 centigram (cg) = 0.01 g
- 1 milligram (mg) = 0.001 g
- 1 gram = 100 centigrams
- 1 gram = 1 000 milligrams

Capacity

- 1 kilolitre (kL) = 1 000 L
- 1 hectolitre (hL) = 100 L
- 1 dekalitre (daL) = 10 L
- 1 litre (L) = 1 L
- 1 decilitre (dL) = 0.1 L
- 1 centilitre (cL) = 0.01 L
- 1 millilitre (mL) = 0.001 L
- 1 litre = 100 centilitres
- 1 litre = 1 000 millilitres

Temperature Conversion

- To convert from Fahrenheit temperature, F , to Celsius temperature, C , use the formula $C = \frac{5}{9}(F - 32)$
- To convert from Celsius temperature, C , to Fahrenheit temperature, F , use the formula $F = \frac{9}{5}C + 32$

6.1: Practice Questions

1. Unit Conversion in the Imperial System

In the following exercises, convert between imperial units

- A park bench is 6 feet long. Convert the length to inches.
- A floor tile is 2 feet wide. Convert the width to inches.
- A ribbon is 18 inches long. Convert the length to feet.
- Carson is 45 inches tall. Convert his height to feet.
- A football field is 160 feet wide. Convert the width to yards.
- On a baseball diamond, the distance from home plate to first base is 30 yards. Convert the distance to feet.
- Ulysses lives 1.5 miles from school. Convert the distance to feet.
- Denver, Colorado, is 5,183 feet above sea level. Convert the height to miles.
- A killer whale weighs 4.6 tons. Convert the weight to pounds.
- Blue whales can weigh as much as 150 tons. Convert the weight to pounds.

- k. An empty bus weighs 35 000 pounds. Convert the weight to tons.
- l. At take-off, an airplane weighs 220 000 pounds. Convert the weight to tons.
- m. Rocco waited $1\frac{1}{2}$ hours for his appointment. Convert the time to seconds.
- n. Misty's surgery lasted $2\frac{1}{4}$ hours. Convert the time to seconds.
- o. How many teaspoons are in a pint?
- p. How many tablespoons are in a gallon?
- q. JJ's cat, Posy, weighs 14 pounds. Convert her weight to ounces.
- r. April's dog, Beans, weighs 8 pounds. Convert his weight to ounces.
- s. Crista will serve 20 cups of juice at her son's party. Convert the volume to gallons.
- t. Lance needs 50 cups of water for the runners in a race. Convert the volume to gallons.
- u. Jon is 6 feet 4 inches tall. Convert his height to inches.
- v. Faye is 4 feet 10 inches tall. Convert her height to inches.
- w. The voyage of the *Mayflower* took 2 months and 5 days. Convert the time to days.
- x. Lynn's cruise lasted 6 days and 18 hours. Convert the time to hours.
- y. Baby Preston weighed 7 pounds 3 ounces at birth. Convert his weight to ounces.
- z. Baby Audrey weighed 6 pounds 15 ounces at birth. Convert her weight to ounces.

2. Mixed Units of Measurement in the Imperial System

Solve the following exercises using mixed imperial units.

- a. Eli caught three fish. The weights of the fish were 2 pounds, 4 ounces; 1 pound, 11 ounces; and 4 pounds, 14 ounces. What was the total weight of the three fish?
- b. Judy bought 1 pound 6 ounces of almonds, 2 pounds 3 ounces of walnuts, and 8 ounces of cashews. How many pounds of nuts did Judy buy?
- c. One day, Anya kept track of the number of minutes she spent driving. She recorded 45, 10, 8, 65, 20, and 35. How many hours did Anya spend driving?
- d. Last year, Eric went on 6 business trips. The number of days of each was 5, 2, 8, 12, 6, and 3. How many weeks did Eric spend on business trips last year?
- e. Renee attached a 6-foot, 6-inch extension cord to her computer's 3-foot, 8-inch power cord. What was the total length of the cords?
- f. Fawzi's SUV is 6 feet 4 inches tall. If he puts a 2 feet 10 inch box on top of his SUV, what is the total height of the SUV and the box?
- g. Leilani wants to make 8 placemats. For each placemat, she needs 18 inches of fabric. How many yards of fabric will she need for the 8 placemats?
- h. Mireille needs to cut 24 inches of ribbon for each of the 12 girls in her dance class. How many yards of ribbon will she need altogether?

3. Unit Conversions in the Metric System

In the following exercises, convert between metric units.

- a. Ghalib ran 5 kilometres. Convert the length to metres.
- b. Kitaka hiked 8 kilometres. Convert the length to metres.
- c. Estrella is 1.55 metres tall. Convert her height to centimetres.
- d. The width of the wading pool is 2.45 metres. Convert the width to centimetres.
- e. Mount Whitney is 3 072 metres tall. Convert the height to kilometres.
- f. The depth of the Mariana Trench is 10 911 metres. Convert the depth to kilometres.
- g. June's multivitamin contains 1 500 milligrams of calcium. Convert this to grams.
- h. A typical ruby-throated hummingbird weighs 3 grams. Convert this to milligrams.
- i. One stick of butter contains 91.6 grams of fat. Convert this to milligrams.
- j. One serving of gourmet ice cream has 25 grams of fat. Convert this to milligrams.
- k. The maximum mass of an airmail letter is 2 kilograms. Convert this to grams.
- l. Dimitri's daughter weighed 3.8 kilograms at birth. Convert this to grams.
- m. A bottle of wine contained 750 millilitres. Convert this to litres.
- n. A bottle of medicine contained 300 millilitres. Convert this to litres.

4. Mixed Units of Measurement in the Metric System

Solve the following exercises using mixed metric units.

- a. Matthias is 1.8 metres tall. His son is 89 centimetres tall. How much taller is Matthias than his son?
- b. Stavros is 1.6 metres tall. His sister is 95 centimetres tall. How much taller is Stavros than his sister?
- c. A typical dove weighs 345 grams. A typical duck weighs 1.2 kilograms. What is the difference, in grams, between the weights of a duck and a dove?
- d. Concetta had a 2-kilogram bag of flour. She used 180 grams of flour to make biscotti. How many kilograms of flour are left in the bag?
- e. Harry mailed 5 packages that weighed 420 grams each. What was the total weight of the packages in kilograms?
- f. One glass of orange juice provides 560 milligrams of potassium. Linda drinks one glass of orange juice every morning. How many grams of potassium does Linda get from her orange juice in 30 days?
- g. Jonas drinks 200 millilitres of water 8 times a day. How many litres of water does Jonas drink in a day?
- h. One serving of whole grain sandwich bread provides 6 grams of protein. How many milligrams of protein are provided by 7 servings of whole grain sandwich bread?

5. Converting Between Imperial & Metric Systems of Measurement

In the following exercises, convert between imperial and metric units. Round to the nearest tenth.

- a. Bill is 75 inches tall. Convert his height to centimetres.
- b. Frankie is 42 inches tall. Convert his height to centimetres.
- c. Marcus passed a football 24 yards. Convert the pass length to metres.
- d. Connie bought 9 yards of fabric to make drapes. Convert the fabric length to metres.
- e. According to research conducted by the CRC, Canadians regrettably produce more garbage per capita than any other country on earth, at 2 172.6 pounds per person annually. Convert the waste to kilograms.
- f. An average Canadian will throw away 163 000 pounds of trash over their lifetime. Convert this weight to kilograms.
- g. A 5K run is 5 kilometres long. Convert this length to miles.
- h. Kathryn is 1.6 metres tall. Convert her height to feet.
- i. Dawn's suitcase weighed 20 kilograms. Convert the weight to pounds.
- j. Jackson's backpack weighed 15 kilograms. Convert the weight to pounds.
- k. Ozzie put 14 gallons of gas in his truck. Convert the volume to litres.
- l. Bernard bought 8 gallons of paint. Convert the volume to litres.

6. Converting Between Fahrenheit & Celsius Temperatures

In the following exercises, convert from Fahrenheit to degrees Celsius. Round to the nearest tenth.

a. 86° Fahrenheit	d. 14° Fahrenheit	g. 0° Fahrenheit
b. 77° Fahrenheit	e. 72° Fahrenheit	h. 120° Fahrenheit
c. 104° Fahrenheit	f. 4° Fahrenheit	

In the following exercises, convert from Celsius to degrees Fahrenheit. Round to the nearest tenth.

i. 5° Celsius	l. -15° Celsius	o. 43° Celsius
j. 25° Celsius	m. 22° Celsius	p. 16° Celsius
k. -10° Celsius	n. 8° Celsius	

7. Everyday Math

- a. **Nutrition:** Julian drinks one can of soda every day. Each can of soda contains 40 grams of sugar. How many kilograms of sugar does Julian get from soda in 1 year?
- b. **Reflectors:** The reflectors in each lane-marking stripe on a highway are spaced 16 yards apart. How many reflectors are needed for a one-mile-long lane-marking stripe?

8. Writing Exercises

- a. Some people think that 22° to 28° Celsius is the ideal temperature range.
 - i. What is your ideal temperature range? Why do you think so?
- b. Did you grow up using the Canadian Metric or the Imperial system of measurement?
 - i. Describe two examples in your life when you had to convert between the two systems of measurement.

6.1: Practice Answers

1. Unit conversions in the imperial system.

a. 72 inches	k. $17\frac{1}{2}$ tons	s. $1\frac{1}{4}$ gallons
b. 1.5 feet		
c. 1.5 feet		
d.	l.	t.
e. $53\frac{1}{3}$ yards	m. 5 400 s	u. 76 in.
f.	n.	v.
g. 7 920 feet	o. 96 teaspoons	w. 65 days
h.	p.	x.
i. 9 200 pounds	q. 224 ounces	y. 115 ounces
j.	r.	

2. Mixed units of measurement in the imperial system.

a. 8 lbs. 13 oz.	e. 10 ft. 2 in.
b.	f.
c. 3.05 hours	g. 4 yards
d.	

3. Unit conversions in the metric system.

a. 5 000 metres	g. 1.5 grams	m. 0.75 litres
b.	h.	
c. 155 centimetres	i. 91 600 milligrams	
d.	j.	
e. 3.072 kilometres	k. 2 000 grams	
f.	l.	

4. Mixed units of measurement in the metric system.

a. 91 centimetres	e. 2.1 kilograms
b.	f.
c. 855 grams	g. 1.6 litres
d.	

5. Covertng between imperial and metric systems of measurement.

a. 190.5 centimetres	e. 977.7 kilograms	i. 44 pounds
b.	f.	j.
c. 21.9 metres	g. 3.1 miles	k. 53.2 litres
d.	h.	

6. Converting between Fahrenheit and Celsius temperatures.

a. 30°C	g. -17.8°C	m. 71.6°F
b.	h.	n.
c. 40°C	i. 41°F	o. 109.4°F
d.	j.	
e. 22.2°C	k. 14°F	
f.	l.	

7. Everyday math.

- a. 14.6 kilograms

8. Writing exercises.

- a. Answers will vary.

Attributions

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CHAPTER 7: PERIMETER, AREA, & VOLUME

7.1: Use Properties of Rectangles, Triangles, & Trapezoids

Chapter 7: Learning Outcomes

By the end of this chapter, you should be able to:

- Find perimeters of triangles, squares, rectangles, parallelograms, trapezoids, circles and composite figures using formulas.
- Find areas of the above shapes using formulas.
- Find the surface area of cubes, rectangular solids, right cylinders and cones, spheres, and composite solids using formulas.
- Distinguish between concepts of perimeter and area and their respective units.

Understand Linear, Square, & Cubic Measure

When you measure your height or the length of a garden hose, you use a ruler or tape measure (see Figure 7.1.1). A tape measure might remind you of a line—you use it for linear measure, which measures

length. Inch, foot, yard, mile, centimetre and metre are units of linear measure.

This tape measure measures inches along the top and centimetres along the bottom.



Figure 7.1.1

When you want to know how much tile is needed to cover a floor or the size of a wall to be painted, you need to know the area, a measure of the region needed to cover a surface.

Area is measured in square units. We often use square inches, square feet, square centimetres, or square miles to measure area. A square centimetre is a square that is one centimetre (cm) on each side. A square inch is a square that is one inch on each side (see Figure 7.1.2).

Square measures have sides that are each 1 unit in length.

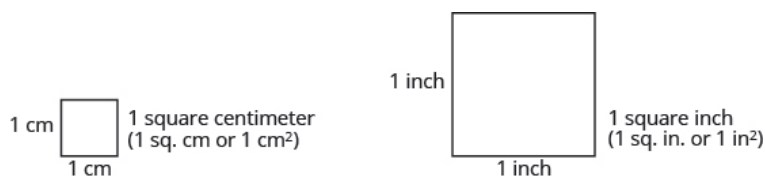


Figure 7.1.2

Figure 7.1.3 shows a rectangular rug that is 2 feet long by 3 feet wide. Each square is 1 foot wide by 1 foot long, or 1 square foot. The rug is made of 6 squares. The area of the rug is 6 square feet.

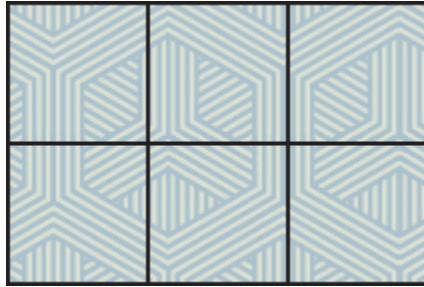


Figure 7.1.3

When you measure how much it takes to fill a container, such as the amount of gasoline that can fit in a tank or the amount of medicine in a syringe, you are measuring volume.

Volume is measured in cubic units such as cubic inches or cubic centimetres. When measuring the volume of a rectangular solid, you measure how many cubes fill the container.

We often use cubic centimetres, cubic inches, and cubic feet. A cubic centimetre is a cube that measures one centimetre on each side, while a cubic inch is a cube that measures one inch on each side (see Figure 7.1.4).

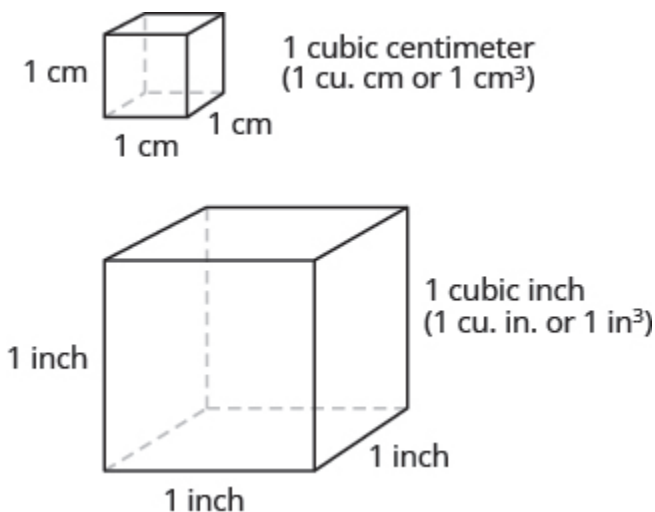


Figure 7.1.4

Suppose the cube in Figure 7.1.5 measures 3 inches on each side and is cut on the lines shown. How many little cubes does it contain? If we were to take the big cube apart, we would find 27 little cubes, each one measuring one inch on all sides. So, each little cube has a volume of 1 cubic inch, and the volume of the big cube is 27 cubic inches.

A cube that measures 3 inches on each side is made up of 27 one-inch cubes, or 27 cubic inches.

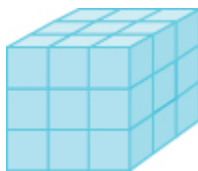


Figure 7.1.5

Example A

For each item, state whether you would use linear, square, or cubic measure:

- a. amount of carpeting needed in a room
- b. extension cord length
- c. amount of sand in a sandbox
- d. length of a curtain rod
- e. amount of flour in a canister
- f. size of the roof of a doghouse

- a. You are measuring how much surface the carpet covers, which is the area. **Square measure**
- b. You are measuring how long the extension cord is, which is the length. **Linear measure**
- c. You are measuring the volume of the sand. **Cubic measure**
- d. You are measuring the length of the curtain rod. **Linear measure**
- e. You are measuring the volume of the flour. **Cubic measure**
- f. You are measuring the area of the roof. **Square measure**

Exercise 1

Determine whether you would use linear, square, or cubic measure for each item.

a. amount of paint in a can	e. size of a piece of sod
b. height of a tree	f. amount of water in a swimming pool
c. floor of your bedroom	
d. diameter of bike wheel	

Exercise 1 Answers

a. cubic	d. linear
b. linear	e. square
c. square	f. cubic

Exercise 2

Determine whether you would use linear, square, or cubic measure for each item.

a. volume of a packing box	d. length of a piece of yarn
b. size of patio	e. size of housing lot
c. amount of medicine in a syringe	f. height of a flagpole

Exercise 2 Answers

a. cubic	d. linear
b. square	e. square
c. cubic	f. linear

Many geometry applications will involve finding the perimeter or area of a figure. There are also many applications of perimeter and area in everyday life, so it is important to make sure you understand what they each mean.

Picture a room that needs new floor tiles. The tiles come in squares that are a foot on each side — one square foot. How many of those squares are needed to cover the floor? This is the area of the floor.

Next, think about putting a new baseboard around the room once the tiles have been laid. To figure out how many strips are needed, you must know the distance around the room. You would use a tape measure to measure the number of feet around the room. This distance is the perimeter.

Perimeter and area:

The perimeter is a measure of the distance around a figure.

The area is a measure of the surface covered by a figure.

Figure 7.1.6 shows a square tile that is 1 inch on each side. If an ant walked around the edge of the tile, it would walk 4 inches. This distance is the perimeter of the tile.

Since the tile is a square that is 1 inch on each side, its area is one square inch. The area of a shape is measured by determining how many square units cover the shape.

Perimeter = 4 inches

Area = 1 square inch

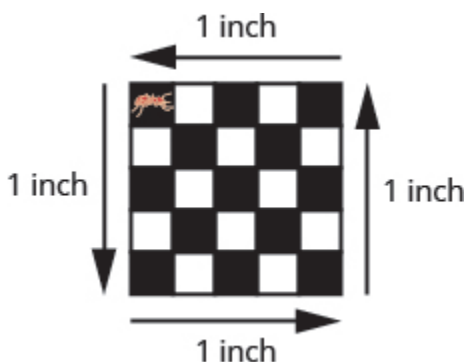


Figure 7.1.6

Example B

Each of the two square tiles is 1 square inch. Two tiles are shown together (see Figure 7.1.7).

- What is the perimeter of the figure?
- What is the area?



Figure 7.1.7

a. Perimeter

The perimeter is the distance around the figure. The perimeter is 6 inches (see Figure 7.1.8).

b. Area

The area is the surface covered by the figure. There are 2 square inch tiles so the area is 2 square inches.

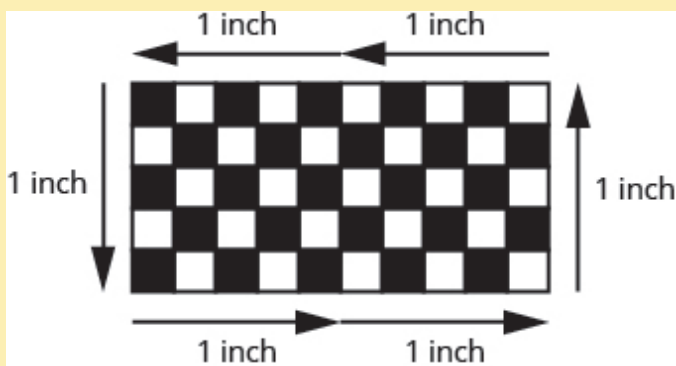


Figure 7.1.8

Exercise 3

For Figure 7.1.9, find the a) perimeter and b) area.



Figure 7.1.9

Exercise 3 Answers

a. 8 inches

b. 3 sq. inches

Exercise 4

For Figure 7.1.10, find the a) perimeter and b) area.

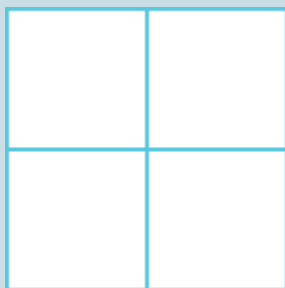


Figure 7.1.10

Exercise 4 Answers

a. 8 centimetres

b. 4 sq. centimetres

Use the Properties of Rectangles

A rectangle has four sides and four right angles. The opposite sides of a rectangle are the same length. We refer to one side of the rectangle as the length, L , and the adjacent side as the width, W (see Figure 7.1.11).

A rectangle has four sides and four right angles. The sides are labelled L for length and W for width.

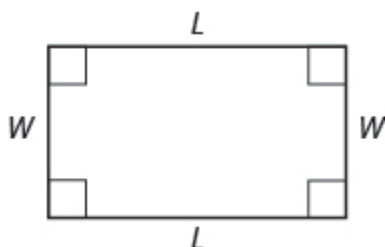


Figure 7.1.11

The perimeter, P , of the rectangle is the distance around the rectangle. If you started at one corner and walked around the rectangle, you would walk $L + W + L + W$ units or two lengths and two widths. The perimeter, then, is

$$P = L + W + L + W$$

or

$$P = 2L + 2W$$

What about the area of a rectangle? Remember the rectangular rug from the beginning of this section. It was 2 feet long by 3 feet wide, and its area was 6 square feet (see Figure 7.1.12). Since $A = 2 \cdot 3$, we see that the area, A , is the length, L , times the width, W , so the area of a rectangle is $A = L \cdot W$.

The area of this rectangular rug is 6 square feet, its length times its width.

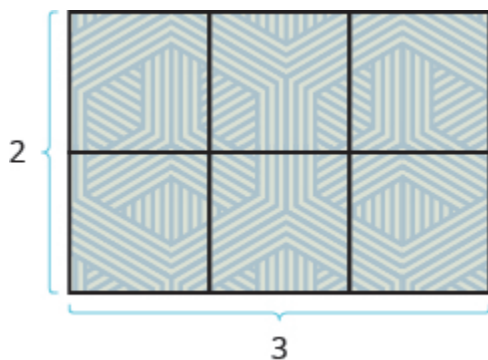


Figure 7.1.12

Properties of rectangles:

- Rectangles have four sides and four right $(90)^\circ$ angles.
- The lengths of opposite sides are equal.
- The perimeter, P , of a rectangle is the sum of twice the length and twice the width (see Figure 7.1.12). $P = 2L + 2W$
- The area, A , of a rectangle is the length times the width. $A = L \cdot W$

For easy reference, as we work through the examples in this section, we will state the problem-solving strategy for geometry applications here.

The problem-solving strategy for geometry applications is:

1. **Read** the problem and make sure you understand all the words and ideas. Draw the figure and label it with the given information.
2. **Identify** what you are looking for.
3. **Name** what you are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer to the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Example C

The length of a rectangle is 32 metres, and the width is 20 metres. Find the a) perimeter and b) area.

a. Perimeter

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.1.13).

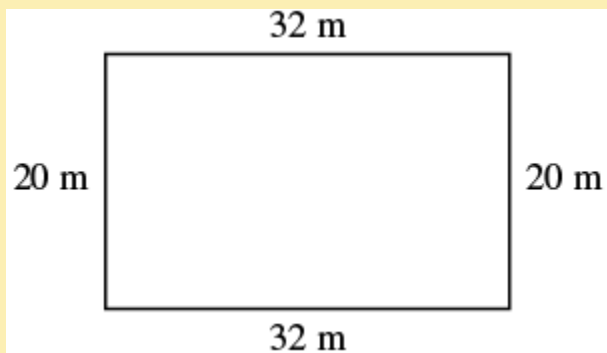


Figure 7.1.13

Step 2: Identify what you are looking for.

The perimeter of a rectangle.

Step 3: Name. Choose a variable to represent it.

Let P = the perimeter

Step 4: Translate. Write the appropriate formula.
Substitute.

$$P = 2L + 2W$$

$$P = 2(32) + 2(20)$$

Step 5: Solve the equation.

$$\begin{aligned} P &= 64 + 40 \\ &= 104 \end{aligned}$$

Step 6: Check.

$$P = 104?$$

$$20 + 32 + 20 + 32 = 104?$$

$$104 = 104 \checkmark$$

Step 7: Answer the question.

The perimeter of the rectangle is 104 metres.

b. Area

Step 1: Read the problem. Draw the figure and label it with the given information (same as in part a).

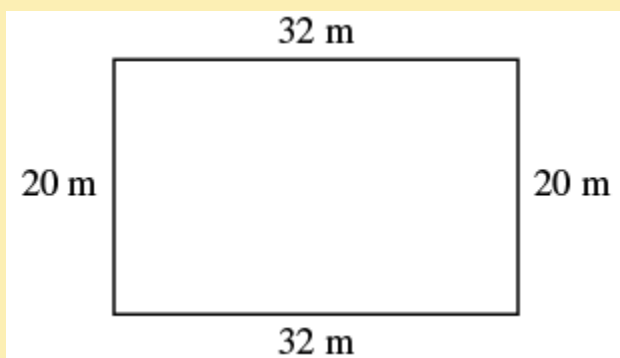


Figure 7.1.13

Step 2: Identify what you are looking for.

The area of a rectangle.

Step 3: Name. Choose a variable to represent it.

Let A = the area

Step 4: Translate. Write the appropriate formula.
Substitute.

$$A = L \cdot W$$

$$A = 32 \text{ m} \cdot 20 \text{ m}$$

Step 5: Solve the equation.

$$A = 640$$

Step 6: Check.

$$A = 640?$$

$$32 \cdot 20 = 640?$$

$$640 = 640 \checkmark$$

Step 7: Answer the question.

The area of the rectangle is 60 square metres.

Exercise 5

The length of a rectangle is 120 yards, and the width is 50 yards. Find the a) perimeter and b) area.

Exercise 5 Answers

a. 340 yd

b. 6000 sq. yd

Exercise 6

The length of a rectangle is 62 feet, and the width is 48 feet. Find the a) perimeter and b) area.

Exercise 6 Answers

a. 220 ft

b. 2976 sq. ft

Example D

Find the length of a rectangle with a perimeter of 50 inches and a width of 10 inches.

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.1.14).

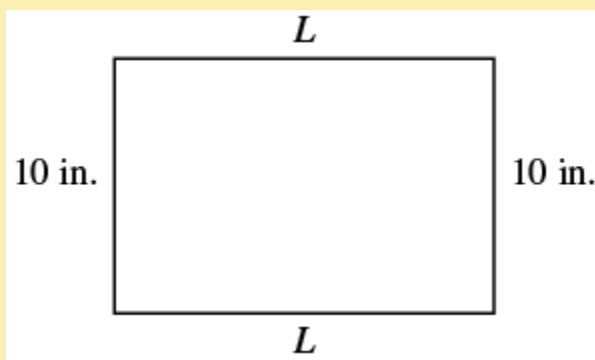


Figure 7.1.14

Step 2: Identify what you are looking for.

The length of the rectangle.

Step 3: Name. Choose a variable to represent it.

Let L = the length

Step 4: Translate. Write the appropriate formula.
Substitute.

$$P = 2L + 2W$$
$$50 = 2L + 2(10)$$

Step 5: Solve the equation.

$$50 - 20 = 2L + 20 - 20$$

$$30 = 2L$$

$$\frac{30}{2} = \frac{2L}{2}$$

$$15 = L$$

Step 6: Check.

$$P = 50$$

$$15 + 10 + 15 + 10 = 50?$$

$$50 = 50 \checkmark$$

Step 7: Answer the question.

The length is 15 inches.

Exercise 7

Find the length of a rectangle with a perimeter of 80 inches and a width of 25 inches.

Exercise 7 Answer

15 in.

Exercise 8

Find the length of a rectangle with a perimeter of 30 yards and a width of 6 yards.

Exercise 8 Answer

9 yd

In the next example, the width is defined in terms of the length. We'll wait to draw the figure until we write an expression for the width so that we can label one side with that expression.

Example E

The width of a rectangle is two inches less than the length. The perimeter is 52 inches. Find the length and width.

Step 1: Read the problem.

Step 2: Identify what you are looking for.

The length and width of the rectangle.

Step 3: Name. Choose a variable to represent it.

Since the width is defined in terms of the length, we let L = length. The width is two feet less than the length, so we let $L - 2$ = width

Now, we can draw a figure using these expressions for the length and width (see Figure 7.1.15).

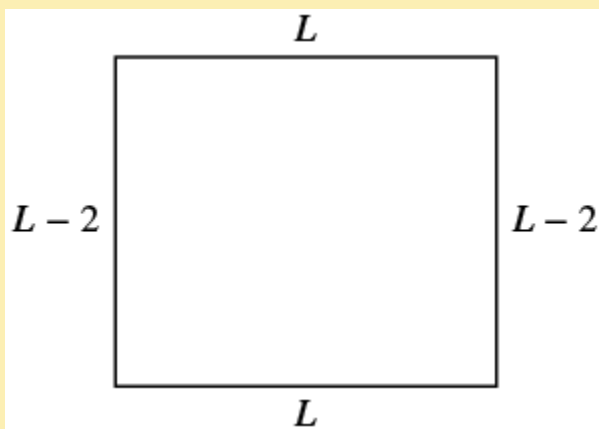


Figure 7.1.15

Step 4: Translate. Write the appropriate formula. The formula for the perimeter of a rectangle relates all the information. Substitute in the given information.

Step 5: Solve the equation.

1. Combine like terms.
2. Add 4 to each side.
3. Divide by 4.

$$52 = 2L + 2L - 4$$

$$52 = 4L - 4$$

$$56 = 4L$$

$$\frac{56}{4} = \frac{4L}{4}$$

$$14 = L$$

The length is 14 inches.

Now, we need to find the width. The width is $L - 2$.

$$L - 2$$

$$14 - 2$$

$$12$$

The width is 12 inches.

Step 6: Check.

Since $14 + 12 + 14 + 12 = 52$, this works!

Step 7: Answer the question.

The length is 14 feet, and the width is 12 feet.

Exercise 9

The width of a rectangle is seven metres less than the

length. The perimeter is 58 metres. Find the length and width.

Exercise 9 Answers

18 m, 11 m

Exercise 10

The length of a rectangle is eight feet more than the width. The perimeter is 60 feet. Find the length and width.

Exercise 10 Answers

11 ft, 19 ft

Example F

The length of a rectangle is four centimetres more than twice the width. The perimeter is 32 centimetres. Find the length and width.

Step 1: Read the problem.

Step 2: Identify what you are looking for.

The length and width.

Step 3: Name. Choose a variable to represent it.

Let W = width

The length is four more than twice the width.

$$2w + 4 = \text{length}$$

Now, you can draw your figure (see Figure 7.1.16).

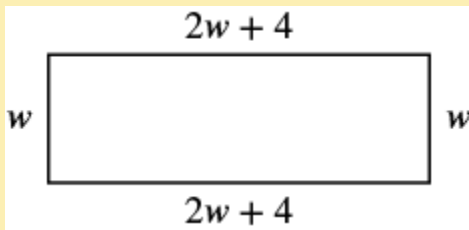


Figure 7.1.16

Step 4: Translate. Write the appropriate formula and substitute the given information into it.

$$P = 2L + 2W$$

$$32 = 2(2w + 4) + 2w$$

Step 5: Solve the equation.

$$32 = 4w + 8 + 2w$$

$$32 = 6w + 8$$

$$24 = 6w$$

$$4 = w$$

The width is 4.

$$2w + 4$$

$$2(4) + 4$$

$$12$$

The length is 12.

Step 6: Check.

$$P = 2L + 2W$$

$$32 = 2 \cdot 12 + 2 \cdot 4?$$

$$32 = 32 \checkmark$$

Step 7: Answer the question.

The length is 12 cm and the width is 4 cm.

Exercise 11

The length of a rectangle is eight more than twice the width. The perimeter is 64 feet. Find the length and width.

Exercise 11 Answers

8 ft, 24 ft

Exercise 12

The width of a rectangle is six less than twice the length. The perimeter is 18 centimetres. Find the length and width.

Exercise 12 Answers

5 cm, 4 cm

Example G

The area of a rectangular room is 168 square feet. The length is 14 feet. What is the width?

Step 1: Read the problem. Draw the figure and label it with the given information. (see Figure 7.1.17).

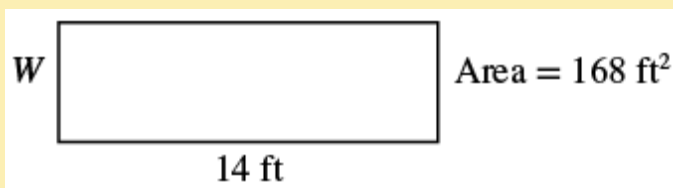


Figure 7.1.17

Step 2: Identify what you are looking for.

The width of a rectangular room.

Step 3: Name. Choose a variable to represent it.

Let W = width

Step 4: Translate. Write the appropriate formula and substitute in the given information.

$$A = LW$$
$$168 = 14W$$

Step 5: Solve the equation.

$$\frac{168}{14} = \frac{14W}{14}$$
$$12 = W$$

Step 6: Check.

$$A = LW$$
$$168 = 14 \cdot 12?$$
$$168 = 168 \checkmark$$

Step 7: Answer the question.

The width of the room is 12 feet.

Exercise 13

The area of a rectangle is 598 square feet. The length is 23 feet. What is the width?

Exercise 13 Answers

26 ft

Exercise 14

The width of a rectangle is 21 metres. The area is 609 square metres. What is the length?

Exercise 14 Answers

29 m

Example H

The perimeter of a rectangular swimming pool is 150 feet. The length is 15 feet more than the width. Find the length and width.

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.1.18).

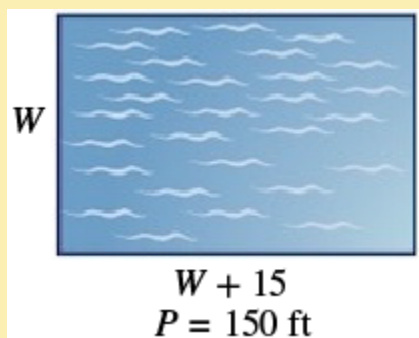


Figure 7.1.18

Step 2: Identify what you are looking for.

The length and width of the pool

Step 3: Name. Choose a variable to represent it.

The length is 15 feet more than the width.

Let W = width

$$W + 15 = \text{length}$$

Step 4: Translate. Write the appropriate formula and substitute.

$$P = 2L + 2W$$

$$150 = 2(w + 15) + 2w$$

Step 5: Solve the equation.

$$150 = 2w + 30 + 2w$$

$$150 = 4w + 30$$

$$120 = 4w$$

$$30 = w$$

The width is 30.

$$w + 15$$

$$30 + 15$$

$$45$$

The length is 45

Step 6: Check.

$$P = 2L + 2W$$

$$150 = 2(45) + 2(30)?$$

$$150 = 150 \checkmark$$

Step 7: Answer the question.

The length of the pool is 45 feet, and the width is 30 feet.

Exercise 15

The perimeter of a rectangular swimming pool is 200 feet. The length is 40 feet more than the width. Find the length and width.

Exercise 15 Answers

30 ft, 70 ft

Exercise 16

The length of a rectangular garden is 30 yards more than the width. The perimeter is 300 yards. Find the length and width.

Exercise 16 Answers

60 yd, 90 yd

Use the Properties of Triangles

We now know how to find the area of a rectangle. We can use this

fact to help us visualize the formula for the area of a triangle. In the rectangle in Figure 7.1.19, we've labelled the length b and the width h , so its area is bh .

The area of a rectangle is the base, b , times the height, h .

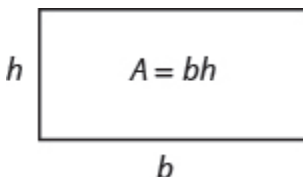


Figure 7.1.19

We can divide this rectangle into two congruent triangles (see Figure 7.1.20). Triangles that are congruent have identical side lengths and angles, and so their areas are equal. The area of each triangle is one-half the area of the rectangle, or $\frac{1}{2}bh$. This example helps us see why the formula for the area of a triangle is $A = \frac{1}{2}bh$.

A rectangle can be divided into two triangles of equal area. The area of each triangle is one-half the area of the rectangle.

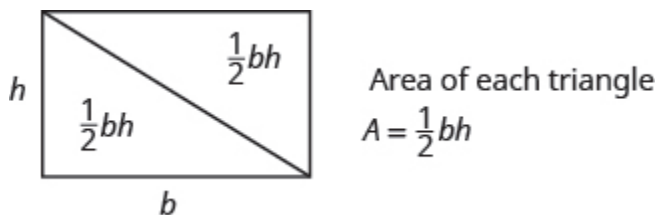


Figure 7.1.20

The formula for the area of a triangle is $A = \frac{1}{2}bh$, where b is the base and h is the height.

To find the area of the triangle, you need to know its base and height.

The base is the length of one side of the triangle, usually the side at the bottom. The height is the length of the line that connects the base to the opposite vertex and makes a 90° angle with the base. Figure 7.1.21 shows three triangles with the base and height of each marked.

The height h of a triangle is the length of a line segment that connects the base to the opposite vertex and makes a 90° angle with the base.

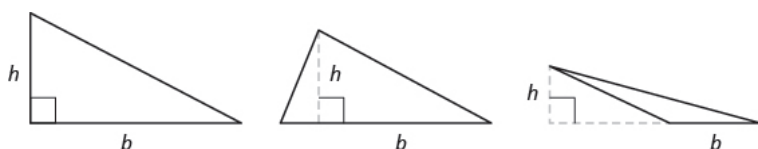


Figure 7.1.21

Properties of triangles:

For any triangle $\triangle ABC$, the sum of the measures of the angles is 180° .

$$m\angle A + m\angle B + m\angle C = 180^\circ$$

The perimeter of a triangle is the sum of the lengths of the sides.

$$P = a + b + c$$

The area of a triangle is one-half the base, b , times the height, h .

$$A = \frac{1}{2}bh$$

Figure 7.1.22 shows an example triangle.

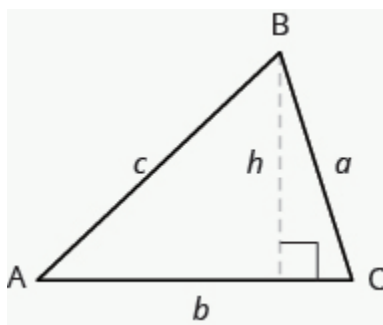


Figure 7.1.22

Example I

Find the area of a triangle whose base is 11 inches and whose height is 8 inches.

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.1.23).

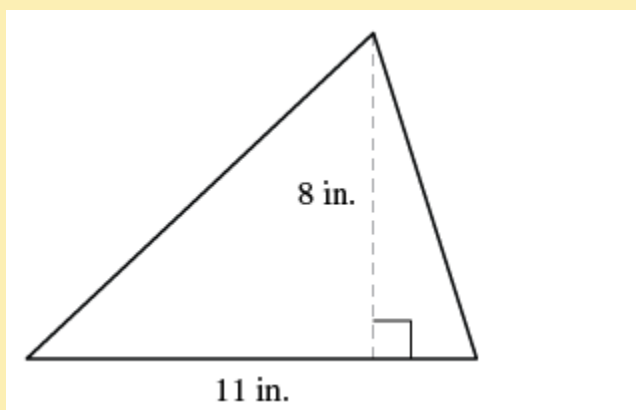


Figure 7.1.23

Step 2: Identify what you are looking for.

The area of the triangle.

Step 3: Name. Choose a variable to represent it.

Let A = area of the triangle

Step 4: Translate. Write the appropriate formula.
Substitute.

$$A = \frac{1}{2} \cdot b \cdot h$$

$$A = \frac{1}{2} \cdot 11 \cdot 8$$

Step 5: Solve the equation.

$$A = 44 \text{ square inches}$$

Step 6: Check.

$$A = \frac{1}{2}bh$$

$$44 = \frac{1}{2}(11)(8)?$$

$$44 = 44 \checkmark$$

Step 7: Answer the question.

The area is 44 square inches.

Exercise 17

Find the area of a triangle with a base of 13 inches and a height of 2 inches.

Exercise 17 Answer

13 sq. in.

Exercise 18

Find the area of a triangle with a base of 14 inches and a height of 7 inches.

Exercise 18 Answer

49 sq. in.

Example J

The perimeter of a triangular garden is 24 feet. The lengths of two sides are 4 feet and 9 feet. How long is the third side?

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.1.24).

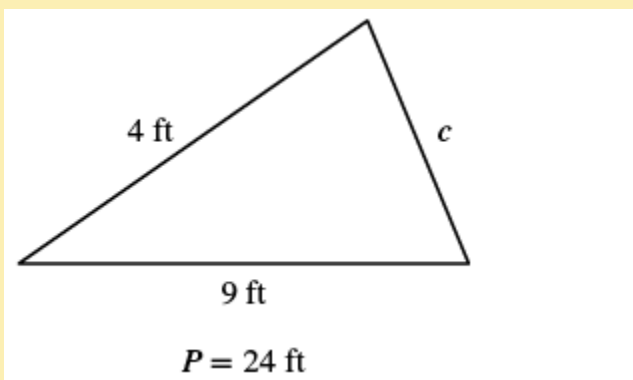


Figure 7.1.24

Step 2: Identify what you are looking for.

The length of the third side of a triangle.

Step 3: Name. Choose a variable to represent it.

Let c = the third side

Step 4: Translate. Write the appropriate formula.

Substitute in the given information.

$$P = a + b + c$$

$$24 = 4 + 9 + c$$

Step 5: Solve the equation.

$$24 = 13 + c$$

$$11 = c$$

Step 6: Check.

$$\begin{aligned}P &= a + b + c \\24 &= 4 + 9 + 11 \\24 &= 24 \checkmark\end{aligned}$$

Step 7: Answer the question.

The third side is 11 feet long.

Exercise 19

The perimeter of a triangular garden is 24 feet. The lengths of the two sides are 18 feet and 22 feet. How long is the third side?

Exercise 19 Answer

8 ft

Exercise 20

The lengths of the two sides of a triangular window are 7 feet and 5 feet. The perimeter is 18 feet. How long is the third side?

Exercise 20 Answer

6 ft

Example K

The area of a triangular church window is 90 square metres. The base of the window is 15 metres. What is the window's height?

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.1.25).

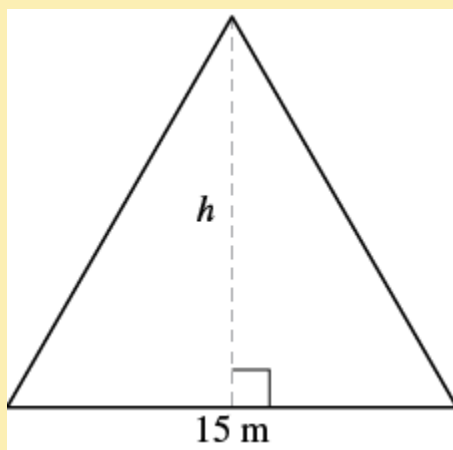


Figure 7.1.25

Step 2: Identify what you are looking for.

The height of a triangle.

Step 3: Name. Choose a variable to represent it.

Let h = the height

Step 4: Translate. Write the appropriate formula.

Substitute in the given information.

$$A = \frac{1}{2} \cdot b \cdot h$$
$$90 = \frac{1}{2} \cdot 15 \cdot h$$

Step 5: Solve the equation.

$$90 = \frac{15}{2}h$$

$$12 = h$$

Step 6: Check.

$$A = \frac{1}{2}bh$$

$$90 = \frac{1}{2} \cdot 15 \cdot 12?$$

$$90 = 90 \checkmark$$

Step 7: Answer the question.

The height of the triangle is 12 metres.

Exercise 21

The area of a triangular painting is 126 square inches. The base is 18 inches. What is the height?

Exercise 21 Answer

14 in.

Exercise 22

A triangular tent door has an area of 15 square feet. The height is 5 feet. What is the base?

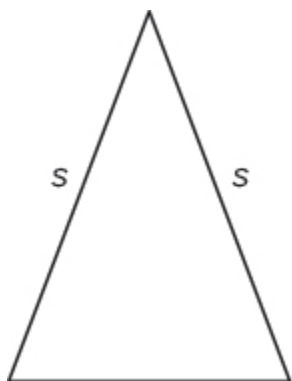
Exercise 22 Answer

6 ft

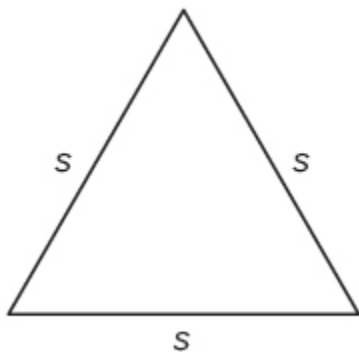
Isosceles & Equilateral Triangles

Besides the right triangle, some other triangles have special names. A triangle with two sides of equal length is called an isosceles triangle. A triangle that has three sides of equal length is called an equilateral triangle. Figure 7.1.26 shows both types of triangles.

In an isosceles triangle, two sides have the same length, and the third side is the base. In an equilateral triangle, all three sides have the same length.



isosceles triangle



equilateral triangle

Figure 7.1.26

An **isosceles** triangle has two sides the same length.

An **equilateral** triangle has three sides of equal length.

Example L

The perimeter of an equilateral triangle is 93 inches. Find the length of each side.

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.1.27).

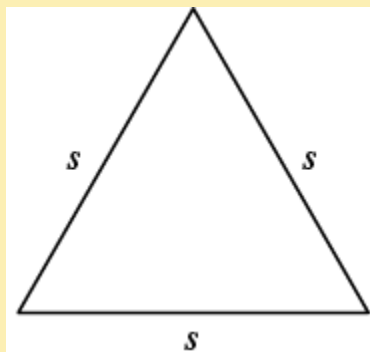


Figure 7.1.27

Perimeter = 93 in.

Step 2: Identify what you are looking for.

The length of the sides of an equilateral triangle.

Step 3: Name. Choose a variable to represent it.

Let s = length of each side

Step 4: Translate. Write the appropriate formula.
Substitute.

$$P = a + b + c$$

$$93 = s + s + s$$

Step 5: Solve the equation.

$$93 = 3s$$

$$31 = s$$

Step 6: Check.

Figure 7.1.28 shows the completed figure.

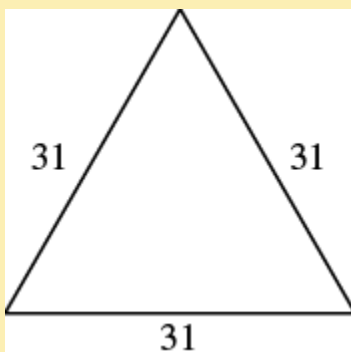


Figure 7.1.28

$$93 = 31 + 31 + 31?$$

$$93 = 93 \checkmark$$

Step 7: Answer the question.

Each side is 31 inches.

Exercise 23

Find the length of each side of an equilateral triangle with a perimeter of 39 inches.

Exercise 23 Answer

13 in.

Exercise 24

Find the length of each side of an equilateral triangle with a perimeter of 51 centimetres.

Exercise 24 Answer

17 cm

Example M

Arianna has 156 inches of beading to use as trim around a scarf. The scarf will be an isosceles triangle with a base of 60 inches. How long can she make the two equal sides?

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.1.29).

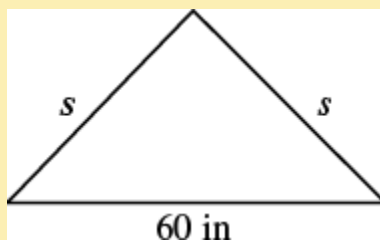


Figure 7.1.29

$$P = 156 \text{ in.}$$

Step 2: Identify what you are looking for.

The lengths of the two equal sides.

Step 3: Name. Choose a variable to represent it.

Let s = the length of each side

Step 4: Translate. Write the appropriate formula.
Substitute.

$$P = a + b + c$$
$$156 = s + 60 + s$$

Step 5: Solve the equation.

$$156 = 2s + 60$$
$$96 = 2s$$
$$48 = s$$

Step 6: Check.

$$156 = 48 + 60 + 48?$$
$$156 = 156 \checkmark$$

Step 7: Answer the question.

Arianna can make each of the two equal sides 48 inches long.

Exercise 25

A backyard deck is in the shape of an isosceles triangle with a base of 20 feet. The perimeter of the deck is 48 feet. How long is each of the equal sides of the deck?

Exercise 25 Answer

14 ft

Exercise 26

A boat's sail is an isosceles triangle with a base of 8 metres. The perimeter is 22 metres. How long is each of the equal sides of the sail?

Exercise 26 Answer

7 m

Use the Properties of Trapezoids

A trapezoid is a four-sided figure, a quadrilateral, with two sides that are parallel and two sides that are not. The parallel sides are called the bases. We call the length of the smaller base b and the length of the bigger base B . The height, h , of a trapezoid is the distance between the two bases (see Figure 7.1.30).

A trapezoid has a larger base, B , and a smaller base, b . The height h is the distance between the bases.

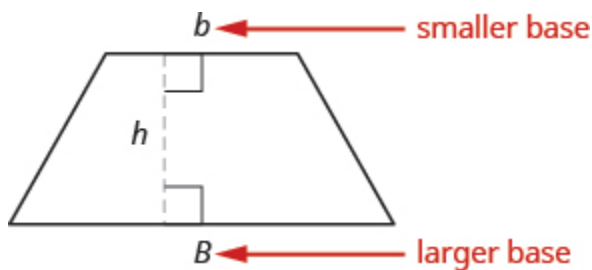


Figure 7.1.30

Formula for the area of a trapezoid:

$$\text{Area}_{\text{trapezoid}} = \frac{1}{2}h(b + B)$$

Splitting the trapezoid into two triangles may help us understand the formula. The area of the trapezoid is the sum of the areas of the two triangles (see Figure 7.1.31).

Splitting a trapezoid into two triangles may help you understand the formula for its area.

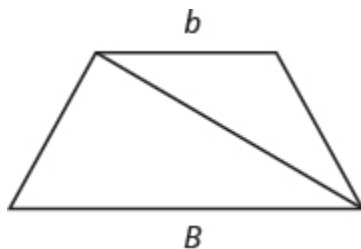


Figure 7.1.31

The height of the trapezoid is also the height of each of the two triangles (see Figure 7.1.32).

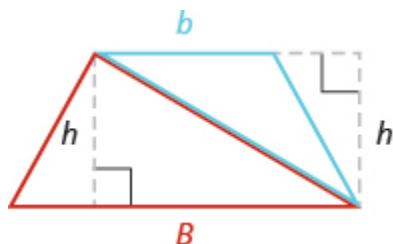


Figure 7.1.32

The formula for the area of a trapezoid is:

Formula does not parse

If we distribute, we get:

Formula does not parse

Properties of trapezoids:

- A trapezoid has four sides.
- Two of its sides are parallel, and two sides are not.
- The area, A , of a trapezoid is

$$A = \frac{1}{2}h(b + B).$$

Example N

Find the area of a trapezoid whose height is 6 inches and whose bases are 14 and 11 inches.

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.1.33).

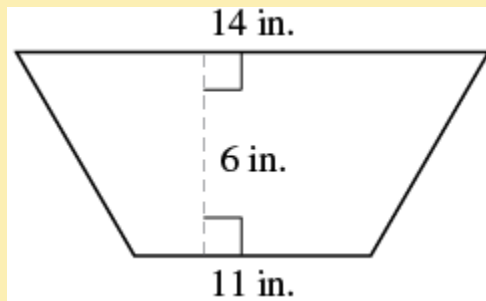


Figure 7.1.33

Step 2: Identify what you are looking for.

The area of the trapezoid.

Step 3: Name. Choose a variable to represent it.

Let A = the area

Step 4: Translate. Write the appropriate formula.
Substitute.

$$A = \frac{1}{2} \cdot h \cdot (b + B)$$

$$A = \frac{1}{2} \cdot 6 \cdot (11 + 14)$$

Step 5: Solve the equation.

$$A = \frac{1}{2} \cdot 6(25)$$

$$A = 3(25)$$

$$A = 25 \text{ square inches}$$

Step 6: Check. Is this answer reasonable?

If we draw a rectangle around the trapezoid with the same big base B and a height h , its area should be greater than that of the trapezoid.

If we draw a rectangle inside the trapezoid with the same little base b and a height h , its area should be smaller than that of the trapezoid.

See Figure 7.1.34 for more details.

		
$A_{\text{rectangle}} = bh$	$A_{\text{trapezoid}} = \frac{1}{2} h(b + B)$	$A_{\text{rectangle}} = bh$
$A_{\text{rectangle}} = 14 \cdot 6$	$A_{\text{trapezoid}} = \frac{1}{2} \cdot 6(11 + 14)$	$A_{\text{rectangle}} = 11 \cdot 6$
$A_{\text{rectangle}} = 84 \text{ sq. in.}$	$A_{\text{trapezoid}} = 75 \text{ sq. in.}$	$A_{\text{rectangle}} = 66 \text{ sq. in.}$

Figure 7.1.34

The area of the larger rectangle is 84 square inches, and

the area of the smaller rectangle is 66 square inches. So, it makes sense that the area of the trapezoid is between 84 and 66 square inches.

Step 7: Answer the question.

The area of the trapezoid is 75 square inches.

Exercise 27

The height of a trapezoid is 14 yards, and the bases are 7 and 16 yards. What is the area?

Exercise 27 Answer

161 sq. yd

Exercise 28

The height of a trapezoid is 18 centimetres, and the bases are 17 and 8 centimetres. What is the area?

Exercise 28 Answer

225 sq. cm

Example O

Find the area of a trapezoid whose height is 5 feet and whose bases are 10.3 and 13.7 feet.

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.1.35).

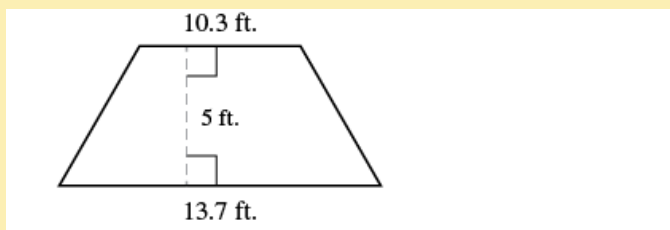


Figure 7.135

Step 2: Identify what you are looking for.

The area of the trapezoid.

Step 3: Name. Choose a variable to represent it.

Let A = the area

Step 4: Translate. Write the appropriate formula.
Substitute.

$$A = \frac{1}{2} \cdot h \cdot (b + B)$$
$$A = \frac{1}{2} \cdot 5 \cdot (10.3 + 13.7)$$

Step 5: Solve the equation.

$$A = \frac{1}{2} \cdot 5(24)$$
$$A = 12 \cdot 5$$
$$A = 60 \text{ square feet}$$

Step 6: Check. Is this answer reasonable?

The area of the trapezoid should be less than the area of a rectangle with base 13.7 and height 5, but more than the

area of a rectangle with base 10.3 and height 5 (see Figure 7.1.36).

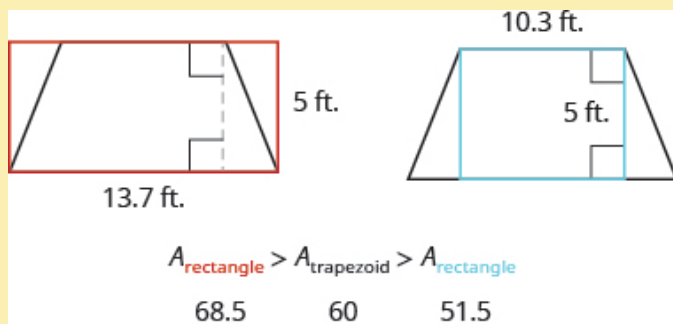


Figure 7.1.36

Step 7: Answer the question.

The area of the trapezoid is 60 square feet.

Exercise 29

The height of a trapezoid is 7 centimetres, and the bases are 4.6 and 7.4 centimetres. What is the area?

Exercise 29 Answer

42 sq. cm

Exercise 30

The height of a trapezoid is 9 metres, and the bases are 6.2 and 7.8 metres. What is the area?

Exercise 30 Answer

63 sq. m

Example P

Vinny has a garden that is shaped like a trapezoid. The trapezoid has a height of 3.4 yards, and the bases are 8.2 and 5.6 yards. How many square yards will be available to plant?

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.1.37).

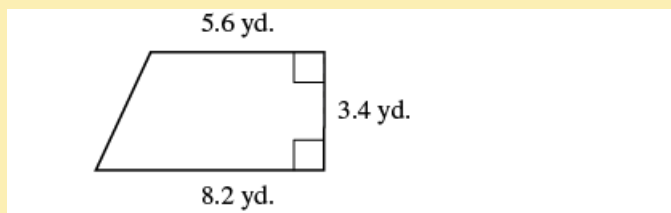


Figure 7.1.37

Step 2: Identify what you are looking for.

The area of the trapezoid.

Step 3: Name. Choose a variable to represent it.

Let A = the area

Step 4: Translate. Write the appropriate formula.
Substitute.

$$A = \frac{1}{2} \cdot h \cdot (b + B)$$

$$A = \frac{1}{2} \cdot 3.4 \cdot (5.6 + 8.2)$$

Step 5: Solve the equation.

$$A = \frac{1}{2}(3.4)(13.8)$$

$$A = 23.46 \text{ square yards}$$

Step 6: Check. Is this answer reasonable?

Yes. The area of the trapezoid is less than the area of a rectangle with a base of 8.2 yd and height 3.4 yd, but more than the area of a rectangle with base 5.6 yd and height 3.4 yd (see Figure 7.1.38).

$A_{\text{rectangle}} = Bh$ $= (8.2)(3.4)$ $= 27.88 \text{ yd}^2$	$A_{\text{trapezoid}} = \frac{1}{2}(3.4 \text{ yd})(5.6 + 8.2)$ $= 23.46 \text{ yd}^2$	$A_{\text{rectangle}} = bh$ $= (5.6)(3.4)$ $= 19.04 \text{ yd}^2$
$A_{\text{rectangle}} > A_{\text{trapezoid}} > A_{\text{rectangle}}$ 27.88 23.46 19.04		

Figure 7.1.38

Step 7: Answer the question.

Vinny has 23.46 square yards in which he can plan.

Exercise 31

Lin wants to sod his lawn, which is shaped like a trapezoid. The bases are 10.8 yards and 6.7 yards, and the height is 4.6 yards. How many square yards of sod does he need?

Exercise 31 Answer

40.25 sq. yd

Exercise 32

Kira wants to cover his patio with concrete pavers. If the patio is shaped like a trapezoid whose bases are 18 feet and 14 feet and whose height is 15 feet, how many square feet of pavers will he need?

Exercise 32 Answer

240 sq. ft.

Additional online resources:

- **Perimeter of a Rectangle**
 - Ex: Determine the Perimeter of a Rectangle Involving Whole Numbers [2:41 min] (Mathispower4u, 2011c).
- **Area of a Rectangle**
 - Ex: Determine the Area of a Rectangle Involving Whole Numbers [2:06 min] (Mathispower4u, 2011b).
- **Perimeter and Area Formulas**
 - Perimeter and Area Formulas [9:44 min] (Mathispower4u, 2009).
- **Area of a Triangle**
 - Ex: Area of a Triangle (Whole Numbers) [1:56 min] (Mathispower4u, 2011d).
- **Area of a Triangle with Fractions**

- Example: Area of a Triangle Involving Fractions [1:34 min] (Mathispower4u, 2011a).
- **Area of a Trapezoid**
 - Ex: Find the Area of a Trapezoid [2:32 min] (Mathispower4u, 2012).

Key Concepts

Properties of Rectangles

- Rectangles have four sides and four right (90°) angles.
- The lengths of opposite sides are equal.
- The perimeter, P , of a rectangle is the sum of twice the length and twice the width.
 - $P = 2L + 2W$
- The area, A , of a rectangle is the length times the width.
 - $A = L \cdot W$

Triangle Properties

- For any triangle $\triangle ABC$, the sum of the measures of the angles is 180° .
 - $m\angle A + m\angle B + m\angle C = 180^\circ$

- The perimeter of a triangle is the sum of the lengths of the sides.
 - $P = a + b + c$
- The area of a triangle is one-half the base, b , times the height, h .
 - $A = \frac{1}{2}bh$

Glossary

- **Area** — The area is a measure of the surface covered by a figure.
- **Equilateral triangle** — A triangle with all three sides of equal length is called an equilateral triangle.
- **Isosceles triangle** — A triangle with two sides of equal length is called an isosceles triangle.
- **Perimeter** — The perimeter is a measure of the distance around a figure.
- **Rectangle** — A rectangle is a geometric figure with four sides and four right angles.
- **Trapezoid** — A trapezoid is a four-sided figure, a quadrilateral, with two sides that are parallel and two sides that are not.

7.1 Practice Questions

1. Understand Linear, Square, & Cubic Measure

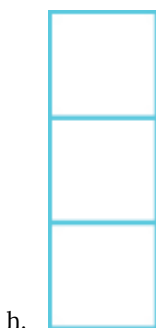
In the following exercises, determine whether you would measure each item using linear, square, or cubic units.

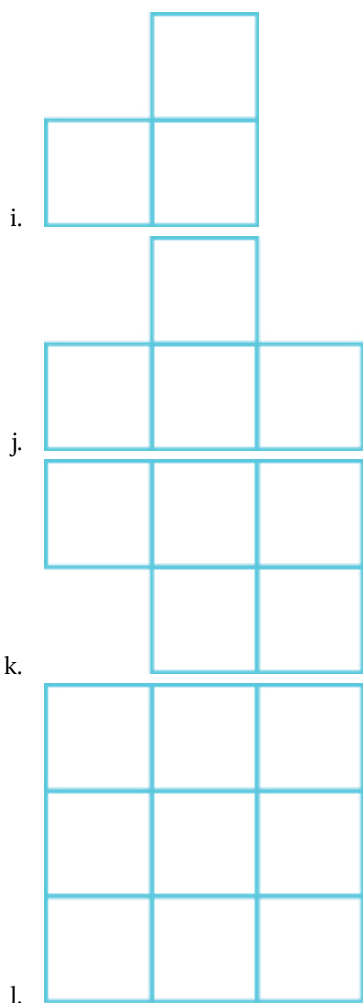
a. amount of water in a fish tank	d. floor space of a bathroom tile
b. length of dental floss	e. height of a doorway
c. living area of an apartment	f. capacity of a truck trailer

In the following exercises, find the a) perimeter and b) area of each figure. Assume each side of the square is 1 cm.



**Figure
7.1.39**





2. Use the Properties of Rectangles

In the following exercises, find the a) perimeter and b) area of each rectangle.

- a. The length of a rectangle is 85 feet, and the width is 45 feet.
- b. The length of a rectangle is 26 inches, and the width is 58 inches.
- c. A rectangular room is 15 feet wide by 14 feet long.
- d. A driveway is in the shape of a rectangle 20 feet wide by 35 feet long.

In the following exercises, solve.

- e. Find the length of a rectangle with a perimeter of 124 inches and a width of 38 inches.
- f. Find the length of a rectangle with a perimeter of 124 inches and a width of 38 inches.
- g. Find the width of a rectangle with a perimeter of 92 metres and a length of 19 metres.
- h. Find the width of a rectangle with a perimeter of 16.2 metres and a length of 3.2 metres.
- i. The area of a rectangle is 414 square metres. The length is 18 metres. What is the width?
- j. The area of a rectangle is 782 square centimetres. The width is 17 centimetres. What is the length?
- k. The length of a rectangle is 9 inches more than the width. The perimeter is 46 inches. Find the length and the width.
- l. The width of a rectangle is 8 inches more than the length. The perimeter is 52 inches. Find the length and the width.
- m. The perimeter of a rectangle is 58 metres. The width of the rectangle is 5 metres less than the length. Find the length and the width of the rectangle.
- n. The perimeter of a rectangle is 62 feet. The width is 7 feet less than the length. Find the length and the width.
- o. The width of the rectangle is 0.7 metres less than the length. The perimeter of a rectangle is 52.6 metres. Find the dimensions of the rectangle.
- p. The length of the rectangle is 1.1 metres less than the width. The perimeter of a rectangle is 49.4 metres. Find the

dimensions of the rectangle.

- q. The perimeter of a rectangle is 150 feet. The length of the rectangle is twice the width. Find the length and width of the rectangle.
- r. The length of a rectangle is three times the width. The perimeter is 72 feet. Find the length and width of the rectangle.
- s. The length of a rectangle is 3 metres less than twice the width. The perimeter is 36 metres. Find the length and width.
- t. The length of a rectangle is 5 inches more than twice the width. The perimeter is 34 inches. Find the length and width.
- u. The width of a rectangular window is 24 inches. The area is 624 square inches. What is the length?
- v. The length of a rectangular poster is 28 inches. The area is 1316 square inches. What is the width?
- w. The area of a rectangular roof is 2310 square metres. The length is 42 metres. What is the width?
- x. The area of a rectangular tarp is 132 square feet. The width is 12 feet. What is the length?
- y. The perimeter of a rectangular courtyard is 160 feet. The length is 10 feet more than the width. Find the length and the width.
- z. The perimeter of a rectangular painting is 306 centimetres. The length is 17 centimetres more than the width. Find the length and the width.
- aa. The width of a rectangular window is 40 inches less than the height. The perimeter of the doorway is 224 inches. Find the length and the width.
- ab. The width of a rectangular playground is 7 metres less than the length. The perimeter of the playground is 46 metres. Find the length and the width.

3. Use the Properties of Triangles

In the following exercises, solve using the properties of triangles.

- a. Find the area of a triangle with a base of 12 inches and a height of 5 inches.
- b. Find the area of a triangle with a base of 45 centimetres and a height of 30 centimetres.
- c. Find the area of a triangle with a base of 8.3 metres and a height of 6.1 metres.
- d. Find the area of a triangle with a base of 24.2 feet and a height of 20.5 feet.
- e. A triangular flag has a base of 1 foot and a height of 1.5 feet. What is its area?
- f. A triangular window has a base of 8 feet and a height of 6 feet. What is its area?
- g. If a triangle has sides of 6 feet and 9 feet and the perimeter is 23 feet, how long is the third side?
- h. If a triangle has sides of 14 centimetres and 18 centimetres and the perimeter is 49 centimetres, how long is the third side?
- i. What is the base of a triangle with an area of 207 square inches and a height of 18 inches?
- j. What is the height of a triangle with an area of 893 square inches and a base of 38 inches?
- k. The perimeter of a triangular reflecting pool is 36 yards. The lengths of the two sides are 10 yards and 15 yards. How long is the third side?
- l. A triangular courtyard has a perimeter of 120 metres. The lengths of the two sides are 30 metres and 50 metres. How long is the third side?
- m. An isosceles triangle has a base of 20 centimetres. If the perimeter is 76 centimetres, find the length of each of the other sides.
- n. An isosceles triangle has a base of 25 inches. If the perimeter is

95 inches, find the length of each of the other sides.

- o. Find the length of each side of an equilateral triangle with a perimeter of 51 yards.
- p. Find the length of each side of an equilateral triangle with a perimeter of 54 metres.
- q. The perimeter of an equilateral triangle is 18 metres. Find the length of each side.
- r. The perimeter of an equilateral triangle is 42 miles. Find the length of each side.
- s. The perimeter of an isosceles triangle is 42 feet. The length of the shortest side is 12 feet. Find the length of the other two sides.
- t. The perimeter of an isosceles triangle is 83 inches. The length of the shortest side is 24 inches. Find the length of the other two sides.
- u. A dish is in the shape of an equilateral triangle. Each side is 8 inches long. Find the perimeter.
- v. A floor tile is in the shape of an equilateral triangle. Each side is 1.5 feet long. Find the perimeter.
- w. A road sign in the shape of an isosceles triangle has a base of 36 inches. If the perimeter is 91 inches, find the length of each of the other sides.
- x. A scarf in the shape of an isosceles triangle has a base of 0.75 metres. If the perimeter is 2 metres, find the length of each of the other sides.
- y. The perimeter of a triangle is 39 feet. One side of the triangle is 1 foot longer than the second side. The third side is 2 feet longer than the second side. Find the length of each side.
- z. The perimeter of a triangle is 35 feet. One side of the triangle is 5 feet longer than the second side. The third side is 3 feet longer than the second side. Find the length of each side.
- aa. One side of a triangle is twice the smallest side. The third side is 5 feet more than the shortest side. The perimeter is 17 feet. Find the lengths of all three sides.
- ab. One side of a triangle is three times the smallest side. The third

side is 3 feet more than the shortest side. The perimeter is 13 feet. Find the lengths of all three sides.

4. Use the Properties of Trapezoids

In the following exercises, solve using the properties of trapezoids.

- a. The height of a trapezoid is 12 feet, and the bases are 9 and 15 feet. What is the area?
- b. The height of a trapezoid is 24 yards, and the bases are 18 and 30 yards. What is the area?
- c. Find the area of a trapezoid with a height of 51 metres and bases of 43 and 67 metres.
- d. Find the area of a trapezoid with a height of 62 inches and bases of 58 and 75 inches.
- e. The height of a trapezoid is 15 centimetres, and the bases are 12.5 and 18.3 centimetres. What is the area?
- f. The height of a trapezoid is 48 feet, and the bases are 38.6 and 60.2 feet. What is the area?
- g. Find the area of a trapezoid with a height of 4.2 metres and bases of 8.1 and 5.5 metres.
- h. Find the area of a trapezoid with a height of 32.5 centimetres and bases of 54.6 and 41.4 centimetres.
- i. Laurel is making a banner shaped like a trapezoid. The height of the banner is 3 feet and the bases are 4 and 5 feet. What is the area of the banner?
- j. Niko wants to tile the floor of his bathroom. The floor is shaped like a trapezoid with a width of 5 feet and lengths of 5 feet and 8 feet. What is the area of the floor?
- k. Theresa needs a new top for her kitchen counter. The counter is shaped like a trapezoid with a width of 18.5 inches and lengths 62 and 50 inches. What is the area of the counter?
- l. Elena is knitting a scarf. The scarf will be shaped like a

trapezoid with a width of 8 inches and lengths of 48.2 inches and 56.2 inches. What is the area of the scarf?

5. Everyday Math

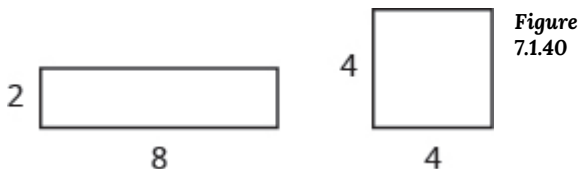
- a. **Fence:** Jose just removed the children's playset from his backyard to make room for a rectangular garden. He wants to put a fence around the garden to keep out the dog. He has a 50-foot roll of fence in his garage that he plans to use. To fit in the backyard, the width of the garden must be 10 feet. How long can he make the other side if he wants to use the entire roll of fence?
- b. **Gardening:** Lupita wants to fence in her tomato garden. The garden is rectangular, and the length is twice the width. It will take 48 feet of fencing to enclose the garden. Find the length and width of her garden.
- c. **Fence:** Christa wants to put a fence around her triangular flowerbed. The sides of the flowerbed are 6 feet, 8 feet, and 10 feet. The fence costs \$10 per foot. How much will it cost for Christa to fence in her flowerbed?
- d. **Painting:** Caleb wants to paint one wall of his attic. The wall is shaped like a trapezoid with a height of 8 feet and bases of 20 feet and 12 feet (like Figure 7.1.39). The cost of painting one square foot of wall is about 0.05. About how much will it cost for Caleb to paint the attic wall?



**Figure
7.1.39**

6. Writing Exercises

- a. Look at the two figures in Figure 7.1.40.



- Which figure looks like it has the larger area? Which looks like it has the larger perimeter?
 - Now, calculate the area and perimeter of each figure. Which has the larger area? Which has the larger perimeter?
- b. The length of a rectangle is 5 feet more than the width. The area is 50 square feet. Find the length and the width.
- Write the equation you would use to solve the problem.
 - Why can't you solve this equation with the methods you learned in the previous chapter?

7.1: Practice Answers

1. Understand linear, square, and cubic measure

a. cubic	e. linear	i. 8 cm, 3 sq. cm
b.	f.	j.
c. square	g. 10 cm and 4 sq. cm	k. 10 cm, 5 sq. cm
d.	h.	

2. Use the properties of rectangles

a. 260 ft, 3825 ft	c. 58 ft, 210 sq. ft	e. 24 inches
b.	d.	f.

g. 27 metres	o. 13.5 m, 12.8 m	w. 55 m
h.	p.	x.
i. 23 m	q. 25 ft, 50 ft	y. 35 ft, 45 ft
j.	r.	z.
k. 7 in., 16 in.	s. 7 m, 11 m	aa. 76 in., 36 in.
l.	t.	
m. 17 m, 12 m	u. 26 in.	
n.	v.	

3. Use the properties of triangles

a. 30 sq. in.	k. 11 yds.	u. 24 in.
b.	l.	v.
c. 25.315 sq. m	m. 28 cm	w. 27.5 in.
d.	n.	x.
e. 0.75 sq. ft	o. 17 yds.	y. 12 ft, 13 ft, 14 ft
f.	p.	z.
g. 8 ft	q. 6 m	aa. 3 ft, 6 ft, 8 ft
h.	r.	
i. 23 in.	s. 15 ft	
j.	t.	

4. Use the properties of trapezoids

a. 144 sq. ft	e. 231 sq. cm	i. 13.5 sq. ft
b.	f.	j.
c. 2805 sq. m	g. 28.56 sq. m	k. 1036 sq. in.
d.	h.	

5. Everyday math

a. 15 ft	c. \$240
b.	

6. Writing exercises

a. Answers will vary.	b. Answers will vary.

References

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[Video]. YouTube. <https://youtu.be/ZASBmoylCPc?si=qVwtR8CyNr1fWdn4>.

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Attributions

All figures in this chapter are from 3.2 Use Properties of Rectangles, Triangles, and Trapezoids in *Introductory Algebra* by Izabela Mazur, via BCcampus.

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7.2: Solve Geometry Applications — Volume & Surface Area

In this section, we will find the volume and surface area of some three-dimensional figures. Since we will be solving applications, we will once again show our Problem-Solving Strategy for Geometry Applications.

The problem-solving strategy for geometry applications:

1. **Read** the problem and make sure you understand all the words and ideas. Draw the figure and label it with the given information.
2. **Identify** what you are looking for.
3. **Name** what you are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer in the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Find Volume & Surface Area of Rectangular Solids

A cheerleading coach wants their squad to squad paint wooden crates with the school colours to stand on at the games (see Figure 7.2.1). The amount of paint needed to cover the outside of each box is the surface area, a square measure of the total area of all the sides. The amount of space inside the crate is the volume, a cubic measure.

This wooden crate is in the shape of a rectangular solid.



Figure 7.2.1

Each crate is in the shape of a rectangular solid. Its dimensions are the length, width, and height. The rectangular solid shown in Figure 7.2.2 has a length of 4 units, a width of 2 units, and a height of 3 units. Can you tell how many cubic units there are altogether? Let's look layer by layer.

Breaking a rectangular solid into layers makes it easier to visualize

the number of cubic units it contains. Figure 7.2.2 is a 4 by 2 by 3 rectangular solid with 24 cubic units.

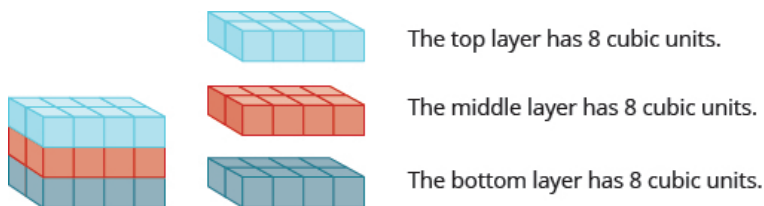


Figure 7.2.2

Altogether there are 24 cubic units. Notice that 24 is the **length** $\times$ **width** $\times$ **height**.

$$V = L \cdot W \cdot H$$

$$24 = 4 \cdot 2 \cdot 3$$

The volume, V , of any rectangular solid is the product of the length, width, and height.

$$V = LWH$$

We could also write the formula for the volume of a rectangular solid in terms of the area of the base. The area of the base, B , is equal to **length** $\times$ **width**.

$$B = L \cdot W$$

We can substitute B for $L \cdot W$ in the volume formula to get another form of the volume formula.

$$V = L \cdot W \cdot H$$

$$V = (L \cdot W) \cdot H$$

$$V = Bh$$

We now have another version of the volume formula for rectangular solids. Let's see how this works with the $4 \times 2 \times 3$ rectangular solid we started with (see Figure 7.2.3).

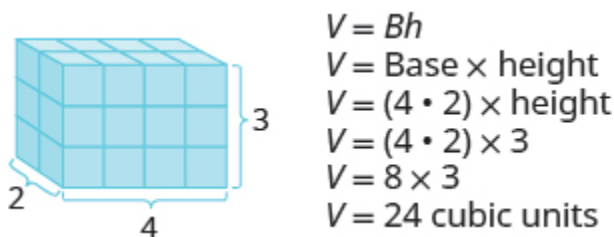


Figure 7.2.3

To find the **surface area** of a rectangular solid, think about finding the area of each of its faces. How many faces does the rectangular solid above have? You can see three of them.

$$\begin{array}{lll}
 A_{\text{front}} = L \times W & A_{\text{side}} = L \times W & A_{\text{top}} = L \times W \\
 A_{\text{front}} = 4 \cdot 3 & A_{\text{side}} = 2 \cdot 3 & A_{\text{top}} = 4 \cdot 2 \\
 A_{\text{front}} = 12 & A_{\text{side}} = 6 & A_{\text{top}} = 8
 \end{array}$$

Notice that for each of the three faces you see, there is an identical opposite face that does not show.

$$S = (\text{front} + \text{back}) + (\text{left side} + \text{right side}) + (\text{top} + \text{bottom})$$

$$S = (2 \cdot \text{front}) + (2 \cdot \text{left side}) + (2 \cdot \text{top})$$

$$S = 2 \cdot 12 + 2 \cdot 6 + 2 \cdot 8$$

$$S = 24 + 12 + 16$$

$$S = 52 \text{ sq. units}$$

The surface area S of the rectangular solid shown in (Figure 7.2.3) is 52 square units.

In general, to find the surface area of a rectangular solid, remember that each face is a rectangle, so its area is the product of its length and width (see Figure 7.2.4). Find the area of each face you see and then multiply each area by two to account for the face on the opposite side.

$$S = 2LH + 2LW + 2WH$$

For each face of the rectangular solid facing you, there is another face on the opposite side. There are 6 faces in all.

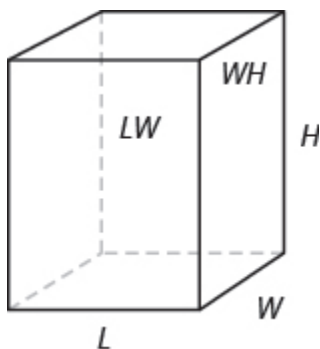


Figure 7.2.4

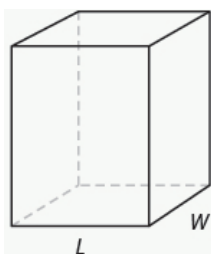
Volume and surface area of a rectangular solid:

For a rectangular solid with length L , width W , and height H :

- **Volume:** $V = LWH$
- **Surface Area:**

$$S = 2LH + 2LW + 2WH$$

(see Figure 7.2.5)



$$\text{Volume: } V = LWH$$

$$\text{Surface Area: } S = 2LH + 2LW + 2WH$$

Figure 7.2.5

Example A

For a rectangular solid with a length of 14 cm, a height of 17 cm, and a width of 9 cm, find the a) volume and b) surface area.

Step 1 is the same for both a) and b), so we will show it just once.

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.2.6).

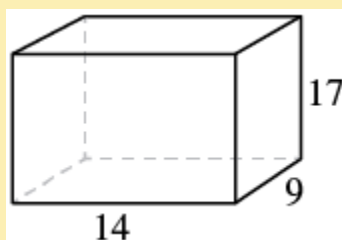


Figure 7.2.6

a. Volume

Step 2: Identify what you are looking for.

The volume of the rectangular solid.

Step 3: Name. Choose a variable to represent it.

Let V = volume

Step 4: Translate. Write the appropriate formula.
Substitute.

$$V = LWH$$

$$V = 14 \cdot 9 \cdot 17$$

Step 5: Solve the equation.

$$V = 2\,142$$

Step 6: Check. We leave it to you to check your calculations.

Step 7: Answer the question.

The volume is 2 142 cubic centimetres.

b. Surface Area

Step 2: Identify what you are looking for.

The surface area of the solid.

Step 3: Name. Choose a variable to represent it.

Let S = surface area

Step 4: Translate. Write the appropriate formula.
Substitute.

$$S = 2LH + 2LW + 2WH$$

$$S = 2(14 \cdot 17) + 2(14 \cdot 9) + 2(9 \cdot 17)$$

Step 5: Solve the equation.

$$S = 1\,034$$

Step 6: Check. Double-check with a calculator.

Step 7: Answer the question.

The surface area is 1 034 square centimetres.

Exercise 1

Find the a) volume and b) surface area of a rectangular solid with a length of 8 feet, a width of 9 feet, and a height of 11 feet.

Exercise 1 Answers

a. 792 cu. ft

b. 518 sq. ft

Exercise 2

Find the a) volume and b) surface area of a rectangular solid with a length of 15 feet, a width of 12 feet, and a height of 8 feet.

Exercise 2 Answers

a. 1440 cu. ft

b. 792 sq. ft

Example B

A rectangular crate has a length of 30 inches, a width of 25 inches, and a height of 20 inches. Find its a) volume and b) surface area.

Step 1 is the same for both a) and b), so we will show it just once.

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.2.7).

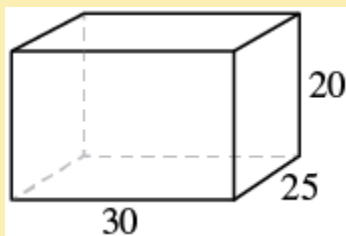


Figure 7.2.7

a. Volume

Step 2: Identify what you are looking for.

The volume of the crate.

Step 3: Name. Choose a variable to represent it.

Let V = volume

Step 4: Translate. Write the appropriate formula.
Substitute.

$$V = LWH$$
$$V = 30 \cdot 25 \cdot 20$$

Step 5: Solve the equation.

$$V = 15\,000$$

Step 6: Check. Double-check your math.

Step 7: Answer the question.

The volume is 15 000 cubic inches.

b. Surface Area

Step 2: Identify what you are looking for.

The surface area of the crate.

Step 3: Name. Choose a variable to represent it.

Let S = surface area

Step 4: Translate. Write the appropriate formula.
Substitute.

$$S = 2LH + 2LW + 2WH$$
$$S = 2(30 \cdot 20) + 2(30 \cdot 25) + 2(25 \cdot 20)$$

Step 5: Solve the equation.

$$S = 3\,700$$

Step 6: Check. Check it yourself!

Step 7: Answer the question.

The surface area is 3 700 square inches.

Exercise 3

A rectangular box has a length of 9 feet, a width of 4 feet, and a height of 6 feet. Find its a) volume and b) surface area.

Exercise 3 Answers

a. 216 cu. ft

b. 228 sq. ft

Exercise 4

A rectangular suitcase has a length of 22 inches, a width of 14 inches, and a height of 9 inches. Find its a) volume and b) surface area.

Exercise 4 Answers

a. 2 772 cu. in.

b. 1 264 sq. in.

Volume & Surface Area of Cubes

A cube is a rectangular solid whose length, width, and height are equal. The formulas for the volume and surface area of a cube are given below. Substituting s for the length, width and height into the formulas for volume and surface area of a rectangular solid, we get:

$$V = LWH$$

$$S = 2LH + 2LW + 2WH$$

$$V = s \cdot s \cdot s$$

$$S = 2s \cdot s + 2s \cdot s + 2s \cdot s$$

$$V = s^3$$

$$S = 2s^2 + 2s^2 + 2s^2$$

$$S = 6s^2$$

So for a cube, the formulas for volume and surface area are $V = s^3$ and $S = 6s^2$.

Volume and surface area of a cube:

For any cube with sides of length s :

- **Volume:** $V = s^3$

- **Surface Area:** $S = 6s^2$

(see Figure 7.2.8)

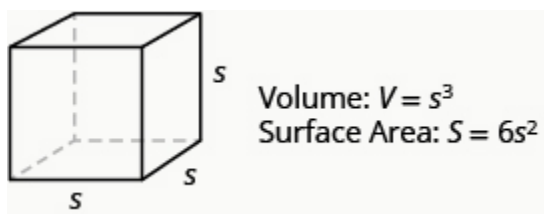


Figure 7.2.8

Example C

A cube is 2.5 inches on each side. Find its a) volume and b) surface area.

Step 1 is the same for both a) and b), so we will show it just once.

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.2.9).

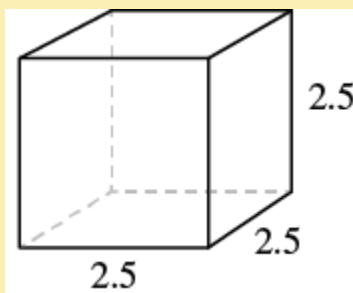


Figure 7.2.9

a. Volume

Step 2: Identify what you are looking for.

The volume of the cube.

Step 3: Name. Choose a variable to represent it.

Let V = volume

Step 4: Translate. Write the appropriate formula.
Substitute.

$$V = s^3$$

$$V = (2.5)^3$$

Step 5: Solve the equation.

$$V = 15.625$$

Step 6: Check. Check your work.

Step 7: Answer the question.

The volume is 15.625 cubic inches.

b. Surface Area

Step 2: Identify what you are looking for.

The surface area of the cube.

Step 3: Name. Choose a variable to represent it.

Let S = surface area

Step 4: Translate. Write the appropriate formula.
Substitute.

$$S = 6s^2$$

$$S = 6 \cdot (2.5)^2$$

Step 5: Solve the equation.

$$S = 37.5$$

Step 6: Check. The check is left to you.

Step 7: Answer the question.

The surface area is 37.5 square inches.

Exercise 5

For a cube with sides of 4.5 metres, find the a) volume and b) surface area of the cube.

Exercise 5 Answers

a. 91.125 cu. m

b. 121.5 sq. m

Exercise 6

For a cube with sides of 7.3 yards, find the a) volume and b) surface area of the cube.

Exercise 6 Answers

a. 389.017 cu. yd.

b. 319.74 sq. yd.

Example D

A notepad cube measures 2 inches on each side. Find its
a) volume and b) surface area.

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.2.10).

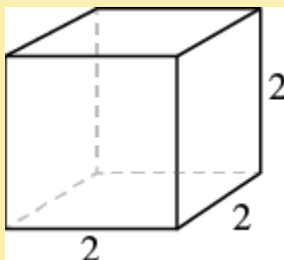


Figure 7.2.10

a. Volume

Step 2: Identify what you are looking for.

The volume of the cube.

Step 3: Name. Choose a variable to represent it.

Let V = volume

Step 4: Translate. Write the appropriate formula.
Substitute.

$$V = s^3$$

$$V = (2)^3$$

Step 5: Solve the equation.

$$V = 8$$

Step 6: Check. Check that you did the calculations correctly.

Step 7: Answer the question.

The volume is 8 cubic inches.

b. Surface Area

Step 2: Identify what you are looking for.

The surface area of the cube.

Step 3: Name. Choose a variable to represent it.

Let S = surface area

Step 4: Translate. Write the appropriate formula.
Substitute.

$$S = 6s^2$$

$$S = 6 \cdot (2)^2$$

Step 5: Solve the equation.

$$S = 24$$

Step 6: Check. The check is left to you.

Step 7: Answer the question.

The surface area is 24 square inches.

Exercise 7

A packing box is a cube measuring 4 feet on each side.
Find its a) volume and b) surface area.

Exercise 7 Answers

a. 64 cu. ft

b. 96 sq. ft

Exercise 8

A packing box is a cube measuring 4 feet on each side.
Find its a) volume and b) surface area.

Exercise 8 Answers

a. 64 cu. ft

b. 96 sq. ft

Find the Volume & Surface Area of Spheres

A sphere is the shape of a basketball, like a three-dimensional circle. Just like a circle, the size of a sphere is determined by its radius, which is the distance from the centre of the sphere to any point on its surface. The formulas for the volume and surface area of a sphere are given below.

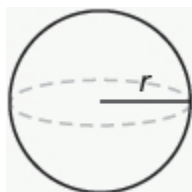
Showing where these formulas come from, like we did for a rectangular solid, is beyond the scope of this course. We will approximate π with 3.14.

Volume and surface area of a sphere:

For a sphere with radius r :

- **Volume:** $V = \frac{4}{3}\pi r^3$
- **Surface Area:** $S = 4\pi r^2$

(see Figure 7.2.11)



$$\text{Volume: } V = \frac{4}{3}\pi r^3$$
$$\text{Surface Area: } S = 4\pi r^2$$

Figure 7.2.11

Example E

A sphere has a radius 6 inches. Find its a) volume and b) surface area.

Step 1 is the same for both a) and b), so we will show it just once.

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.2.12).

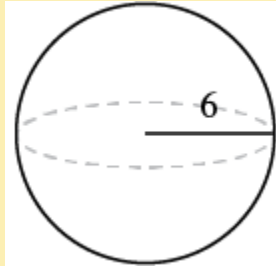


Figure 7.2.12

a. Volume

Step 2: Identify what you are looking for.

The volume of the sphere.

Step 3: Name. Choose a variable to represent it.

Let V = volume

Step 4: Translate. Write the appropriate formula.

Substitute. (Use 3.14 for π)

$$V = \frac{4}{3}\pi r^3$$

$$V \approx \frac{4}{3}(3.14)6^3$$

Step 5: Solve the equation.

$$V \approx 904.32$$

Step 6: Check. Double-check your math on a calculator.

Step 7: Answer the question.

The volume is approximately 904.32 cubic inches.

b. Surface Area

Step 2: Identify what you are looking for.

The surface area of the sphere.

Step 3: Name. Choose a variable to represent it.

Let S = surface area

Step 4: Translate. Write the appropriate formula.
Substitute. (Use 3.14 for π)

$$S = 4\pi r^2$$

$$S \approx 4(3.14)6^2$$

Step 5: Solve the equation.

$$S \approx 452.16$$

Step 6: Check. Double-check your math on a calculator.

Step 7: Answer the question.

The surface area is approximately 452.16 square inches.

Exercise 9

Find the a) volume and b) surface area of a sphere with a radius of 3 centimetres.

Exercise 9 Answers

a. 113.04 cu. cm

b. 113.04 sq. cm

Exercise 10

Find the a) volume and b) surface area of each sphere with a radius of 1 foot.

Exercise 10 Answers

a. 4.19 cu. ft

b. 12.56 sq. ft

Example F

A globe of Earth is in the shape of a sphere with a radius of 14 centimetres. Find its a) volume and b) surface area. Round the answer to the nearest hundredth.

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.2.13).



Figure 7.2.13

a. Volume

Step 2: Identify what you are looking for.

The volume of the sphere.

Step 3: Name. Choose a variable to represent it.

Let V = volume

Step 4: Translate. Write the appropriate formula.
Substitute. (Use 3.14 for π)

$$V = \frac{4}{3}\pi r^3$$

$$V \approx \frac{4}{3}(3.14)14^3$$

Step 5: Solve the equation.

$$V \approx 11\,488.21$$

Step 6: Check. We leave it to you to check your calculations.

Step 7: Answer the question.

The volume is approximately 11 488.21 cubic inches.

b. Surface Area

Step 2: Identify what you are looking for.

The surface area of the sphere.

Step 3: Name. Choose a variable to represent it.

Let S = surface area

Step 4: Translate. Write the appropriate formula.
Substitute. (Use 3.14 for π)

$$S = 4\pi r^2$$

$$S \approx 4(3.14)14^2$$

Step 5: Solve the equation.

$$S \approx 2\,461.76$$

Step 6: Check. We leave it to you to check your calculations.

Step 7: Answer the question.

The surface area is approximately 2 461.76 square inches.

Exercise 11

A beach ball is in the shape of a sphere with a radius of 9 inches. Find its a) volume and b) surface area.

Exercise 11 Answers

a. 3 052.08 cu. in.

b. 1 017.36 sq. in.

Exercise 12

A Roman statue depicts Atlas holding a globe with a

radius of 1.5 feet. Find the a) volume and b) surface area of the globe.

Exercise 12 Answers

a. 14.13 cu. ft

b. 28.26 sq. ft

Find the Volume & Surface Area of Cylinders

If you have ever seen a can of soda, you know what a cylinder looks like. A cylinder is a solid figure with two parallel circles of the same size at the top and bottom (see Figure 7.2.14). The top and bottom of a cylinder are called the bases. The height h of a cylinder is the distance between the two bases. For all the cylinders we will work with here, the sides and the height, h , will be perpendicular to the bases.

A cylinder has two circular bases of equal size. The height is the distance between the bases.

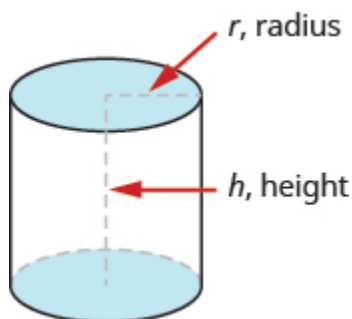


Figure 7.2.14

Rectangular solids and cylinders are somewhat similar because they both have two bases and a height. The formula for the volume of a rectangular solid, $V = Bh$, can also be used to find the volume of a cylinder.

For the rectangular solid, the area of the base, B , is the area of the rectangular base, length $\times$ width. For a cylinder, the area of the base, B , is the area of its circular base, πr^2 (see Figure 7.2.15) compares how the formula $V = Bh$ is used for rectangular solids and cylinders.

Seeing how a cylinder is similar to a rectangular solid may make it easier to understand the formula for the volume of a cylinder.

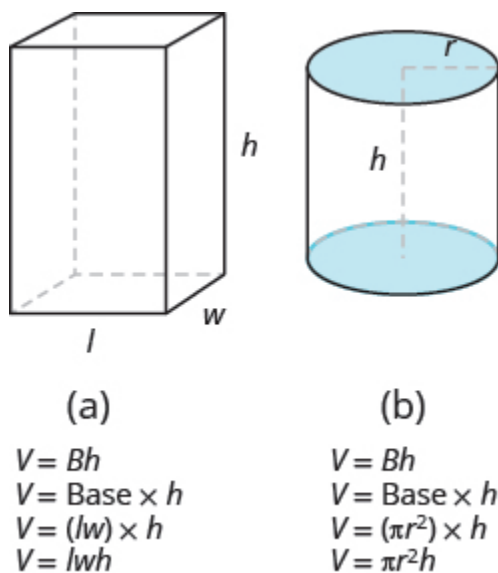


Figure 7.2.15

To understand the formula for the surface area of a cylinder, think of a can of vegetables. It has three surfaces: the top, the bottom, and the piece that forms the sides of the can. If you carefully cut the label off the side of the can and unroll it, you will see that it is a rectangle (see Figure 7.2.16).

By cutting and unrolling the label of a can of vegetables, we can see that the surface of a cylinder is a rectangle. The length of the rectangle is the circumference of the cylinder's base, and the width is the height of the cylinder.

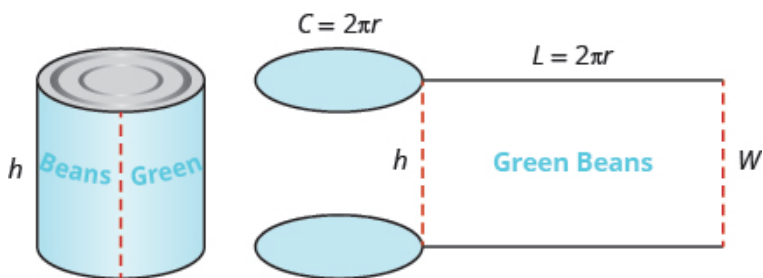


Figure 7.2.16

The distance around the edge of the can is the circumference of the cylinder's base. It is also the length L of the rectangular label. The height of the cylinder is the width W of the rectangular label. So, the area of the label can be represented as:

Formula does not parse

To find the total surface area of the cylinder, we add the areas of the two circles to the area of the rectangle (see Figure 7.2.17).



$$S = A_{\text{top circle}} + A_{\text{bottom circle}} + A_{\text{rectangle}}$$

$$S = \underbrace{\pi r^2 + \pi r^2}_{2 \cdot \pi r^2} + 2\pi r \cdot h$$

$$S = 2 \cdot \pi r^2 + 2\pi rh$$

$$S = 2\pi r^2 + 2\pi rh$$

Figure 7.2.17

$$S = A_{\text{top circle}} + A_{\text{bottom circle}} + A_{\text{rectangle}}$$

$$S = \pi r^2 + 2\pi r \cdot h$$

$$S = 2 \cdot \pi r^2 + 2\pi rh$$

$$S = 2\pi r^2 + 2\pi rh$$

The surface area of a cylinder with radius r and height h is

$$S = 2\pi r^2 + 2\pi rh$$

Volume and surface area of a cylinder:

For a cylinder with radius r and height h :

- **Volume:** $V = \pi r^2 h$ or $V = Bh$
- **Surface Area:** $S = 2\pi r^2 + 2\pi rh$

(see Figure 7.2.18)

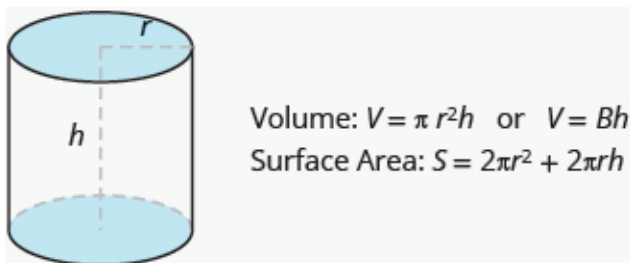


Figure 7.2.18

Example G

A cylinder has a height of 5 centimetres and a radius of 3 centimetres. Find the a) volume and b) surface area.

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.2.19).

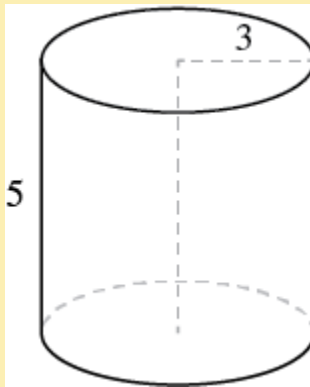


Figure 7.2.19

a. Volume

Step 2: Identify what you are looking for.

The volume of the cylinder.

Step 3: Name. Choose a variable to represent it.

Let V = volume

Step 4: Translate. Write the appropriate formula.
Substitute. (Use 3.14 for π)

$$V = \pi r^2 h$$
$$V \approx (3.14)3^2 \cdot 5$$

Step 5: Solve the equation.

$$V \approx 141.3$$

Step 6: Check. We leave it to you to check your calculations.

Step 7: Answer the question.

The volume is approximately 141.3 cubic inches.

b. Surface Area

Step 2: Identify what you are looking for.

The surface area of the cylinder.

Step 3: Name. Choose a variable to represent it.

Let S = surface area

Step 4: Translate. Write the appropriate formula.
Substitute. (Use 3.14 for π)

$$S = 2\pi r^2 + 2\pi r h$$
$$S \approx 2(3.14)3^2 + 2(3.14)(3)5$$

Step 5: Solve the equation.

$$S \approx 150.72$$

Step 6: Check. We leave it to you to check your calculations.

Step 7: Answer the question.

The surface area is approximately 150.72 square inches.

Exercise 13

Find the a) volume and b) surface area of a cylinder with a radius of 4 cm and a height of 7 cm.

Exercise 13 Answers

a. 351.68 cu. cm

b. 276.32 sq. cm

Exercise 14

Find the a) volume and b) surface area of a cylinder with a radius of 2 ft and a height of 8 ft.

Exercise 14 Answers

a. 100.48 cu. ft

b. 125.6 sq. ft

Example H

Find the a) volume and b) surface area of a can of soda. The radius of the base is 4 centimetres, and the height is 13 centimetres. Assume the can is shaped exactly like a cylinder.

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.2.20).



Figure 7.2.20

a. Volume

Step 2: Identify what you are looking for.

The volume of the cylinder.

Step 3: Name. Choose a variable to represent it.

Let V = volume

Step 4: Translate. Write the appropriate formula.

Substitute. (Use 3.14 for π)

$$V = \pi r^2 h$$

$$V \approx (3.14)4^2 \cdot 13$$

Step 5: Solve the equation.

$$V \approx 653.12$$

Step 6: Check. We leave it to you to check.

Step 7: Answer the question.

The volume is approximately 653.12 cubic centimetres.

b. Surface Area

Step 2: Identify what you are looking for.

The surface area of the cylinder.

Step 3: Name. Choose a variable to represent it.

Let S = surface area

Step 4: Translate. Write the appropriate formula.

Substitute. (Use 3.14 for π)

$$S = 2\pi r^2 + 2\pi rh$$

$$S \approx 2(3.14)4^2 + 2(3.14)(4)13$$

Step 5: Solve the equation.

$$S \approx 427.04$$

Step 6: Check. We leave it to you to check your calculations.

Step 7: Answer the question.

The surface area is approximately 427.04 square centimetres.

Exercise 15

Find the a) volume and b) surface area of a can of paint

with a radius of 8 centimetres and a height of 19 centimetres. Assume the can is shaped exactly like a cylinder.

Exercise 15 Answers

a. 3 818.24 cu. cm

b. 1 356.48 sq. cm

Exercise 16

Find the a) volume and b) surface area of a cylindrical drum with a radius of 2.7 feet and a height of 4 feet. Assume the drum is shaped exactly like a cylinder.

Exercise 16 Answers

a. 91.5624 cu. ft

b. 113.6052 sq. ft

Find the Volume of Cones

The first image many of us have when we hear the word ‘cone’ is an ice cream cone. There are many other applications for cones (but most are not as tasty as ice cream cones). In this section, we will see how to find the volume of a cone.

In geometry, a cone is a solid figure with one circular base and a vertex. The height of a cone is the distance between its base and the vertex. The cones that we will look at in this section will always have a height perpendicular to the base (see Figure 7.2.21).

The height of a cone is the distance between its base and vertex.

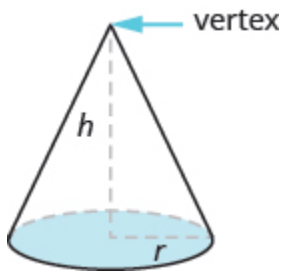


Figure 7.2.21

Earlier in this section, we saw that the volume of a cylinder is $V = \pi r^2 h$. We can think of a cone as part of a cylinder. Figure 7.2.22 shows a cone placed inside a cylinder with the same height and same base. If we compare the volume of the cone and the cylinder, we can see that the volume of the cone is less than that of the cylinder.

The volume of a cone is less than the volume of a cylinder with the same base and height.

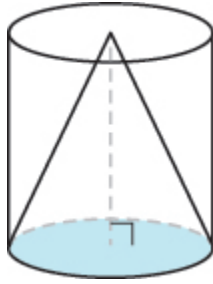


Figure 7.2.22

In fact, the volume of a cone is exactly one-third of the volume of a cylinder with the same base and height. The volume of a cone is

$$V = \frac{1}{3}Bh$$

Since the base of a cone is a circle, we can substitute the formula of the area of a circle, πr^2 , for B to get the formula for the volume of a cone.

$$V = \frac{1}{3}\pi r^2 h$$

In this book, we will only find the volume of a cone and not its surface area.

Volume of a cone:

For a cone with radius r and height h .

- **Volume:** $V = \frac{1}{3}\pi r^2 h$

(see Figure 7.2.23)

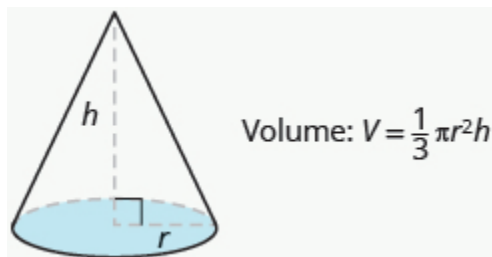


Figure 7.2.23

Example I

Find the volume of a cone with a height of 6 inches and a base with a radius of 2 inches.

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.2.24).

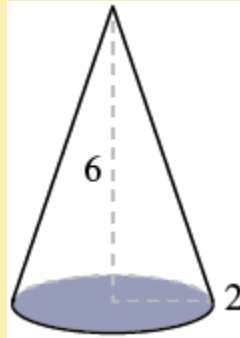


Figure 7.2.24

Step 2: Identify what you are looking for.

The volume of the cone.

Step 3: Name. Choose a variable to represent it.

Let V = volume

Step 4: Translate. Write the appropriate formula.

Substitute. (Use 3.14 for π)

$$V = \frac{1}{3}\pi r^2 h$$

$$V \approx \frac{1}{3}(3.14)(2)^2(6)$$

Step 5: Solve the equation.

$$V \approx 25.12$$

Step 6: Check. We leave it to you to check your calculations.

Step 7: Answer the question.

The volume is approximately 25.12 cubic inches.

Exercise 17

Find the volume of a cone with a height of 7 inches and a radius of 3 inches.

Exercise 17 Answer

65.94 cu. in.

Exercise 18

Find the volume of a cone with a height of 9 centimetres and a radius of 5 centimetres.

Exercise 18 Answer

235.5 cu. cm

Example J

Marty's favourite gastro pub serves french fries in a paper wrap shaped like a cone. What is the volume of a conic wrap that is 8 inches tall and 5 inches in diameter? Round the answer to the nearest hundredth.

Step 1: Read the problem. Draw the figure and label it with the given information. Notice here that the base is the circle at the top of the cone (see Figure 7.2.25).

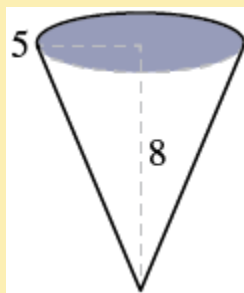


Figure 7.2.25

Step 2: Identify what you are looking for.

The volume of the cone.

Step 3: Name. Choose a variable to represent it.

Let V = volume

Step 4: Translate. Write the appropriate formula.

Substitute. (Use 3.14 for π)

Notice that we were given the distance across the circle, which is its diameter. The radius is 2.5 inches.

$$V = \frac{1}{3}\pi r^2 h$$

$$V \approx \frac{1}{3}(3.14)(2.5)^2(8)$$

Step 5: Solve the equation.

$$V \approx 52.33$$

Step 6: Check. We leave it to you to check your calculations.

Step 7: Answer the question.

The volume of the wrap is approximately 52.33 cubic inches.

Exercise 19

How many cubic inches of candy will fit in a cone-shaped piñata that is 18 inches long and 12 inches across its base? Round the answer to the nearest hundredth.

Exercise 19 Answer

678.24 cu. in.

Exercise 20

What is the volume of a cone-shaped party hat that is 10 inches tall and 7 inches across at the base? Round the answer to the nearest hundredth.

Exercise 20 Answer

128.2 cu. in.

Additional online resources:

- **Volume of a Cone** — Ex: Determine Volume of a Cone [3:44 min] (Mathispower4u, 2011).

Key Concepts

Volume & Surface Area of a Rectangular Solid

- $V = LWH$
- $S = 2LH + 2LW + 2WH$

Volume & Surface Area of a Cube

- $V = s^3$
- $S = 6s^2$

Volume & Surface Area of a Sphere

- $V = \frac{4}{3}\pi r^3$
- $S = 4\pi r^2$

Volume & Surface Area of a Cylinder

- $V = \pi r^2 h$
- $S = 2\pi r^2 + 2\pi r h$

Volume of a Cone

- For a cone with radius r and height h :

$$\text{Volume: } V = \frac{1}{3}\pi r^2 h$$

Glossary

- **Cone** — A cone is a solid figure with one circular base and a vertex.
- **Cube** — A cube is a rectangular solid whose length, width, and height are equal.
- **Cylinder** — A cylinder is a solid figure with two parallel circles of the same size at the top and bottom.

7.2: Practice Questions

1. Find Volume & Surface Area of Rectangular Solids

In the following exercises, find a) the volume and b) the surface area of the rectangular solid with the given dimensions.

- Length 2 metres, width 1.5 metres, height 3 metres
- Length 5 feet, width 8 feet, height 2.5 feet
- Length 3.5 yards, width 2.1 yards, height 2.4 yards
- Length 8.8 centimetres, width 6.5 centimetres, height 4.2 centimetres

In the following exercises, solve.

- e. **Moving van** A rectangular moving van has length 16 feet, width 8 feet, and height 8 feet. Find its a) volume and b) surface area.
- f. **Gift box** A rectangular gift box has length 26 inches, width 16 inches, and height 4 inches. Find its a) volume and b) surface area.
- g. **Carton** A rectangular carton has length 21.3 cm, width 24.2 cm, and height 6.5 cm. Find its a) volume and b) surface area.
- h. **Shipping container** A rectangular shipping container has length 22.8 feet, width 8.5 feet, and height 8.2 feet. Find its a) volume and b) surface area.

In the following exercises, find a) the volume and b) the surface area of the cube with the given side length.

- i. 5 centimetres
- j. 6 inches
- k. 10.4 feet
- l. 12.5 metres

In the following exercises, solve.

- m. **Science center:** Each side of the cube at the Discovery Science Center in Santa Ana is 64 feet long. Find its a) volume and b) surface area.
- n. **Museum:** A cube-shaped museum has sides 45 metres long. Find its a) volume and b) surface area.
- o. **Base of a statue:** The base of a statue is a cube with sides 2.8 metres long. Find its a) volume and b) surface area.
- p. **Tissue box:** A box of tissues is a cube with sides 4.5 inches long. Find its a) volume and b) surface area.

2. Find the Volume & Surface Area of Spheres

In the following exercises, find a) the volume and b) the surface area

of the sphere with the given radius. Round answers to the nearest hundredth.

- a. 3 centimetres
- b. 9 inches
- c. 7.5 feet
- d. 2.1 yards

In the following exercises, solve. Round answers to the nearest hundredth.

- e. **Exercise ball:** An exercise ball has a radius of 15 inches. Find its a) volume and b) surface area.
- f. **Balloon ride:** The Great Park Balloon is a big orange sphere with a radius of 36 feet. Find its a) volume and b) surface area.
- g. **Golf ball:** A golf ball has a radius of 4.5 centimetres. Find its a) volume and b) surface area.
- h. **Baseball:** A baseball has a radius of 2.9 inches. Find its a) volume and b) surface area.

3. Find the Volume & Surface Area of a Cylinder

In the following exercises, find a) the volume and b) the surface area of the cylinder with the given radius and height. Round answers to the nearest hundredth.

- a. Radius 3 feet, height 9 feet
- b. Radius 5 centimetres, height 15 centimetres
- c. Radius 1.5 metres, height 4.2 metres
- d. Radius 1.3 yards, height 2.8 yards

In the following exercises, solve. Round answers to the nearest hundredth.

- e. **Coffee can:** A can of coffee has a radius of 5 cm and a height of 13 cm. Find its a) volume and b) surface area.
- f. **Snack pack:** A snack pack of cookies is shaped like a cylinder

with a radius of 4 cm and a height of 3 cm. Find its a) volume and b) surface area.

- g. **Barber shop pole:** A cylindrical barber shop pole has a diameter of 6 inches and height of 24 inches. Find its a) volume and b) surface area.
- h. **Architecture:** A cylindrical column has a diameter of 8 feet and a height of 28 feet. Find its a) volume and b) surface area.

4. Find the Volume of Cones

In the following exercises, find the volume of the cone with the given dimensions. Round answers to the nearest hundredth.

- a. Height 9 feet and radius 2 feet
- b. Height 8 inches and radius 6 inches
- c. Height 12.4 centimetres and radius 5 cm
- d. Height 15.2 metres and radius 4 metres

In the following exercises, solve. Round answers to the nearest hundredth.

- e. **Teepee:** What is the volume of a cone-shaped teepee tent that is 10 feet tall and 10 feet across at the base?
- f. **Popcorn cup:** What is the volume of a cone-shaped popcorn cup that is 8 inches tall and 6 inches across at the base?
- g. **Silo:** What is the volume of a cone-shaped silo that is 50 feet tall and 70 feet across at the base?
- h. **Sand pile:** What is the volume of a cone-shaped pile of sand that is 12 metres tall and 30 metres across at the base?

5. Everyday Math

- a. **Street light post:** The post of a street light is shaped like a truncated cone, as shown below in Figure 7.2.26. It is a large cone minus a smaller top cone. The large cone is 30 feet tall with a base radius of 1 foot. The smaller cone is 10 feet tall with a base radius of 0.5 feet. Round to the nearest tenth.

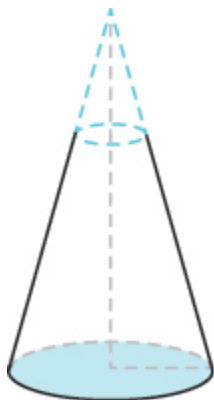


Figure 7.2.26

- Find the volume of the large cone.
 - Find the volume of the small cone.
 - Find the volume of the post by subtracting the volume of the small cone from the volume of the large cone.
- b. **Ice cream cones:** A regular ice cream cone is 4 inches tall and has a diameter of 2.5 inches. A waffle cone is 7 inches tall and has a diameter of 3.25 inches. Round to the nearest hundredth.
- Find the volume of the regular ice cream cone.
 - Find the volume of the waffle cone.
 - How much more ice cream fits in the waffle cone compared to the regular cone?

6. Writing Exercises

- The formulas for the volume of a cylinder and a cone are similar. Explain how you can remember which formula goes with which shape.
- Which has a larger volume, a cube of sides of 8 feet or a sphere with a diameter of 8 feet? Explain your reasoning.

7.2: Practice Answers

- Find volume and surface area of rectangular solids.

a. $V = 9 \text{ cu. m}$ $SA = 27 \text{ sq. m}$	g. $V = 3\,350.49 \text{ cu. cm}$ $SA = 1,622.42 \text{ sq. cm}$	m. $V = 262,144 \text{ cu. ft}$ $SA = 24,576 \text{ sq. ft}$
b.	h.	n.
c. $V = 17.64 \text{ cu. yd.}$ $SA = 41.58 \text{ sq. yd.}$	i. $V = 125 \text{ cu. cm}$ $SA = 150 \text{ sq. cm}$	o. $V = 21.952 \text{ cu. m}$ $SA = 47.04 \text{ sq. m}$
d.	j.	
e. $V = 1\,024 \text{ cu. ft}$ $SA = 640 \text{ sq. ft}$	k. $V = 1\,124.864 \text{ cu. ft}$ $SA = 648.96 \text{ sq. ft}$	
f.	l.	

- Find the volume and surface area of spheres.

a. $V = 113.04 \text{ cu. cm}$ $SA = 113.04 \text{ sq. cm}$	e. $V = 14\,130 \text{ cu. in.}$ $SA = 2\,826 \text{ sq. in.}$
b.	f.
c. $V = 1\,766.25 \text{ cu. ft}$ $SA = 706.5 \text{ sq. ft}$	g. $V = 381.51 \text{ cu. cm}$ $SA = 254.34 \text{ sq. cm}$
d.	

- Find the volume and surface area of a cylinder.

a. $V = 254.34 \text{ cu. ft}$ $SA = 226.08 \text{ sq. ft}$	e. $V = 1\,020.5 \text{ cu. cm}$ $SA = 565.2 \text{ sq. cm}$
b.	f.
c. $V = 29.673 \text{ cu. m}$ $SA = 53.694 \text{ sq. m}$	g. $V = 678.24 \text{ cu. in.}$ $SA = 508.68 \text{ sq. in.}$
d.	

4. Find the volume of cones.

a. 37.68 cu. ft	e. 261.67 cu. ft
b.	f.
c. 324.47 cu. cm	g. 64 108.33 cu. ft
d.	

5. Everyday math.

- a. Street light post
- i. 31.4 cu. ft
 - ii. 2.6 cu. ft
 - iii. 28.8 cu. ft

6. Writing exercises.

- a. Answers will vary.

References

Mathispower4u. (2011, September 2). Ex: Determine volume of a cone [Video]. YouTube. <https://youtu.be/7Y0ZMnCcVGs?si=W2NB6e3r2SSZZtkh>.

Attributions

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7.3: Solve Geometry Applications — Circles & Irregular Figures

In this section, we'll continue working with geometry applications. We will add several new formulas to our collection of formulas. To help you as you do the examples and exercises in this section, we will show the problem-solving strategy for geometry applications here.

Problem-solving strategy for geometry applications:

1. **Read** the problem and make sure you understand all the words and ideas. Draw the figure and label it with the given information.
2. **Identify** what you are looking for.
3. **Name** what you are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer to the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Use the Properties of Circles

We will refer to the properties of circles as we use them to solve applications.

Properties of circles:

- r is the length of the radius.
- d is the length of the diameter.
- $d = 2r$
- Circumference is the perimeter of a circle. The formula for circumference is $C = 2\pi r$
- The formula for the area of a circle is $A = \pi r^2$

(see Figure 7.3.1)

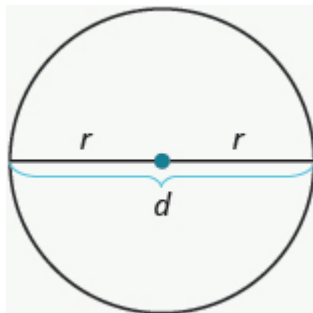


Figure 7.3.1

Remember: We approximate π with 3.14 or $\frac{22}{7}$ depending on whether the radius of the circle is given as a decimal or a fraction.

If you use the π key on your calculator to do the calculations in this section, your answers will be slightly different from the answers shown. That is because the π key uses more than two decimal places.

Example A

A circular sandbox has a radius of 2.5 feet. Find the a) circumference and b) area of the sandbox.

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.3.2).

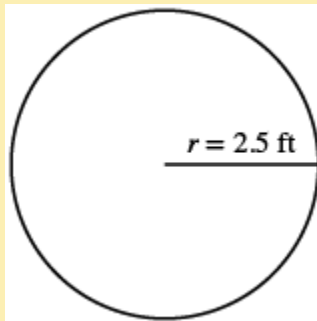


Figure 7.3.2

a. Circumference

Step 2: Identify what you are looking for.

The circumference of the circle.

Step 3: Name. Choose a variable to represent it.

Let c = circumference of the circle

Step 4: Translate. Write the appropriate formula.
Substitute. (Use 3.14 for π)

$$C = 2\pi r$$

$$C = 2(3.14)(2.5)$$

Step 5: Solve the equation.

$$C \approx 15$$

Step 6: Check. Does this answer make sense?

Yes. If we draw a square around the circle, its sides would be 5 ft (twice the radius), so its perimeter would be 20 ft (see Figure 7.3.3). This is slightly more than the circle's circumference, 15.7 ft.

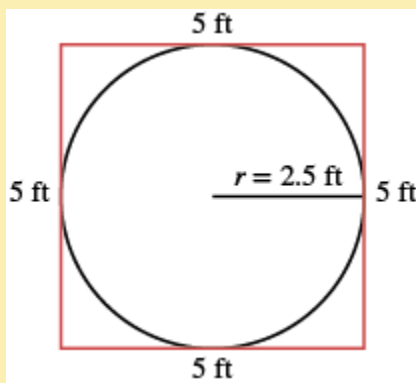


Figure 7.3.3

Step 7: Answer the question.

The circumference of the sandbox is 15.7 feet.

b. Area

Step 2: Identify what you are looking for.

The area of the circle.

Step 3: Name. Choose a variable to represent it.

Let A = the area of the circle

Step 4: Translate. Write the appropriate formula.

Substitute. (Use 3.14 for π)

$$A = \pi r^2$$

$$A \approx (3.14)2.5^2$$

Step 5: Solve the equation.

$$A \approx 19.625$$

Step 6: Check. Does this answer make sense?

Yes. If we draw a square around the circle, its sides would be 5 ft, as shown in part a). So the area of the square would be 25 sq. ft. This is slightly more than the circle's area, 19.625 sq. ft.

Step 7: Answer the question.

The area of the circle is 19.625 square feet.

Exercise 1

A circular mirror has a radius of 5 inches. Find the a) circumference and b) area of the mirror.

Exercise 1 Answers

a. 31.4 in.

b. 78.5 sq. in.

Exercise 2

A circular spa has a radius of 4.5 feet. Find the a) circumference and b) area of the spa.

Exercise 2 Answers

a. 28.26 ft

b. 63.585 sq. ft

We usually see the formula for circumference in terms of the radius r of the circle:

$$C = 2\pi r$$

However, since the diameter of a circle is two times the radius, we could write the formula for the circumference in terms of d .

$$C = 2\pi r$$

Using the commutative property, we get

$$C = \pi \cdot 2r$$

Then substituting $d = 2r$

$$C = \pi \cdot d$$

So

$$C = \pi d$$

We will use this form of the circumference when we're given the length of the diameter instead of the radius.

Example B

A circular table has a diameter of four feet. What is the circumference of the table?

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.3.4).

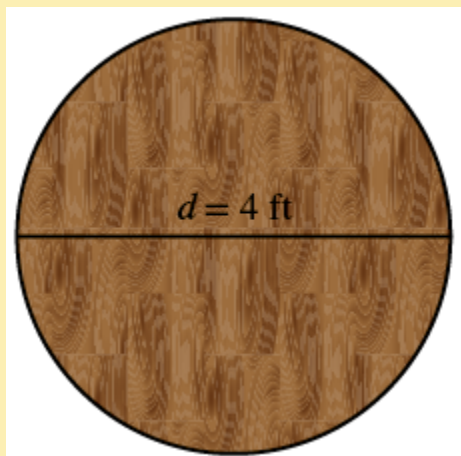


Figure 7.3.4

Step 2: Identify what you are looking for.

The circumference of the table.

Step 3: Name. Choose a variable to represent it.

Let c = the circumference of the table

Step 4: Translate. Write the appropriate formula.
Substitute. (Use 3.14 for π)

$$C = \pi d$$

$$C \approx (3.14)(4)$$

Step 5: Solve the equation.

$$C \approx 12.56$$

Step 6: Check.

If we put a square around the circle, its side would be 4 (see Figure 7.3.5).

The perimeter would be 16. It makes sense that the circumference of the circle, 12.56, is a little less than 16.

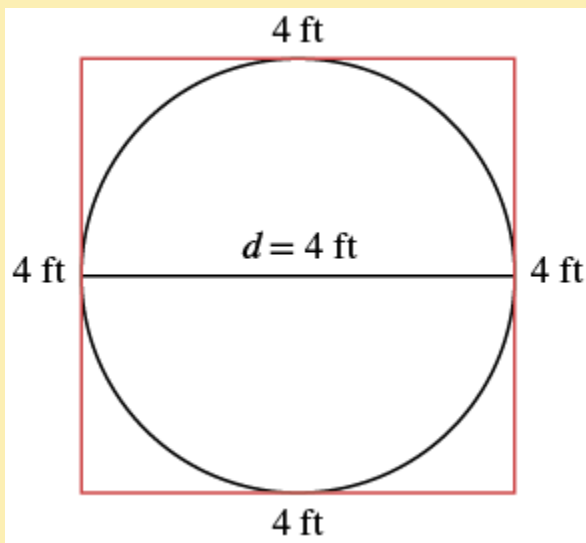


Figure 7.3.5

Step 7: Answer the question.

The diameter of the table is 12.56 square feet.

Exercise 3

Find the circumference of a circular fire pit whose diameter is 5.5 feet.

Exercise 3 Answer

17.27 ft

Exercise 4

If the diameter of a circular trampoline is 12 feet, what is its circumference?

Exercise 4 Answer

37.68 ft

Example C

Find the diameter of a circle with a circumference of 47.1 centimetres.

Step 1: Read the problem. Draw the figure and label it with the given information (see Figure 7.3.6).

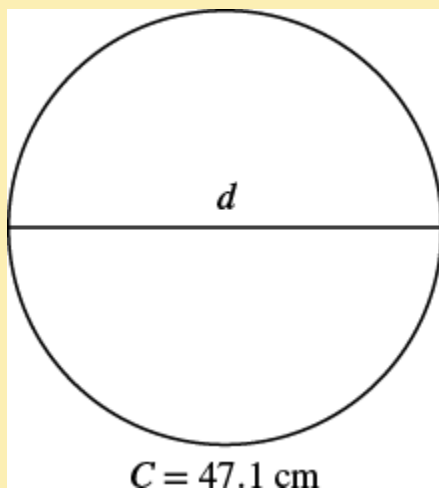


Figure 7.3.6

Step 2: Identify what you are looking for.

The diameter of the circle.

Step 3: Name. Choose a variable to represent it.

Let d = the diameter of the circle

Step 4: Translate. Write the appropriate formula.
Substitute. (Use 3.14 for π)

$$C = \pi d$$

$$47.1 \approx (3.14)d$$

Step 5: Solve.

$$\frac{47.1}{3.14} \approx \frac{3.14d}{3.14}$$

$$15 \approx d$$

Step 6: Check.

$$C \approx \pi d$$
$$47.1 = (3.14)(15)?$$
$$47.1 = 47.1 \checkmark$$

Step 7: Answer the question.

The diameter of the circle is approximately 15 centimetres.

Exercise 5

Find the diameter of a circle with a circumference of 94.2 centimetres.

Exercise 5 Answer

30 cm

Exercise 6

Find the diameter of a circle with a circumference of 345.4 feet.

Exercise 6 Answer

110 ft

Find the Area of Irregular Figures

So far, we have found the areas for rectangles, triangles, trapezoids, and circles. An irregular figure is a figure that is not a standard geometric shape. Its area cannot be calculated using any of the standard area formulas.

However, some irregular figures are made up of two or more standard geometric shapes. To find the area of one of these irregular figures, we can split it into figures whose formulas we know and then add the areas of the figures.

Example D

Find the area of the shaded region in Figure 7.3.7.

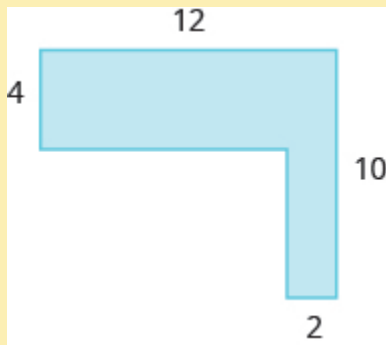


Figure 7.3.7

The given figure is irregular, but we can break it into two rectangles. The area of the shaded region will be the sum of the areas of both rectangles (see Figure 7.3.8).

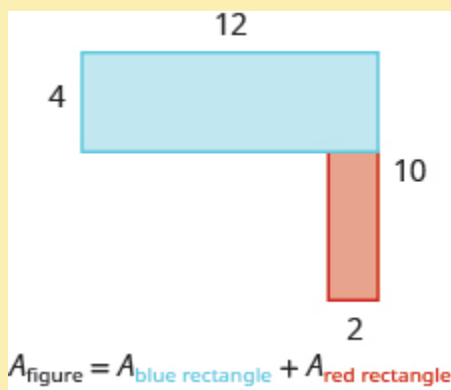


Figure 7.3.8

The blue rectangle has a width of 12 and a length of 4. The red rectangle has a width of 2, but its length is not labelled. The right side of the figure is the length of the red rectangle plus the length of the blue rectangle. Since the right side of the blue rectangle is 4 units long, the length of the red rectangle must be 6 units (see Figure 7.3.9).

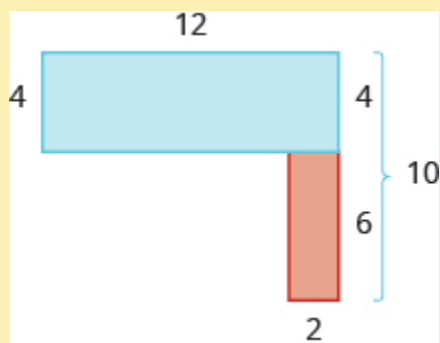


Figure 7.3.9

Formula does not parse

The area of the figure is 60 square units.

Is there another way to split this figure into two rectangles? Try it, and make sure you get the same area.

Exercise 7

Find the area of each shaded region in Figure 7.3.10.

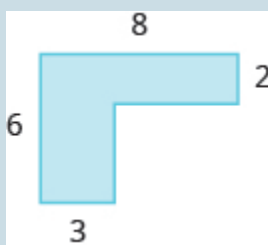


Figure 7.3.10

Exercise 7 Answer

28 sq. units

Exercise 8

Find the area of each shaded region in Figure 7.3.11.

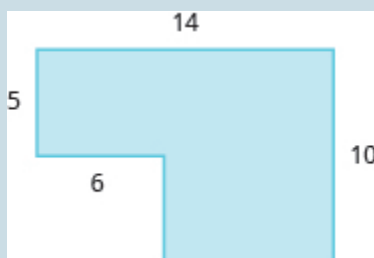


Figure 7.3.11

Exercise 8 Answer

110 sq. units

Example E

Find the area of the shaded region in Figure 7.3.12.

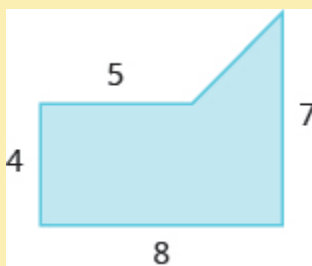


Figure 7.3.12

Step 1: We can break this irregular figure into a triangle and a rectangle. The area of the figure will be the sum of the areas of the triangle and rectangle.

The rectangle has a length of 8 units and a width of 4 units.

We need to find the base and height of the triangle.

Since both sides of the rectangle are 4, the vertical side of the triangle is 3, which is $7 - 4$.

The length of the rectangle is 8, so the base of the triangle will be 3, which is $8 - 4$ (see Figure 7.3.13).

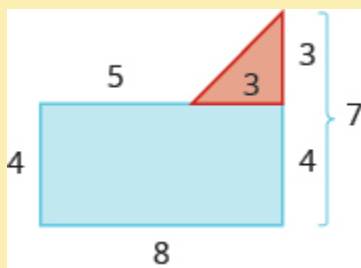


Figure 7.3.13

Step 2: Now, we can add the areas to find the area of the irregular figure.

Formula does not parse

The area of the figure is 36.5 square units.

Exercise 9

Find the area of each shaded region in Figure 7.3.14.

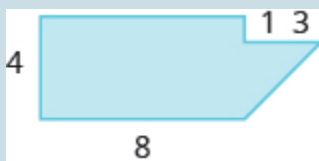


Figure 7.3.14

Exercise 9 Answer

36.5 sq. units

Exercise 10

Find the area of each shaded region in Figure 7.3.15.

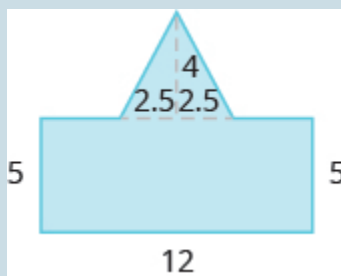


Figure 7.3.15

Exercise 10 Answer

70 sq. units

Example F

A high school track is shaped like a rectangle with a semi-circle (half a circle) on each end (see Figure 7.3.16). The rectangle has a length of 105 metres and a width of 68 metres. Find the area enclosed by the track. Round your answer to the nearest hundredth.



Figure 7.3.16

We will break the figure into a rectangle and two semi-circles. The area of the figure will be the sum of the areas of the rectangle and the semi-circles.

The rectangle has a length of 105 m and a width of 68 m. The semi-circles have a diameter of 68 m, so each has a radius of 34 m (see Figure 7.3.17).

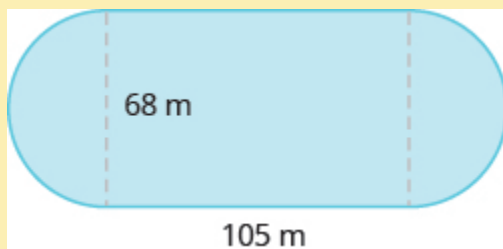


Figure 7.3.17

Formula does not parse

The area of the track is $10\,769.84\text{ m}^2$.

Exercise 11

Find the area of Figure 7.3.18.

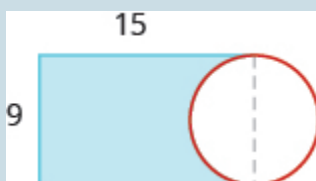


Figure 7.3.18

Exercise 11 Answer

103.2 sq. units

Exercise 12

Find the area of Figure 7.3.19.

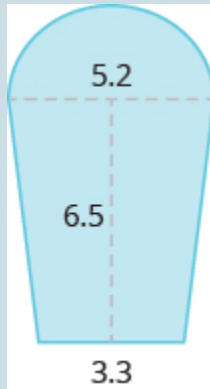


Figure 7.3.19

Exercise 12 Answer

38.24 sq. units

Additional online resources:

- **Circumference of a Circle** — Examples:
Determine the Circumference of a Circle [2:59 min] (Mathispower4u, 2011e).
- **Area of a Circle** — Example: Determine the Area of a Circle [2:57 min] (Mathispower4u, 2011d).
- **Area of an L-shaped Polygon** — Ex: Find the Area of an L-Shaped Polygon Involving Whole Numbers [3:19 min] (Mathispower4u, 2011f).
- **Area of an L-shaped Polygon with Decimals** — Example: Determine the Area of an L-Shaped Polygon Using Decimals [5:01 min] (Mathispower4u, 2011a).
- **Perimeter Involving a Rectangle and Circle** — Example: Determine Perimeter Involving a Rectangle and Circle [3:05 min] (Mathispower4u, 2011c).
- **Area Involving a Rectangle and Circle** — Example: Determine an Area Involving a Rectangle and Circle [3:10 min] (Mathispower4u, 2011b).

Key Concepts

Problem Solving Strategy for Geometry Applications

1. **Read** the problem and make sure you understand all the words and ideas. Draw the figure and label it with the given information.
2. **Identify** what you are looking for.
3. **Name** what you are looking for. Choose a variable to represent that quantity.
4. **Translate** into an equation by writing the appropriate formula or model for the situation. Substitute in the given information.
5. **Solve** the equation using good algebra techniques.
6. **Check** the answer to the problem and make sure it makes sense.
7. **Answer** the question with a complete sentence.

Properties of Circles

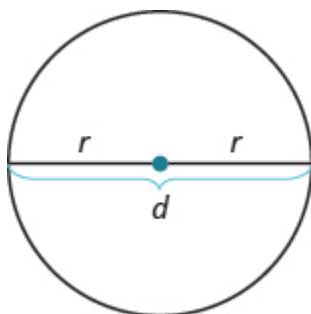


Figure 7.3.20

- $d = 2r$
- **Circumference:** $C = 2\pi r$ or $C = \pi d$
- **Area:** $A = \pi r^2$
- (see Figure 7.3.20)

Glossary

- **Irregular figure** — An irregular figure is a figure that is not a standard geometric shape. Its area cannot be calculated using any of the standard area formulas.

7.3: Practice Questions

1. Use the Properties of Circles

In the following exercises, solve using the properties of circles.

- The lid of a paint bucket is a circle with a radius of 7 inches. Find the a) circumference and b) area of the lid.
- An extra-large pizza is a circle with a radius of 8 inches. Find the a) circumference and b) area of the pizza.
- A farm sprinkler spreads water in a circle with a radius of 8.5 feet. Find the a) circumference and b) area of the watered circle.
- A circular rug has a radius of 3.5 feet. Find the a) circumference and b) area of the rug.
- A reflecting pool is in the shape of a circle with a diameter of 20 feet. What is the circumference of the pool?
- A turntable is a circle with a diameter of 10 inches. What is the

circumference of the turntable?

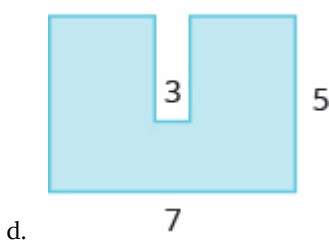
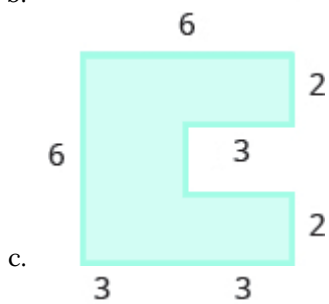
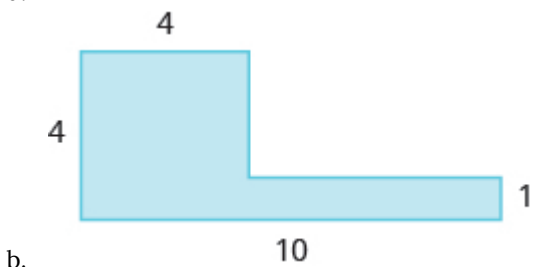
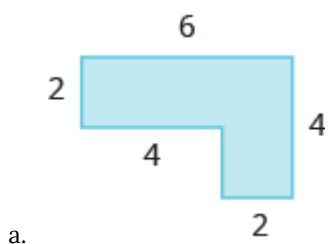
- g. A circular saw has a diameter of 12 inches. What is the circumference of the saw?
- h. A round coin has a diameter of 3 centimetres. What is the circumference of the coin?
- i. A barbecue grill is a circle with a diameter of 2.2 feet. What is the circumference of the grill?
- j. The top of a pie tin is a circle with a diameter of 9.5 inches. What is the circumference of the top?
- k. A circle has a circumference of 163.28 inches. Find the diameter.
- l. A circle has a circumference of 59.66 feet. Find the diameter.
- m. A circle has a circumference of 17.27 metres. Find the diameter.
- n. A circle has a circumference of 80.07 centimetres. Find the diameter.

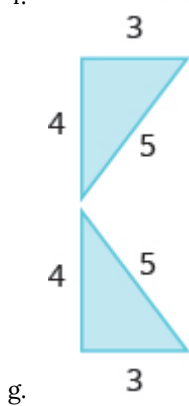
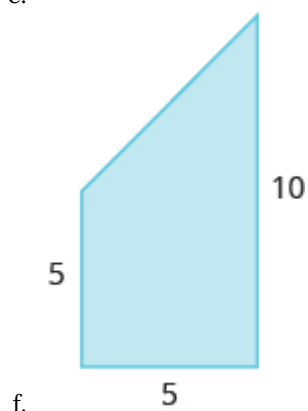
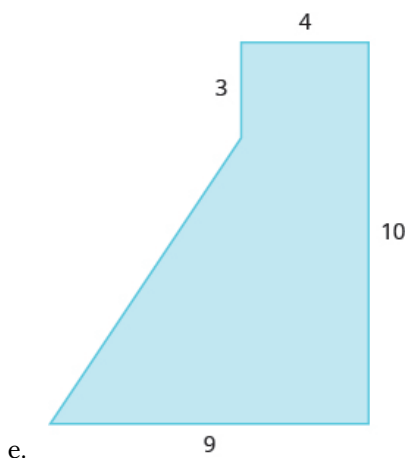
In the following exercises, find the radius of the circle with the given circumference.

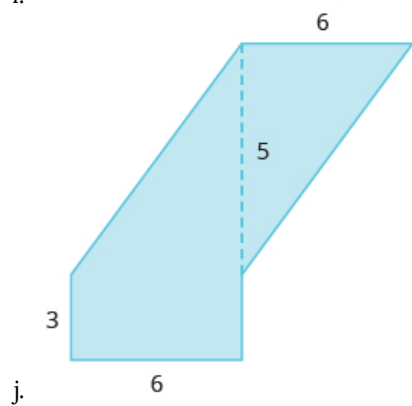
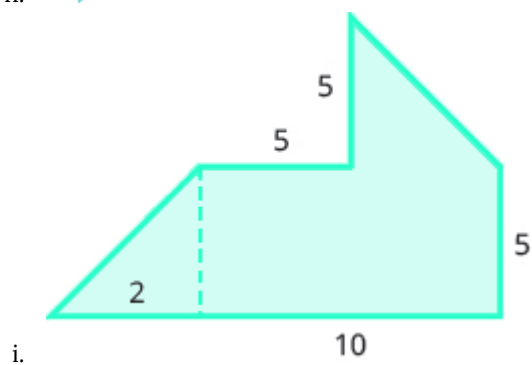
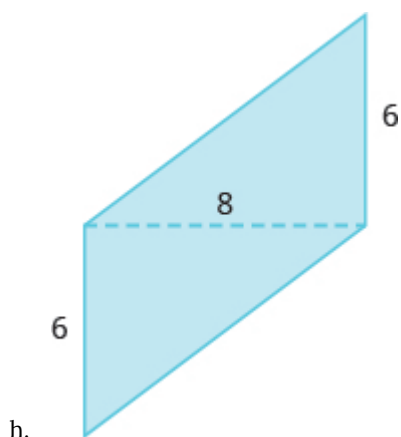
- o. A circle has a circumference of 150.72 feet.
- p. A circle has a circumference of 251.2 centimetres.
- q. A circle has a circumference of 40.82 miles.
- r. A circle has a circumference of 78.5 inches.

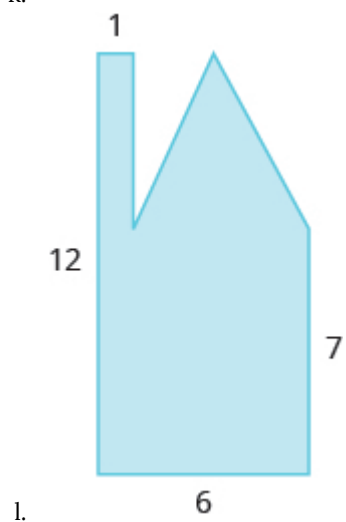
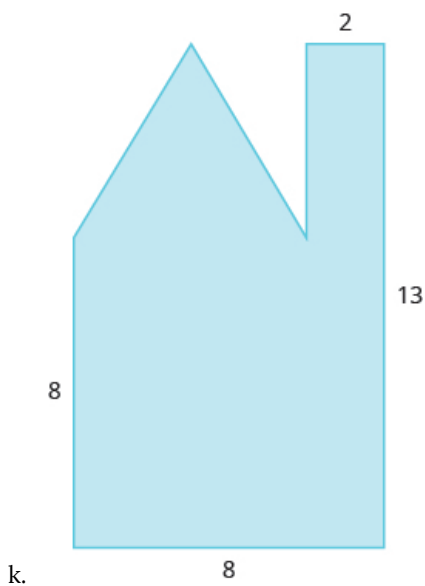
2. Find the Area of Irregular Figures

In the following exercises, find the area of the irregular figure. Round your answers to the nearest hundredth.







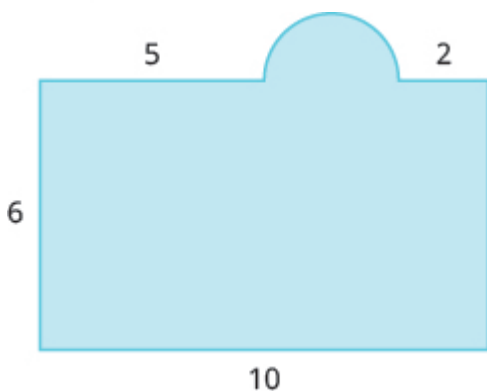




m.

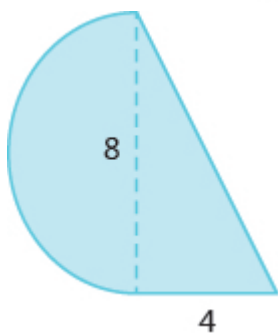
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7



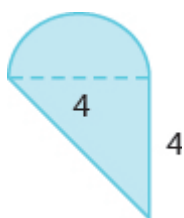
n.

10

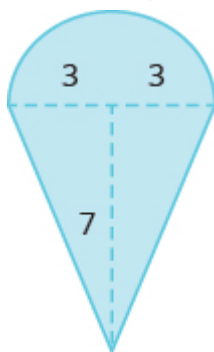


o.

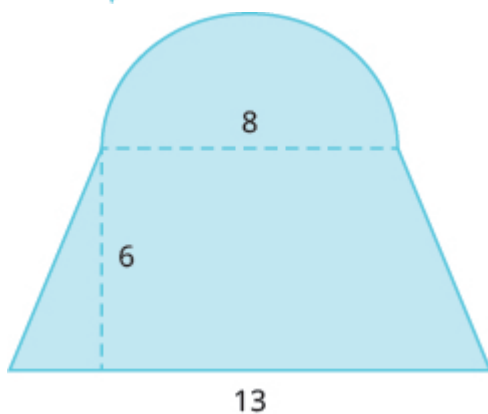
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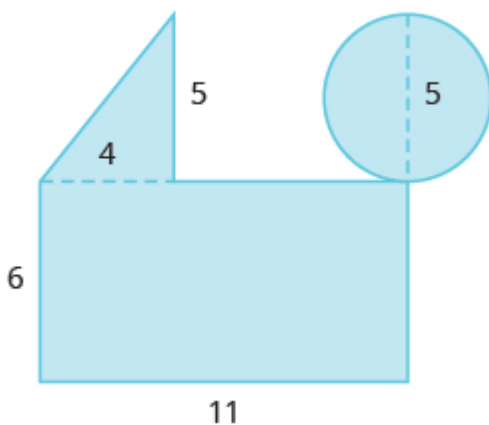
p.



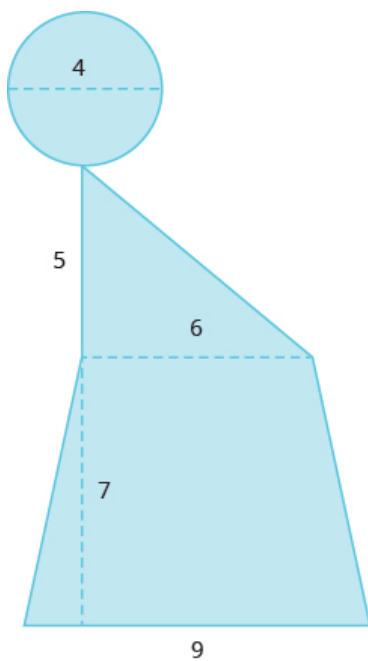
q.



r.



s.



t.

In the following exercises, solve.

- u. A city park covers one block plus parts of four more blocks, as shown in Figure 7.3.21. The block is a square with sides 250 feet long, and the triangles are isosceles right triangles. Find the

area of the park.



**Figure
7.3.21**

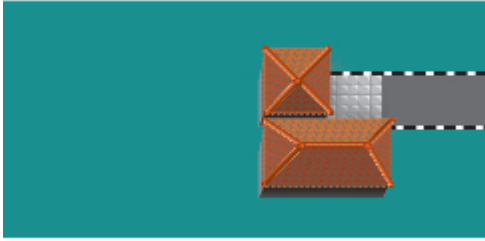
- v. A gift box will be made from a rectangular piece of cardboard measuring 12 inches by 20 inches, with squares cut out of the corners of the sides, as shown in Figure 7.3.22. The sides of the squares are 3 inches. Find the area of the cardboard after the corners are cut out.



**Figure
7.3.22**

- w. Perry needs to put in a new lawn. His lot is a rectangle with a length of 120 feet and a width of 100 feet. The house is rectangular and measures 50 feet by 40 feet. His driveway is rectangular and measures 20 feet by 30 feet, as shown in Figure 7.3.23. Find the area of Perry's lawn.

**Figure
7.3.23**



- x. Denise is planning to put a deck in her backyard. The deck will be a 20-ft by 12-ft rectangle with a semicircle of diameter 6 feet, as shown in Figure 7.3.24. Find the area of the deck.

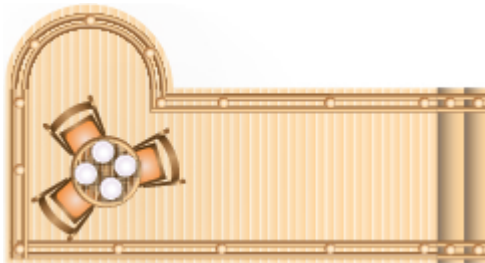


**Figure
7.3.24**

3. Everyday Math

- a. **Area of a Tabletop:** Yuki bought a drop-leaf kitchen table. The rectangular part of the table is a 1-ft by 3-ft rectangle with a semicircle at each end, as shown in Figure 7.3.25.

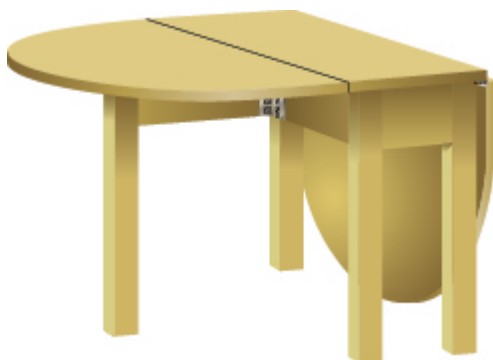


Figure
7.3.25

- i. Find the area of the table with one leaf up.
 - ii. Find the area of the table with both leaves up.
- b. **Painting:** Leora wants to paint the nursery in her house. The nursery is an 8-ft by 10-ft rectangle, and the ceiling is 8 feet tall. There is a 3-ft by 6.5-ft door on one wall, a 3-ft by 6.5-ft closet door on another wall, and one 4-ft by 3.5-ft window on the third wall. The fourth wall has no doors or windows. If she will only paint the four walls and not the ceiling or doors, how many square feet will she need to paint?

4. Writing Exercises

- a. Describe two different ways to find the area of Figure 7.3.26, and then show your work to make sure both ways give the same area.

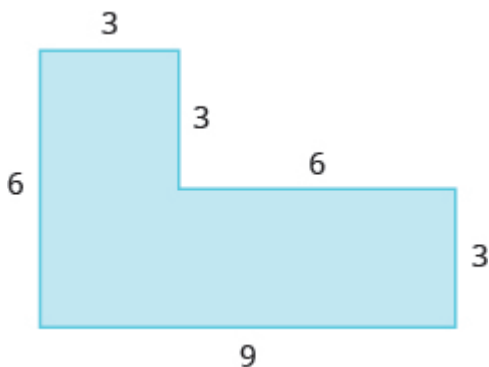


**Figure
7.3.26**

- b. A circle has a diameter of 14 feet. Find the area of the circle
- Using 3.14 for π .
 - Using $\frac{22}{7}$ for π .
 - Which calculation to do prefer? Why?

7.3: Practice Answers

1. Use the properties of circles

a. $C = 43.96$ in.	g. 37.68 in.	o. 24 ft
A = 153.86 sq. in.	h.	p.
b. $C = 53.38$ ft	i. 6.908 ft	q. 6.5 mi
A = 226.865 sq. ft	j.	
d. 62.8 ft	k. 52 in.	
e. 62.8 ft	l.	
f.	m. 5.5 m	
	n.	

2. Find the area of irregular figures

a. 16 sq. units	g. 12 sq. units	m. 44.81 sq. units
b.	h.	n.
c. 30 sq. units	i. 67.5 sq. units	o. 41.12 sq. units
d.	j.	p.
e. 57.5 sq. units	k. 89 sq. units	q. 35.13 sq. units
f.	l.	r.

s. 95.625 sq. units	u. 187,500 sq. ft	w. 9400 sq. ft
t.	v.	

3. Everyday math.
 - a. Area of a Tabletop.
 - i. 6.5325 sq. ft
 - ii. 10.065 sq. ft
4. Writing exercises
 - a. Answers will vary.

References

Mathispower4u. (2011a, June 17). *Example: Determine the area of an L-shaped polygon using decimals* [Video]. YouTube. <https://youtu.be/roRp3BjHwbU?si=gFXn80n90TVE2tM6>.

Mathispower4u. (2011b, June 21). *Example: Determine an area involving a rectangle and circle* [Video]. YouTube. https://youtu.be/085t_Fmje4o?si=p8ElBx_v3oqEPsrw.

Mathispower4u. (2011c, June 21). *Example: Determine perimeter involving a rectangle and circle* [Video]. YouTube. <https://youtu.be/fW2Ly8WFHzU?si=2codTCp-I45piJ3d>.

Mathispower4u. (2011d, June 21). *Example: Determine the area of a circle* [Video]. YouTube. <https://youtu.be/SIKkWLqt2mQ?si=1ggAEqaNrQbdZ1DI>.

Mathispower4u. (2011e, June 21). *Examples: Determine the circumference of a circle* [Video]. YouTube. <https://youtu.be/sHtsnC2Mgnk?si=L9jRfnkFQm2U7ClR>.

Mathispower4u. (2011f, August 5). *Ex: Find the area of an L-shaped polygon involving whole numbers* [Video]. YouTube. https://youtu.be/lx8vweYTPpg?si=PET4FPjfwI-p_xyZ.

Attributions

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CHAPTER 8: ALGEBRA

8.1: Use the Language of Algebra

Chapter 8: Learning Outcomes

By the end of this chapter, you should be able to:

- Explain the use of variables.
- Evaluate algebraic expressions using substitution.
- Combine like terms and remove parentheses.
- Solve first-degree equations in one variable.

Use Variables & Algebraic Symbols

Greg and Alex have the same birthday but were born in different years. This year, Greg is 20 years old, and Alex is 23, so Alex is 3 years older than Greg. When Greg was 12, Alex was 15. When Greg is 35, Alex will be 38. No matter what Greg's age is, Alex's age will always be 3 years more, right?

In the language of algebra, we say that Greg's age and Alex's age are variable, and the three is a constant. The ages change or vary, so age is a variable. The 3 years between them always stay the same, so the age difference is the constant.

In algebra, letters of the alphabet are used to represent variables.

Suppose we call Greg’s age g . Then, we could use $g + 3$ to represent Alex’s age (see Table 8.1.1).

Table 8.1.1: Greg’s Age vs. Alex’s Age

Greg’s Age	Alex’s Age
12	15
20	23
35	38
g	$g + 3$

Letters are used to represent variables. Letters often used for variables are x , y , a , b , and c .

Variables and constants:

A variable is a letter that represents a number or quantity whose value may change.

A constant is a number whose value always stays the same.

To write algebraically, we need some symbols as well as numbers and variables. There are several types of symbols we will be using. Table 8.1.2 summarizes them, along with the words we use for the operations and the result.

Table 8.1.2: Algebraic Symbols

Operation	Notation	Say	The Result Is
Addition	$a + b$	a plus b	The sum of a and b
Subtraction	$a - b$	a minus b	The difference of a and b
Multiplication	$a \cdot b, (a)(b), (a)b, a(b)$	a times b	The product of a and b
Division	$a \div b, a/b, \frac{a}{b}, b \overline{)a}$	a divided by b	The quotient of a and b

In algebra, the cross symbol, $\times$, is not used to show multiplication because that symbol may cause confusion. Does $3xy$ mean $3 \times y$ (three times y) or $3 \cdot x \cdot y$ (three times x times y)? To make it clear, use $\cdot$ or parentheses for multiplication.

We perform these operations on two numbers. When translating from symbolic form to words, or from words to symbolic form, pay attention to the words *of* or *and* to help you find the numbers.

- The *sum of 5 and 3* means add 5 plus 3, which we write as $5 + 3$.
- The *difference of 9 and 2* means subtract 9 minus 2, which we write as $9 - 2$.
- The *product of 4 and 8* means multiply 4 times 8, which we can write as $4 \cdot 8$.
- The *quotient of 20 and 5* means divide 20 by 5, which we can write as $20 \div 5$.

Example A

Translate from algebra to words:

a. $12 + 14$

b. $(30)(5)$

c. $64 \div 8$

d. $x - y$

a. $12 + 14$

12 plus 14.

The sum of twelve and fourteen.

b. $(30)(5)$

30 times 5.

The product of thirty and five.

c. $64 \div 8$

64 divided by 8.

The quotient of sixty-four and eight.

d. $x - y$

x minus y .
The difference of x and y .

Exercise 1

Translate from algebra to words.

a. $18 + 11$

c. $84 \div 7$

b. $(27)(9)$

d. $p - q$

Exercise 1 Answers

a. 18 plus 11
The sum of eighteen and eleven

b. 27 times 9
The product of twenty-seven and nine

c. 84 divided by 7;
The quotient of eighty-four and seven

d. p minus q
The difference of p and q

Exercise 2

Translate from algebra to words.

a. $47 - 19$

b. $72 \div 9$

c. $m + n$

d. $(13)(7)$

Exercise 2 Answers

a. 47 minus 19

The difference of forty-seven
and nineteen

b. 72 divided by 9

The quotient of seventy-two
and nine

c. m plus n

The sum of m and n

d. 13 times 7

The product of thirteen and
seven

When two quantities have the same value, we say they are equal and connect them with an **equal sign**.

Equality symbol:

$a = b$ is read as a is equal to b .

The symbol $=$ is called the equal sign.

An inequality is used in algebra to compare two quantities that may have different values. The number line can help you understand inequalities.

Remember: On the number line, the numbers get larger as they go from left to right.

So, if we know that b is greater than a , it means that b is to the right of a on the number line. We use the symbols $<$ and $>$ for inequalities.

Inequality:

$a < b$ is read a is less than b

a is to the left of b on the number line (see Figure 8.1.1).

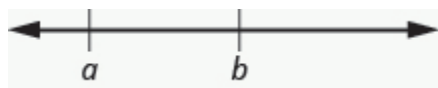


Figure 8.1.1

$a > b$ is read a is greater than b

a is to the right of b on the number line (see Figure 8.1.2).



Figure 8.1.2

The expressions $a < b$ and $a > b$ can be read from left-to-right or right-to-left, though, in English, we usually read from left-to-right. In general:

- $a < b$ is equivalent to $b > a$.
 - E.g. $7 < 11$ is equivalent to $11 > 7$.
- $a > b$ is equivalent to $b < a$.
 - E.g. $17 > 4$ is equivalent to $4 < 17$.

When we write an inequality symbol with a line under it, such as $a \leq b$, it means $a < b$ or $a = b$. We read this as a is less than or equal to b . Also, if we put a slash through an equal sign, $\neq$, it means not equal.

We summarize the symbols of equality and inequality below in Table 8.1.3.

Table 8.1.3: Symbols of Equality and Inequality

Algebraic Notation	Say
$a = b$	a is equal to b
$a \neq b$	a is not equal to b
$a < b$	a is less than b
$a > b$	a is greater than b
$a \leq b$	a is less than or equal to b
$a \geq b$	a is greater than or equal to b

Symbols $<$ and $>$:

The symbols $<$ and $>$ each have a smaller side and a larger side.

- smaller side $<$ larger side
- larger side $>$ smaller side

The smaller side of the symbol faces the smaller number, and the larger one faces the larger number.

Example B

Translate from algebra to words:

- a. $20 \leq 35$
- b. $11 \neq 15 - 3$
- c. $9 > 10 \div 2$
- d. $x + 2 < 10$

a. $20 \leq 35$

20 is less than or equal to 35.

b. $11 \neq 15 - 3$

11 is not equal to 15 minus 3.

c. $9 > 10 \div 2$

9 is greater than 10 divided by 2.

d. $x + 2 < 10$

x plus 2 is less than 10.

Exercise 3

Translate from algebra to words.

a. $14 \leq 27$

c. $12 > 4 \div 2$

b. $19 - 2 \neq 8$

d. $x - 7 < 1$

Exercise 3 Answers

a. Fourteen is less than or equal to twenty-seven.	c. Twelve is greater than four divided by two.
b. Nineteen minus two is not equal to eight.	d. x minus seven is less than one.

Exercise 4

Translate from algebra to words.

a. $19 \geq 15$

c. $15 \div 3 < 8$

b. $7 = 12 - 5$

d. $y - 3 > 6$

Exercise 4 Answers

- a. Nineteen is greater than or equal to fifteen. c. Fifteen divided by three is less than eight.
b. Seven is equal to twelve minus five. d. y minus three is greater than six.

Example C

Figure 8.1.3 compares the fuel economy in miles-per-gallon (mpg) of several cars. Write the appropriate symbol

=, < or >. in each expression to compare the fuel economy of the cars.

Car	Prius	Mini Cooper	Toyota Corolla	Versa	Honda Fit
Fuel economy (mpg)	48	27	28	26	27

Figure 8.1.3: (credit: modification of work by Bernard Goldbach, Wikimedia Commons)

- MPG of Prius_____ MPG of Mini Cooper
- MPG of Versa_____ MPG of Fit
- MPG of Mini Cooper_____ MPG of Fit
- MPG of Corolla_____ MPG of Versa
- MPG of Corolla_____ MPG of Prius

a. MPG of Prius_____ MPG of Mini Cooper

Step 1: Find the values in the chart.

48_____27

Step 2: Compare.

48_____27

MPG of Prius > MPG of Mini Cooper

b. MPG of Versa_____ MPG of Fit

Step 1: Find the values in the chart.

26_____27

Step 2: Compare.

$$26 < 27$$

MPG of Versa < MPG of Fit

c. MPG of Mini Cooper _____ MPG of Fit

Step 1: Find the values in the chart.

$$27 \text{ _____ } 27$$

Step 2: Compare.

$$27 = 27$$

MPG of Mini Cooper = MPG of Fit

d. MPG of Corolla _____ MPG of Versa

Step 1: Find the values in the chart.

$$28 \text{ _____ } 26$$

Step 2: Compare.

$$28 > 26$$

MPG of Corolla > MPG of Versa

e. MPG of Corolla _____ MPG of Prius

Step 1: Find the values in the chart.

$$28 \text{ _____ } 48$$

Step 2: Compare.

$$28 < 48$$

MPG of Corolla < MPG of Prius

Exercise 5

Use Figure 8.1.3 to fill in the appropriate $=$, $<$, or $>$.

- a. MPG of Prius _____ MPG of Versa
- b. MPG of Mini Cooper _____ MPG of Corolla

Exercise 5 Answers

a. $>$

b. $<$

Exercise 6

Use Figure 8.1.3 to fill in the appropriate $=$, $<$, or $>$.

- a. MPG of Fit _____ MPG of Prius
- b. MPG of Corolla _____ MPG of Fit

Exercise 6 Answers

a. $<$

b. $<$

Grouping symbols in algebra is much like the commas, colons, and other punctuation marks in written language. They indicate which expressions are to be kept together and separate from other expressions.

Table 8.1.4 below lists three of the most commonly used grouping symbols in algebra.

Table 8.1.4: Common Grouping Symbols

Name	Symbol
parentheses	()
brackets	[]
braces	{ }

Here are some examples of expressions that include grouping symbols. We will simplify expressions like these later in this section.

$$8(14 - 8) \qquad 21 - 3[2 + 4(9 - 8)] \qquad 24 \div \{13 - 2[1(6 - 5) + 4]\}$$

Identify Expressions & Equations

What is the difference in English between a phrase and a sentence? A phrase expresses a single thought that is incomplete by itself, but

a sentence makes a complete statement. “Running very fast” is a phrase, but “The football player was running very fast” is a sentence. A sentence has a subject and a verb.

In algebra, we have **expressions** and **equations**. An expression is like a phrase. Table 8.1.5 some examples of expressions and how they relate to word phrases.

Table 8.1.5: Expression Examples

Expression	Words	Phrase
$3 + 5$	3 plus 5	The sum of three and five
$n - 1$	n minus one	The difference of n and one
$6 \cdot 7$	6 times 7	The product of six and seven
$\frac{x}{y}$	x divided by y	The quotient of x and y

Notice that the phrases do not form a complete sentence because the phrase does not have a verb. An equation is two expressions linked with an equal sign. When you read the words the symbols represent in an equation, you have a complete sentence in English. The equal sign gives the verb. Table 8.1.6 shows some examples of equations.

Table 8.1.6: Equation Examples

Equation	Sentence
$3 + 5 = 8$	The sum of three and five is equal to eight.
$n - 1 = 14$	n minus one equals fourteen.
$6 \cdot 7 = 42$	The product of six and seven is equal to forty-two.
$x = 53$	x is equal to fifty-three.
$y + 9 = 2y - 3$	y plus nine is equal to two y minus three.

Expressions and equations:

An expression is a number, a variable, or a combination of numbers, variables, and operation symbols.

An equation is made up of two expressions connected by an equal sign.

Example D

Determine if each is an expression or an equation:

- a. $16 - 6 = 10$
- b. $4 \cdot 2 + 1$
- c. $x \div 25$
- d. $y + 8 = 40$

a. $16 - 6 = 10$

This is an equation — two expressions are connected with an equal sign.

b. $4 \cdot 2 + 1$

This is an expression — no equal sign.

c. $x \div 25$

This is an expression — no equal sign.

d. $y + 8 = 40$

This is an equation — two expressions are connected with an equal sign.

Exercise 7

For each one, determine if it is an expression or an equation.

a. $23 + 6 = 29$

b. $7 \cdot 3 - 7$

Exercise 7 Answers

a. equation

b. expression

Exercise 8

For each one, determine if it is an expression or an equation.

a. $y \div 14$

b. $x - 6 = 21$

Exercise 8 Answers

a. expression

b. equation

Attributions

All figures in this chapter are from 1.2 Use the Language of Algebra in Introductory Algebra by Izabela Mazur, via BCcampus.

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8.2: Evaluate, Simplify, & Translate Expressions

Evaluate Algebraic Expressions

In the last section, we simplified expressions using the order of operations. In this section, we'll evaluate expressions — again following the order of operations.

To evaluate an algebraic expression means to find the value of the expression when the variable is replaced by a given number. To evaluate an expression, we substitute the given number for the variable in the expression and then simplify the expression using the order of operations.

Example A

Evaluate $x + 7$ when:

- a. $x = 3$
- b. $x = 12$

a. $x = 3$

To evaluate, substitute 3 for x in the expression, and then simplify.

$$x + 7$$

$$\text{Substitute: } 3 + 7$$

$$\text{Add: } 10$$

When $x = 3$, the expression $x + 7$ has a value of 10.

b. $x = 12$

To evaluate, substitute 12 for x in the expression, and then simplify.

$$x + 7$$

$$\text{Substitute: } 12 + 7$$

$$\text{Add: } 19$$

When $x = 12$, the expression $x + 7$ has a value of 19.

Notice that we got different results for parts a) and b) even though we started with the same expression. This is because the values used for x were different. When we evaluate an expression, the value varies depending on the value used for the variable.

Exercise 1

Evaluate $y + 4$ when:

a. $y = 6$

b. $y = 15$

Exercise 1 Answers

a. 10

b. 19

Exercise 2

Evaluate $a - 5$ when:

a. $a = 9$

b. $a = 17$

Exercise 2 Answers

a. 4

b. 12

Example B

Evaluate $9x - 2$ when:

a. $x = 5$

b. $x = 1$

Remember ab means a times b , so $9x$ means 9 times x .

a. $x = 5$

To evaluate the expression when $x = 5$, we substitute 5 for x , and then simplify.

$$9x - 2$$

Substitute 5 for x : $9 \cdot 5 - 2$

Multiply: $45 - 2$

Subtract: 43

b. $x = 1$

To evaluate the expression when $x = 1$, we substitute 1 for x , and then simplify.

$$9x - 2$$

Substitute 1 for x : $9(1) - 2$

Multiply: $9 - 2$

Subtract: 7

Notice that in part a), we wrote $9 \cdot 5$, and in part b), we wrote $9(1)$. Both the dot and the parentheses tell us to multiply.

Exercise 3

Evaluate $8x - 3$ when:

a. $x = 2$

b. $x = 1$

Exercise 3 Answers

a. 13

b. 5

Exercise 4

Evaluate $4y - 4$ when:

a. $y = 3$

b. $y = 5$

Exercise 4 Answers

a. 8

b. 16

Example C

Evaluate x^2 when $x = 10$.

We substitute 10 for x , and then simplify the expression.

$$x^2$$

$$\text{Substitute 10 for } x : \quad 10^2$$

$$\text{Use the definition of exponent: } 10 \cdot 10$$

$$\text{Multiply: } 100$$

When $x = 10$, the expression x^2 has a value of 100.

Exercise 5

Evaluate x^2 when $x = 8$.

Exercise 5 Answer

64

Exercise 6

Evaluate x^3 when $x = 6$.

Exercise 6 Answer

216

Example D

Evaluate 2^x when $x = 5$.

In this expression, the variable is an exponent.

$$2^x$$

Substitute 5 for x : 2^5

Use the definition of exponent: $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

Multiply: 32

When $x = 5$, the expression 2^x has a value of 32.

Exercise 7

Evaluate 2^x when $x = 6$.

Exercise 7 Answer

64

Exercise 8

Evaluate 3^x when $x = 4$.

Exercise 8 Answer

81

Example E

Evaluate $3x + 4y - 6$ when $x = 10$ and $y = 2$.

This expression contains two variables, so we must make two substitutions.

$$3x + 4y - 6$$

Substitute 10 for x and 2 for y : $3(10) + 4(2) - 6$

$$\text{Multiply: } 30 + 8 - 6$$

$$\text{Add and subtract left to right: } 32$$

When $x = 10$ and $y = 2$, the expression $3x + 4y - 6$ has a value of 32.

Exercise 9

Evaluate $2x + 5y - 4$ when $x = 11$ and $y = 3$.

Exercise 9 Answer

33

Exercise 10

Evaluate $5x - 2y - 9$ when $x = 7$ and $y = 8$.

Exercise 10 Answer

10

Example F

Evaluate $2x^2 + 3x + 8$ when $x = 4$.

We need to be careful when an expression has a variable with an exponent. In this expression, $2x^2$ means $2 \cdot x \cdot x$ and is different from the expression $(2x)^2$, which means $2x \cdot 2x$.

$$2x^2 + 3x + 8$$

$$\text{Substitute 4 for } x : 2(4)^2 + 3(4) + 8$$

$$\text{Simplify } 4^2 : 2(16) + 3(4) + 8$$

$$\text{Multiply: } 32 + 12 + 8$$

$$\text{Add: } 52$$

Exercise 11

Evaluate $3x^2 + 4x + 1$ when $x = 3$.

Exercise 11 Answer

40

Exercise 12

Evaluate $6x^2 - 4x - 7$ when $x = 2$.

Exercise 12 Answer

9

Identify Terms, Coefficients, & Like Terms

Algebraic expressions are made up of **terms**. A term is a constant or the product of a constant and one or more variables. Some examples of terms are 7, y , $5x^2$, $9a$, and $13xy$.

The constant that multiplies the variable(s) in a term is called the coefficient. We can think of the coefficient as the number *in front of* the variable. The coefficient of the term $3x$ is 3. When we write x , the coefficient is 1 since $x = 1 \cdot x$. Table 8.2.1 gives the coefficients for each of the terms in the left column.

Table 8.2.1: Identifying Coefficients of Terms

Term	Coefficient
7	7
$9a$	9
y	1
$5x^2$	5

An algebraic expression may consist of one or more terms added or subtracted. In this chapter, we will only work with terms that are added together. Table 8.2.2 gives some examples of algebraic expressions with various numbers of terms. Notice that we include the operation before a term with it.

Table 8.2.2: Identifying Terms in an Expression

Expression	Terms
7	7
y	y
$x + 7$	$x, 7$
$2x + 7y + 4$	$2x, 7y, 4$
$3x^2 + 4x^2 + 5y + 3$	$3x^2, 4x^2, 5y, 3$

Example G

Identify each term in the expression $9b + 15x^2 + a + 6$. Then, identify the coefficient of each term.

The expression has four terms. They are $9b$, $15x^2$, a , and 6 .

- The coefficient of $9b$ is 9.
- The coefficient of $15x^2$ is 15.
- Remember that if no number is written before a variable, the coefficient is 1. So, the coefficient of a is 1.
- The coefficient of a constant is the constant, so the coefficient of 6 is 6.

Exercise 13

Identify all terms in the given expression, and their coefficients: $4x + 3b + 2$.

Exercise 13 Answer

The terms are $4x$, $3b$, and 2 .
The coefficients are 4 , 3 , and 2 .

Exercise 14

Identify all terms in the given expression, and their coefficients: $9a + 13a^2 + a^3$.

Exercise 14 Answer

The terms are $9a$, $13a^2$, and a^3 .
The coefficients are 9 , 13 , and 1 .

Some terms share common traits. Look at the following terms.
Which ones seem to have traits in common?

$$5x, 7, n^2, 4, 3x, 9n^2$$

Which of these terms are like terms?

- The terms 7 and 4 are both constant terms.
- The terms $5x$ and $3x$ are both terms with x .
- The terms n^2 and $9n^2$ both have n^2 .

Terms are called like terms if they have the same variables and exponents. All constant terms are also like terms. So among the terms $5x$, 7 , n^2 , 4 , $3x$, $9n^2$:

- 7 and 4 are like terms.
- $5x$ and $3x$ are like terms.
- n^2 and $9n^2$ are like terms.

Like terms:

Terms that are either constants or have the same variables with the same exponents are like terms.

Example H

Identify the like terms:

- $y^3, 7x^2, 14, 23, 4y^3, 9x, 5x^2$
- $4x^2 + 2x + 5x^2 + 6x + 40x + 8xy$

a. $y^3, 7x^2, 14, 23, 4y^3, 9x, 5x^2$

Look at the variables and exponents. The expression contains y^3 , x^2 , x , and constants.

- The terms y^3 and $4y^3$ are like terms because they both have y^3 .
- The terms $7x^2$ and $5x^2$ are like terms because they both have x^2 .
- The terms 14 and 23 are like terms because they are both constants.
- The term $9x$ does not have any like terms in this list since no other terms have the variable x raised to the power of 1.

b. $4x^2 + 2x + 5x^2 + 6x + 40x + 8xy$

Look at the variables and exponents. The expression contains the terms $4x^2$, $2x$, $5x^2$, $6x$, $40x$, and $8xy$.

- The terms $4x^2$ and $5x^2$ are like terms because they both have x^2 .
- The terms $2x$, $6x$, and $40x$ are like terms because they all have x .
- The term $8xy$ has no like terms in the given expression because no other terms contain the two variables xy .

Exercise 15

Identify the like terms in the list or the expression:

$$9, 2x^3, y^2, 8x^3, 15, 9y, 11y^2$$

Exercise 15 Answers

9 and 15; $2x^3$ and $8x^3$; y^2 , and $11y^2$

Exercise 16

Identify the like terms in the list or the expression:

$$4x^3 + 8x^2 + 19 + 3x^2 + 24 + 6x^3$$

Exercise 16 Answers

$4x^3$ and $6x^3$; $8x^2$ and $3x^2$; 19 and 24

Simplify Expressions by Combining Like Terms

We can simplify an expression by combining the like terms. What do you think $3x + 6x$ would simplify to? If you thought $9x$, you would be right!

We can see why this works by writing both terms as addition problems.

$$\begin{array}{rcccl} 3x & & + & & 6x \\ x + x + x & + & x + x + x + x + x + x & & \\ 9x & & & & \end{array}$$

Add the coefficients and keep the same variable. It doesn't matter what x is. If you have 3 of something and add 6 more of the same thing, the result is 9 of them. For example, 3 oranges plus 6 oranges is 9 oranges. We will discuss the mathematical properties behind this later.

The expression $3x + 6x$ has only two terms. When an expression contains more terms, it may be helpful to rearrange the terms so that like terms are together.

The Commutative Property of Addition says that we can change the order of addends without changing the sum. So, we could rearrange the following expression before combining like terms.

Formula does not parse

Now, it is easier to see the like terms to be combined.

How to combine like terms:

1. Identify like terms.
2. Rearrange the expression so the like terms are together.
3. Add the coefficients of the like terms.

Example I

Simplify the expression: $3x + 7 + 4x + 5$.

Step 1: Identify the like terms.

Formula does not parse

Step 2: Rearrange the expression so the like terms are together.

Formula does not parse

Step 3: Add the coefficients of the like terms.

Formula does not parse

The original expression is simplified to $7x + 12$.

Exercise 17

Simplify: $7x + 9 + 9x + 8$.

Exercise 17 Answer

$$16x + 17$$

Exercise 18

Simplify: $5y + 2 + 8y + 4y + 5$.

Exercise 18 Answer

$$17y + 7$$

Example J

Simplify the expression: $7x^2 + 8x + x^2 + 4x$.

Step 1: Identify the like terms.

Formula does not parse

Step 2: Rearrange the expression so like terms are together.

Formula does not parse

Step 3: Add the coefficients of the like terms.

Formula does not parse

These are not like terms and cannot be combined. So $8x^2 + 12x$ is in simplest form.

Exercise 19

Simplify: $3x^2 + 9x + x^2 + 5x$

Exercise 19 Answer

$$4x^2 + 14x$$

Exercise 20

Simplify: $11y^2 + 8y + y^2 + 7y$.

Exercise 20 Answer

$$13y^2 + 15y$$

Translate Words to Algebraic Expressions

In the previous section, we listed many operation symbols used in algebra, and then we translated expressions and equations into word phrases and sentences. Now, we'll reverse the process and translate word phrases into algebraic expressions. The symbols and variables we've talked about will help us do that. Table 8.2.3 summarizes them.

Table 8.2.3: Translating Words to Algebraic Expressions

Operation	Phrase	Expression
Addition	<i>a</i> plus <i>b</i> The sum of <i>a</i> and <i>b</i> <i>a</i> increased by <i>b</i> <i>b</i> more than <i>a</i> The total of <i>a</i> and <i>b</i> <i>b</i> added to <i>a</i>	$a + b$
Subtraction	<i>a</i> minus <i>b</i> The difference of <i>a</i> and <i>b</i> <i>b</i> subtracted from <i>a</i> <i>a</i> decreased by <i>b</i> <i>b</i> less than <i>a</i>	$a - b$
Multiplication	<i>a</i> times <i>b</i> The product of <i>a</i> and <i>b</i>	$a \cdot b, ab, a(b), (a)(b)$
Division	<i>a</i> divided by <i>b</i> The quotient of <i>a</i> and <i>b</i> The ratio of <i>a</i> and <i>b</i> <i>b</i> divided into <i>a</i>	$a \div b, a/b, \frac{a}{b}, \overline{b)a}$

Look closely at these phrases using the four operations:

- The sum of *a* and *b*
- The difference of *a* and *b*
- The product of *a* and *b*
- The quotient of *a* and *b*

Each phrase tells you to operate on two numbers. Look for the words **of** and **and** to find the numbers.

Example L

Translate each word phrase into an algebraic expression:

- a. The difference of 20 and 4
- b. The quotient of $10x$ and 3

a. The difference of 20 and 4

The key word is *difference*, which tells us the operation is subtraction. Look for the words of *of* and *and* to find the numbers to subtract.

the difference of 20 and 4

20 minus 4

$$20 - 4$$

b. The quotient of $10x$ and 3

The key word is *quotient*, which tells us the operation is division.

the quotient of $10x$ and 3

divide $10x$ by 3

$$10x \div 3$$

This can also be written as $10x/3$ or $\frac{10x}{3}$.

Exercise 21

Translate the given word phrase into an algebraic expression.

- a. The difference of 47 and 41 b. The quotient of $5x$ and 2

Exercise 21 Answers

- a. $47 - 41$ b. $5x \div 2$

Exercise 22

Translate the given word phrase into an algebraic expression.

- a. The sum of 17 and 19 b. The product of 7 and x

Exercise 22 Answers

- a. $17 + 19$ b. $7x$

How old will you be in eight years? What age is eight more years than your age now? Did you add 8 to your present age? Eight *more than* means eight added to your present age.

How old were you seven years ago? This is seven years less than your age now. You subtract 7 from your present age. Seven *less than* means seven subtracted from your present age.

Example M

Translate each word phrase into an algebraic expression:

- a. Eight more than y
- b. Seven less than $9z$

a. Eight more than y

The keywords are *more than*. They tell us the operation is addition. *More than* means “added to”.

Eight more than y

Eight added to y

$$y + 8$$

b. Seven less than $9z$

The keywords are *less than*. They tell us the operation is subtraction. *Less than* means “subtracted from”.

Seven less than $9z$

Seven subtracted from $9z$

$$9z - 7$$

Exercise 23

Translate each word phrase into an algebraic expression.

- a. Eleven more than x b. Fourteen less than $11a$

Exercise 23 Answers

- a. $x + 11$ b. $11a - 14$

Exercise 24

Translate each word phrase into an algebraic expression.

- a. 19 more than j b. 21 less than $2x$

Exercise 24 Answers

a. $j + 19$

b. $2x - 21$

Example N

Translate each word phrase into an algebraic expression:

- a. Five times the sum of m and n
- b. The sum of five times m and n

a. Five times the sum of m and n

There are two operation words: *times* tells us to multiply, and *sum* tells us to add. Because we are multiplying 5 times the sum, we need parentheses around the sum of m and n .

$$m + n$$

b. The sum of five times m and n

To take a sum, we look for the words *of* and *and* to see

what is being added. Here, we are taking the sum of five times m and n .

$$5m + n$$

Notice how the use of parentheses changes the result. In part a), we add first and in part b), we multiply first.

Exercise 25

Translate the word phrase into an algebraic expression.

- a. Four times the sum of p and q b. The sum of four times p and q

Exercise 25 Answers

a. $4(p + q)$

b. $4p + q$

Exercise 26

Translate the word phrase into an algebraic expression.

- a. The difference of two times x and 8 b. Two times the difference of x and 8

Exercise 26 Answers

- a. $2x - 8$ b. $2(x - 8)$

Later in this course, we'll apply our skills in algebra to solving equations. We'll usually start by translating a word phrase into an algebraic expression. We'll need to be clear about what the expression will represent. We'll see how to do this in the next two examples.

Example O

The height of a rectangular window is 6 inches less than the width. Let w represent the width of the window. Write an expression for the height of the window.

Step 1: Write a phrase about the height.

6 less than the width

Step 2: Substitute w for the width.

6 less than w

Step 3: Rewrite 'less than' as 'subtracted from.'

6 subtracted from w

Step 4: Translate the phrase into algebra.

$$w - 6$$

Exercise 27

The length of a rectangle is 5 inches less than the width.

Let w represent the width of the rectangle. Write an expression for the length of the rectangle.

Exercise 27 Answer

$$w - 5$$

Exercise 28

The width of a rectangle is 2 metres greater than the length. Let l represent the length of the rectangle. Write an expression for the width of the rectangle.

Exercise 27 Answer

$$l + 2$$

Example P

Blanca has dimes and quarters in her purse. The number of dimes is 2 less than 5 times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

Step 1: Write a phrase about the number of dimes.

Two less than five times the number of quarters

Step 2: Substitute q for the number of quarters.

2 less than five times q

Step 3: Translate 5 times q .

2 less than $5q$

Step 4: Translate the phrase into algebra.

$$5q - 2$$

Exercise 29

Geoffrey has dimes and quarters in his pocket. The number of dimes is seven less than six times the number of quarters. Let q represent the number of quarters. Write an expression for the number of dimes.

Exercise 29 Answer

$$6q - 7$$

Exercise 30

Lauren has dimes and nickels in her purse. The number of dimes is eight more than four times the number of nickels. Let n represent the number of nickels. Write an expression for the number of dimes.

Exercise 30 Answer

$$4n + 8$$

Additional online resources:

- **Algebraic Expression Vocabulary** – Algebraic Expression Vocabulary (L2.2) [5:52 min] (Mathispower4u, 2014).

Key Concepts

Combine like terms

1. Identify like terms.
2. Rearrange the expression so like terms are together.
3. Add the coefficients of the like terms.

Glossary

- **Term** — A term is a constant or the product of a constant and one or more variables.
- **Coefficient** — The constant that multiplies the variable(s) in a term is called the coefficient.
- **Like terms** — Terms that are either constants or have the same variables with the same exponents are like terms.
- **Evaluate** — To evaluate an algebraic expression means to find the value of the expression when the variable is replaced by a given number.

8.2: Practice Questions

1. Evaluate Algebraic Expressions

In the following exercises, evaluate the expression for the given value.

a. $7x + 8$ when $x = 2$	l. $x^2 + 5x - 8$ when $x = 6$
b. $9x + 7$ when $x = 3$	m. $2x + 4y - 5$ when $x = 7, y = 8$
c. $5x - 4$ when $x = 6$	n. $6x + 3y - 9$ when $x = 6, y = 9$
d. $8x - 6$ when $x = 7$	o. $x - y^2$ when $x = 10, y = 7$
e. x^2 when $x = 12$	p. $x + y^2$ when $x = 6, y = 9$
f. $6x^3$ when $x = 5$	q. $a^2 + b^2$ when
g. x^5 when $x = 2$	
h. x^4 when $x = 3$	
i. 3^x when $x = 3$	
j. 4^x when $x = 2$	
k. $x^2 + 3x - 7$ when $x = 4$	

	$a = 3, b = 8$		$l = 15, w = 12$	
r.	$r^2 - s^2$	when	$2l + 2w$	when
	$r = 12, s = 5$		$l = 18, w = 14$	
s.	$2l + 2w$	when		

2. Identify Terms, Coefficients, and Like Terms

In the following exercises, list the terms in the given expression.

a.	$15x^2 + 6x + 2$	c.	$10y^3 + y + 2$
b.	$11x^2 + 8x + 5$	d.	$9y^3 + y + 5$

In the following exercises, identify the coefficient of the given term.

e.	$8a$	g.	$5r^2$
f.	$13m$	h.	$6x^3$

In the following exercises, identify all sets of like terms.

i.	$x^3, 8x, 14, 8y, 5, 8x^3$	k.	$9a, a^2, 16ab, 16b^2, 4ab, 9b^2$
j.	$6z, 3w^2, 1, 6z^2, 4z, w^2$	l.	$3, 25r^2, 10s, 10r, 4r^2, 3s$

3. Simplify Expressions by Combining Like Terms

In the following exercises, simplify the given expression by combining like terms.

a.	$10x + 3x$	h.	$8a + 5a + 9$
b.	$15x + 4x$	i.	$7u + 2 + 3u + 1$
c.	$17a + 9a$	j.	$8d + 6 + 2d + 5$
d.	$18z + 9z$	k.	$7p + 6 + 5p + 4$
e.	$4c + 2c + c$	l.	$8x + 7 + 4x - 5$
f.	$6y + 4y + y$	m.	$10a + 7 + 5a - 2 + 7a - 4$
g.	$9x + 3x + 8$	n.	$7c + 4 + 6c - 3 + 9c - 1$

o. $3x^2 + 12x + 11 + 14x^2 + 8x + 5$. $5b^2 + 9b + 10 + 2b^2 + 3b - 4$

4. Translate English Phrases into Algebraic Expressions

In the following exercises, translate the given word phrase into an algebraic expression.

a. The sum of 8 and 12	l. The product of 9 and y
b. The sum of 9 and 1	m. The sum of $8x$ and $3x$
c. The difference of 14 and 9	n. The sum of $13x$ and $3x$
d. 8 less than 19	o. The quotient of y and 3
e. The product of 9 and 7	p. The quotient of y and 8
f. The product of 8 and 7	q. Eight times the difference of y and nine
g. The quotient of 36 and 9	r. Seven times the difference of y and one
h. The quotient of 42 and 7	s. Five times the sum of x and y
i. The difference of x and 4	t. Times five less than twice x
j. 3 less than x	
k. The product of 6 and y	

In the following exercises, write an algebraic expression.

- u. Adele bought a skirt and a blouse. The skirt cost \$15 more than the blouse. Let b represent the cost of the blouse. Write an expression for the cost of the skirt.
- v. Eric has rock and classical CDs in his car. The number of rock CDs is 3 more than the number of classical CDs. Let c represent the number of classical CDs. Write an expression for the number of rock CDs.
- w. The number of girls in a second-grade class is 4 less than the number of boys. Let b represent the number of boys. Write an expression for the number of girls.
- x. Marcella has 6 fewer male cousins than female cousins. Let f represent the number of female cousins. Write an expression for the number of boy cousins.
- y. Marcella has 6 fewer male cousins than female cousins. Let f represent the number of female cousins. Write an expression for the number of boy cousins.

- z. Jeannette has \$5 and \$10 bills in her wallet. The number of fives is three more than six times the number of tens. Let t represent the number of tens. Write an expression for the number of fives.

5. Everyday Math

In the following exercises, use algebraic expressions to solve the problem.

- a. **Car insurance:** Justin's car insurance has a \$750 deductible per incident. This means that he pays \$750, and his insurance company will pay all costs beyond \$750. If Justin files a claim for \$2,100, how much will he pay, and how much will his insurance company pay?
- b. **Home insurance:** Pam and Armando's home insurance has a \$2,500 deductible per incident. This means that they pay \$2,500, and their insurance company will pay all costs beyond \$2,500. If Pam and Armando file a claim for \$19,400, how much will they pay, and how much will their insurance company pay?

6. Writing Exercises

- a. Explain why "the sum of x and y " is the same as "the sum of y and x ," but "the difference of x and y " is not the same as "the difference of y and x ." Try substituting two random numbers for x and y to help you explain.
- b. Explain the difference between "4 times the sum of x and y " and "the sum of 4 times x and y ."

8.2: Practice Answers

1. Evaluate algebraic expressions.

a. 22	i. 27	q. 73
b.	j.	r.
c. 26	k. 21	s. 54
d.	l.	
e. 144	m. 41	
f.	n.	
g. 32	o. 9	
h.	p.	

2. Identify terms, coefficients, and like terms.

a. $15x^2$, $6x$, 2	e. 8	i. x^3 , $8x^3$ and 14, 5
b.	f.	j.
c. $10y^3$, y , 2	g. 5	k. $16ab$ and $4ab$; $16b^2$ and $9b^2$
d.	h.	

3. Simplify expressions by combining like terms.

a. $13x$	g. $12x + 8$	m. $22a + 1$
b.	h.	n.
c. $26a$	i. $10u + 3$	o. $17x^2 + 20x + 16$
d.	j.	
e. $7c$	k. $12p + 10$	
f.	l.	

4. Translate English phrases into algebraic expressions.

a. $8 + 12$	k. $6y$	s. $5(x + y)$
b.	l.	t.
c. $14 - 9$	m. $8x + 3x$	u. $b + 15$
d.	n.	v.
e. $9 \cdot 7$	o. $\frac{y}{3}$	w. $b - 4$
f.		x.
g. $36 \div 9$		y. $2n - 7$
h.	p.	
i. $x - 4$	q. $8(y - 9)$	
j.	r.	

5. Everyday math.

- a. He will pay \$750. His insurance company will pay \$1350.

6. Writing exercises.
 - a. Answers will vary.

References

Mathispower4u. (2014, August 10). *Algebraic expression vocabulary (L2.2)* [Video]. YouTube. https://youtu.be/E6lK-by__rA?si=YMB1kgHMXUfRy4Zl.

Attributions

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8.3: Solve Linear Equations

Verify a Solution of an Equation

Solving an equation is like discovering the answer to a puzzle. The purpose of solving an equation is to find the value or values of the variable that make each side of the equation the same — so that we end up with a true statement. Any value of the variable that makes the equation true is called a solution to the equation. It is the answer to the puzzle!

Solution of an equation:

A **solution of an equation** is a value of a variable that makes a true statement when substituted into the equation.

How to determine whether a number is a solution to an equation:

1. Substitute the number in for the variable in the equation.
2. Simplify the expressions on both sides of the equation.
3. Determine whether the resulting equation is

true (the left side is equal to the right side).

- If it is true, the number is a solution.
- If it is not true, the number is not a solution.

Example A

Determine whether $x = -3$ is a solution of $4x + 5 = -7$.

Since a solution to an equation is a value of the variable that makes the equation true, begin by substituting the value of the solution for the variable.

Step 1: Substitute $x = -3$ for x .

$$4(-3) + 5 = -7$$

Step 2: Multiply.

$$-12 + 5 = -7$$

Step 3: Simplify.

$$-7 = -7$$

Since $x = -3$ results in a true statement (-7 is in fact

equal to -7), -3 is a solution to the equation
 $4x + 5 = -7$.

Exercise 1

Is $y = 2$ a solution of $9y - 2 = 6$?

Exercise 1 Answer

No

There are many types of equations. In this section, we will focus on solving linear equations.

Linear equation:

A **linear equation** is a first-degree equation in one variable that can be written as:

$$ax + b = 0, \text{ where } a \text{ and } b \text{ are real numbers and } a \neq 0.$$

Solve Equations Using the Subtraction & Addition Properties of Equality

Let us review all the properties that will help us to solve equations algebraically. The first one is the **Subtraction Property of Equality**.

Subtraction Property of Equality:

For any numbers a , b , and c :

$$\begin{array}{lcl} \text{If} & a & = b, \\ \text{then} & a - c & = b - c \end{array}$$

When you subtract the same quantity from both sides of an equation, you still have equality.

Let's see how to use this property to solve an equation. Remember, the goal is to isolate the variable on one side of the equation. We check our solutions by substituting the value into the equation to make sure we have a true statement.

Example B

Solve: $y + 37 = -13$

To get y by itself, we will undo the addition of 37 by using the Subtraction Property of Equality.

Step 1: Subtract 37 from each side to 'undo' the addition.

Formula does not parse

Step 2: Simplify.

$$y = -50$$

Check: Substitute $y = -50$.

Formula does not parse

Since $y = -50$ makes $y + 37 = -13$ a true statement, we have the solution to this equation.

Exercise 2

Solve: $x + 19 = -27$

Exercise 2 Answer

$$x = -46$$

What happens when an equation has a number subtracted from the variable, as in the equation $x - 5 = 8$? We use another property to solve equations when a number is subtracted from the variable. We want to isolate the variable, so to ‘undo’ the subtraction, we will add the number to both sides. We use the **Addition Property of Equality**.

Addition Property of Equality:

For any numbers a , b , and c :

$$\begin{array}{lcl} \text{If} & a & = b, \\ \text{then} & a + c & = b + c \end{array}$$

When you add the same quantity to both sides of an equation, you still have equality.

In Example B, 37 was added to the y, so we subtracted 37 to ‘undo’ the addition. In Example C, we will need to ‘undo’ subtraction by using the Addition Property of Equality.

Example C

Solve: $a - 28 = -37$

Step 1: Add 28 to each side to ‘undo’ the subtraction.

Formula does not parse

Step 2: Simplify.

$$a = -9$$

Check: Substitute $a = -9$.

Formula does not parse

The solution to $a - 28 = -37$ is $a = -9$.

Exercise 3

Solve: $n - 61 = -75$

Exercise 3 Answer

$$n = -14$$

Example D

Solve: $x - \frac{5}{8} = \frac{3}{4}$

Step 1: Use the Addition Property of Equality.

Formula does not parse

Step 2: Find the LCD to add the fractions on the right.

$$x - \frac{5}{8} + \frac{5}{8} = \frac{6}{8} + \frac{5}{8}$$

Step 3: Simplify.

$$x = \frac{11}{8}$$

Check:

Formula does not parse

The solution to $x - \frac{5}{8} = \frac{3}{4}$ is $x = \frac{11}{8}$.

Exercise 4

Solve: $p - \frac{2}{3} = \frac{5}{6}$

Exercise 4 Answer

$$p = \frac{9}{6} \text{ or } p = \frac{3}{2}$$

The next example will be an equation with decimals.

Example E

Solve: $n - 0.63 = -4.2$

Step 1: Use the Addition Property of Equality.

Formula does not parse

Step 2: Add.

$$n = -3.57$$

Check:

Formula does not parse

Exercise 5

Solve: $b - 0.47 = -2.1$

Exercise 5 Answer

$$b = -1.63$$

Solve Equations Using the Division & Multiplication Properties of Equality

You may have noticed that all of the equations we have solved so far have been of the form $x + a = b$ or $x - a = b$. We were able to isolate the variable by adding or subtracting the constant term on the side of the equation with the variable.

Now, we will see how to solve equations that have a variable multiplied by a constant, which will require division to isolate the variable. To solve those kinds of equations, we will use the **Division Property of Equality**.

Division Property of Equality:

For any numbers a , b , and c , and $c \neq 0$:

$$\begin{array}{lcl} \text{If} & a & = b, \\ \text{then} & \frac{a}{c} & = \frac{b}{c} \end{array}$$

When you divide both sides of an equation by any non-zero number, you still have equality.

The goal in solving an equation is to ‘undo’ the operation on the variable. In the next example, the variable is multiplied by 5, so we will divide both sides by 5 to ‘undo’ the multiplication.

Example F

Solve: $5x = -27$

Step 1: To isolate x , “undo” the multiplication by dividing both sides by 5.

Formula does not parse

Step 2: Simplify.

$$x = -\frac{27}{5}$$

Check:

Formula does not parse

Exercise 6

Solve: $3y = -41$

Exercise 6 Answer

$$y = \frac{-41}{3}$$

Consider the equation $\frac{x}{4} = 3$. We want to know what number divided by 4 gives 3. So, to “undo” the division, we will need to multiply by 4.

The **Multiplication Property of Equality** will allow us to do this. This property says that if we start with two equal quantities and multiply both by the same number, the results are equal.

Multiplication Property of Equality:

For any numbers a , b , and c ,

$$\begin{array}{lcl} \text{If} & a & = b, \\ \text{then} & ac & = bc \end{array}$$

If you multiply both sides of an equation by the same number, you still have equality.

Example G

Solve: $\frac{y}{-7} = -14$

Here y is divided by -7 . We must multiply by -7 to isolate y .

Step 1: Multiply both sides by -7 .

$$-7\left(\frac{y}{-7}\right) = -7(-14)$$

Step 2: Multiply.

$$\frac{-7y}{7} = 98$$

Step 3: Simplify.

$$y = 98$$

Check:

Formula does not parse

Exercise 7

Solve: $\frac{a}{-7} = -42$

Exercise 7 Answer

$$a = 294$$

Example H

Solve: $\frac{3}{4}x = 12$

Since the product of a number and its reciprocal is 1, our strategy will be to isolate x by multiplying by the reciprocal of $\frac{3}{4}$.

Step 1: Multiply by the reciprocal of $\frac{3}{4}$.

Formula does not parse

Step 2: Reciprocals multiply to 1.

$$1x = \frac{4}{3} \cdot \frac{12}{1}$$

Step 3: Multiply.

$$x = 16$$

Notice that we could have divided both sides of the equation $\frac{3}{4}x = 12$ by $\frac{3}{4}$ to isolate x . While this would work, most people would find multiplying by the reciprocal easier.

Check:

Formula does not parse

Exercise 8

Solve: $\frac{2}{5}n = 14$

Exercise 8 Answer

$$n = 35$$

Now, we have covered all four properties of equality—subtraction, addition, division, and multiplication. We'll list them all together here for easy reference.

Properties of equality:

Subtraction Property of Equality

For any real numbers a , b , and c ,

$$\begin{array}{l} \text{if } a = b, \\ \text{then } a - c = b - c. \end{array}$$

Division Property of Equality

For any numbers a , b , and c , and $c \neq 0$,

$$\begin{array}{l} \text{if } a = b, \\ \text{then } \frac{a}{c} = \frac{b}{c}. \end{array}$$

Addition Property of Equality

For any real numbers a , b , and c ,

$$\begin{array}{l} \text{if } a = b, \\ \text{then } a + c = b + c. \end{array}$$

Multiplication Property of Equality

For any numbers a , b , and c ,

$$\begin{array}{l} \text{if } a = b, \\ \text{then } ac = bc. \end{array}$$

When you add, subtract, multiply, or divide the same quantity from both sides of an equation, you still have equality.

Now, we will use those properties to solve equations in which the variable terms, constant terms, or both are on both sides of the equation.

Solve Equations With Variables & Constants on Both Sides

Our strategy will involve choosing one side of the equation to be the “variable side” and the other side of the equation to be the “constant side.” Then, we will use the Subtraction and Addition Properties of Equality to get all the variable terms together on one side of the equation and the constant terms together on the other side.

By doing this, we will transform the equation that began with variables and constants on both sides into the form $ax = b$. We already know how to solve equations of this form by using the Division or Multiplication Properties of Equality.

Example I

Solve: $8y - 9 = 31$

In this equation, the variable is found only on the left side. It makes sense to call the left side the “variable” side. Therefore, the right side will be the “constant” side. We will write the labels above the equation to help us remember what goes where.

Since the left side is the “variable” side, the 9 is out of place. It is subtracted from the $8y$, so to “undo” subtraction, add 9 to both sides. Remember, whatever you do to the left, you must do to the right.

Formula does not parse

Step 1: Add 9 to both sides.

Formula does not parse

Step 2: Simplify.

$$8y = 40$$

The variables are now on one side and the constants are on the other. We continue from here as we did earlier.

Step 3: Divide both sides by 8.

Formula does not parse

Step 4: Simplify.

$$y = 5$$

Check:

Formula does not parse

Exercise 9

Solve: $5y - 9 = 16$

Exercise 9 Answer

$$y = 5$$

What if there are variables on both sides of the equation? For equations like this, begin as we did above — choose a “variable” side and a “constant” side, and then use the subtraction and addition properties of equality to collect all variables on one side and all constants on the other side.

Example J

Solve: $5y - 9 = 8y$.

The only constant is on the left, and the y's are on both sides. Let's leave the constant on the left and get the variables to the right.

Formula does not parse

Step 1: Subtract 5y from both sides.

Formula does not parse

Step 2: Simplify.

$$-9 = 3y$$

Step 3: We have the y's on the right and the constants on the left. Divide both sides by 3.

Formula does not parse

Step 4: Simplify.

$$-3 = y$$

Check:

Formula does not parse

Exercise 10

Solve: $3p - 14 = 5p$

Exercise 10 Answer

$$p = -7$$

The next example will be the first to have variables and constants on both sides of the equation. It may take several steps to solve this equation, so we need a clear and organized strategy.

Example K

Solve: $7x + 5 = 6x + 2$.

Step 1: Choose which side will be the “variable” side — the other side will be the “constant” side.

The variable terms are $7x$ and $6x$. Since 7 is greater than 6, we will make the left side the “x” side. The right side will be the “constant side.”

Formula does not parse

Step 2: Collect the variable terms to the “variable” side of the equation using the addition or subtraction property of equality.

With the right side as the “constant” side, the $6x$ is out of place, so subtract $6x$ from both sides.

Formula does not parse

Step 3: Combine like terms.

$$x + 5 = 2$$

Now, the variable is only on the left side!

Step 4: Collect all the constants on the other side of the equation, using the Addition or Subtraction Property of Equality.

The right side is the “constant” side, so the 5 is out of place. Subtract 5 from both sides.

Formula does not parse

Step 5: Simplify.

$$x = -3$$

Step 6: Make the coefficient of the variable equal to 1, using the multiplication or division property of equality.

The coefficient of x is one. The equation is solved.

Check:

Formula does not parse

Exercise 11

Solve: $12x + 8 = 6x + 2$

Exercise 11 Answer

$$x = -1$$

We'll list the steps below so you can easily refer to them. But we'll call this the 'Beginning Strategy' because we'll be adding some steps later in this chapter.

Equations with variables and constants on both sides of the equation (Beginning strategy):

1. Choose which side will be the “variable” side — the other side will be the “constant” side.
2. Collect the variable terms to the “variable” side of the equation using the Addition or Subtraction Property of Equality.
3. Collect all the constants to the other side of the equation using the Addition or Subtraction

Property of Equality.

4. Make the coefficient of the variable equal 1, using the Multiplication or Division Property of Equality.
5. Check the solution by substituting it into the original equation.

In Step 1, a helpful approach is to make the “variable” side the side that has the variable with the larger coefficient. This usually makes the arithmetic easier.

Example L

Solve: $7a - 3 = 13a + 7$

Step 1: Choose the variable side by comparing the coefficients of the variables on each side.

Since $13 > 7$, make the right side the “variable” side and the left side the “constant” side.

Formula does not parse

Step 2: Subtract $7a$ from both sides to remove the variable term from the left.

Formula does not parse

Step 3: Combine like terms.

$$-3 = 6a + 7$$

Step 4: Subtract 7 from both sides to remove the constant from the right.

Formula does not parse

Step 5: Simplify.

$$-10 = 6a$$

Step 6: Divide both sides by 6 to make 1 the coefficient of a .

Formula does not parse

Step 7: Simplify.

$$-\frac{5}{3} = a$$

Check:

Formula does not parse

Exercise 12

Solve: $2a - 2 = 6a + 18$

Exercise 12 Answer

$$a = -5$$

In the last example, we could have made the left side the “variable” side, but it would have led to a negative coefficient on the variable term. (Try it!)

While we could work with the negative, there is less chance of errors when working with positives. The strategy outlined above helps avoid the negatives!

To solve an equation with fractions, we just follow the steps of our strategy to get the solution!

Example M

Solve: $\frac{5}{4}x + 6 = \frac{1}{4}x - 2$

Since $\frac{5}{4} > \frac{1}{4}$, make the left side the “variable” side and the right side the “constant” side.

Formula does not parse

Step 1: Subtract $\frac{1}{4}x$ from both sides.

Formula does not parse

Step 2: Combine like terms.

$$x + 6 = -2$$

Step 3: Subtract 6 from both sides.

Formula does not parse

Step 4: Simplify.

$$x = -8$$

Check:

Formula does not parse

Exercise 13

Solve: $\frac{7}{8}x - 12 = -\frac{1}{8}x - 2$

Exercise 13 Answer

$$x = 10$$

We will use the same strategy to find the solution for an equation with decimals.

Example N

Solve: $7.8x + 4 = 5.4x - 8$

Since $7.8 > 5.4$, make the left side the “variable” side and the right side the “constant” side.

Formula does not parse

Step 1: Subtract $5.4x$ from both sides.

Formula does not parse

Step 2: Combine like terms.

$$2.4x + 4 = -8$$

Step 3: Subtract 4 from both sides.

Formula does not parse

Step 4: Simplify.

$$2.4x = -12$$

Step 5: Use the Division Property of Equality.

Formula does not parse

Step 6: Simplify.

$$x = -5$$

Check:

Formula does not parse

Exercise 14

Solve: $2.8x + 12 = -1.4x - 9$

Exercise 14 Answer

$$x = -5$$

Key Concepts

Determine Whether a Number Is a Solution to an Equation

1. Substitute the number in for the variable in the equation.
2. Simplify the expressions on both sides of the equation.
3. Determine whether the resulting statement is true.
 - If it is true, the number is a solution.
 - If it is not true, the number is not a solution.

The Addition Property of Equality

- For any numbers a , b , and c , if $a = b$, then $a + c = b + c$.

The Subtraction Property of Equality

- For any numbers a , b , and c , if $a = b$, then $a - c = b - c$.

The Division Property of Equality

- For any numbers a , b , and c , and $c \neq 0$, if $a = b$, then

$$\frac{a}{c} = \frac{b}{c}.$$

- When you divide both sides of an equation by any non-zero number, you still have equality.

The Multiplication Property of Equality

- For any numbers a , b , and c , if $a = b$, then $ac = bc$.

Glossary

- **Solution of an equation** — A value of a variable that makes a true statement when substituted into the equation.

8.3: Practice Questions

1. Determine whether the given value is a solution to the equation.

a. Is $y = -4$ a solution of $6y + 30 = 6$?

b. Is $u = 3$ a solution of $8u - 4 = 18$?

2. Solve each equation using the Subtraction and Addition Properties of Equality.

a. $x + 24 = 35$ b. $b + \frac{1}{4} = \frac{3}{4}$ c. $a - 45 = 76$
 d. $y + 45 = -66$ e. $m - 18 = -200$
 f. $p + 2.4 = -9.3$

$$\begin{array}{lll} \text{g. } x - \frac{1}{3} = 2 & \text{j. } z + 0.52 = -8.5 & \text{l. } p - \frac{2}{5} = \frac{2}{3} \\ \text{h. } y - 3.8 = 10 & \text{k. } q + \frac{3}{4} = \frac{1}{2} & \\ \text{i. } x - 165 = -420 & & \end{array}$$

3. Solve each equation using the Division and Multiplication Properties of Equality and check the solution.

$$\begin{array}{lll} \text{a. } 8x = 56 & \text{i. } \frac{y}{9} = -16 & \text{o. } -\frac{2}{5} = \frac{1}{10}a \\ \text{b. } -5c = 55 & & \\ \text{c. } -809 = 15y & \text{j. } \frac{m}{-12} = 45 & \text{p. } -\frac{7}{10}x = -\frac{14}{3} \\ \text{d. } -37p = -54 & & \\ \text{e. } 0.25z = 3.25 & \text{k. } -y = 6 & \text{q. } \frac{7}{12} = -\frac{3}{4}p \\ \text{f. } -13x = 0 & \text{l. } -v = -72 & \\ \text{g. } \frac{x}{4} = 35 & \text{m. } \frac{2}{3}y = 48 & \text{r. } -\frac{5}{18} = -\frac{10}{9}u \\ \text{h. } -20 = \frac{q}{-5} & \text{n. } -\frac{5}{8}w = 40 & \end{array}$$

4. Solve the following equations with constants on both sides.

$$\begin{array}{lll} \text{a. } 21k = 20k - 4 & 21 + 18f = 19f + 14 & 11 - \frac{1}{5}a = \frac{4}{5}a + 4 \\ \text{b. } 8x + 27 = 11x & 12q - 5 = 9q - 20 & \\ \text{c. } 5z = 39 - 8z & 8c + 7 = -3c - 37 & \frac{5}{4}a + 15 = \frac{3}{4}a - 5 \\ & 7x - 17 = -8x + 13 & \\ \text{d. } 4x + \frac{3}{4} = 3x & 9p + 14 = 6 + 4p & \frac{3}{5}p + 2 = \frac{4}{5}p - 1 \\ & 3y - 4 = 12 - y & \\ \text{e. } -11r - 8 = -7r & \frac{7}{4}m - 7 = \frac{3}{4}m - 13 & 13z + 6.45 = 8z + 23.75 \\ \text{f. } 6x - 17 = 5x + 2 & & 6.6x - 18.9 = 3.4x + 54.7 \end{array}$$

8.3: Practice Answers

1. Determine whether the given value is a solution to the

equation.

a. Yes

b. No

2. Solve each equation using the Subtraction and Addition Properties of Equality.

a. $x = 11$	f. $m = -182$	k. $q = -\frac{1}{4}$
b. $y = -111$	g. $x = \frac{7}{3}$	
c. $b = \frac{1}{2}$	h. $y = 13.8$	l. $p = \frac{16}{15}$
d. $p = -11.7$	i. $x = -255$	
e. $a = 121$	j. $z = -9.02$	

3. Solve each equation using the Division and Multiplication Properties of Equality and check the solution.

a. $x = 7$	g. $x = 140$	o. $a = -4$
b. $c = -11$	h. $q = 100$	p. $x = \frac{20}{3}$
c. $y = -\frac{809}{15}$	i. $y = -144$	q. $p = -\frac{7}{9}$
d. $p = \frac{541}{37}$	j. $m = -540$	r. $u = \frac{1}{4}$
e. $z = 13$	k. $y = -6$	
f. $x = 0$	l. $v = 72$	
	m. $y = 72$	
	n. $w = -64$	

4. Solve the following equations with constants on both sides.

a. $k = -11$	g. $f = 7$	l. $y = 4$
b. $x = 9$	h. $q = -5$	m. $m = -6$
c. $z = 3$	i. $c = -4$	n. $a = 7$
d. $x = -\frac{3}{4}$	j. $x = 2$	o. $a = -40$
e. $r = -2$	k. $p = -\frac{8}{5}$	p. $p = 15$
f. $x = 19$		q. $z = 3.46$
		r. $x = 23$

Attributions

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